Contributions to the Study of Babylonian Lunar Theory

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Synopsis

The principal part of the present contributions to the study of Babylonian lunar theory is a reconstruction of a scheme for computing the length of time intervals consisting of six consecutive synodic months. The authors give direct textual evidence for this scheme and employ it to bring under control several texts – most of them concerned with eclipses – in which six-month intervals play a role, and which hitherto have defied complete numerical analysis. Further, they take this opportunity to publish some results of their concern with the corpus of lunar texts of System A, most of them edited in ACT: new datings, interpretations, and joins of fragmentary texts.

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To Otto Neugebauer
on the occasion of his eightieth birthday, 26 May 1979,
with admiration, gratitude, and affection.

In the present paper our chief concern is with the precise manner of computing certain time-intervals between like syzygies – conjunctions or oppositions of Sun and Moon – within Babylonian lunar theory according to System A. These intervals are the lengths of five or of six consecutive lunations, both important for the construction of eclipse tables, and they are affected by the variation in apparent lunar and solar velocities. The Babylonian numerical techniques for introducing the latter influence have long been reproducible and understood, and it is the ancient accounting for the effects of lunar anomaly that we are here bringing under exact control for the first time.

Our access to the electronic computers of our institutions, particularly the one at the Computer Center of the University of Illinois at Chicago Circle, is in large measure responsible for our success in reconstructiong the ancient scheme for deriving V and W, as we call the functions of relevance for 5-month and 6-month intervals, respectively.

In what follows, we first outline the background of our problem and present its solution. Next, we draw attention to direct textual support for our reconstructed scheme, and to texts that employ it. Finally, we take this opportunity to publish some new results on the corpus of lunar texts of System A in ACT,[1] though they have nothing directly to do with the functions V and W. In three instances we have joined new fragments to those published in ACT. In the rest we have used the extensive tables generated by our computers to date texts that remained undated in ACT, or to learn more about them in other respects.

All the cuneiform tablets we have occasion to mention happen to derive from the astronomical archive in Babylon – none is from Uruk – and they all came to the British Museum through acquisition from dealers during the last decades of last century. The hitherto unpublished texts, and the photographs, are published here through the courtesy of the Trustees of the British Museum.

A. Aaboe had the opportunity to collate several of the relevant texts in the British Museum in August 1977, enjoying once more the hospitality of the Department of Western Asiatic Antiquities[2] and of their Keeper, Dr. *Edmond Sollberger*.

1. For this and other abbreviations see the Bibliography.

2. *A. Aaboe's* previous visits to the British Museum, during which some of the fragments published below were rejoined, were supported by various grants from the National Science Foundation and by a Guggenheim Fellowship. The photographs are published through the courtesy of the Trustees of the British Museum.

1° Introduction of W and V

Otto Neugebauer's Astronomical Cuneiform Texts – ACT for short – appeared in 1955 and contained editions of the then known cuneiform texts that deal with mathematical astronomy, some 300 in number and almost all of the Seleucid period. Since then particular interest has been centred on a group of new texts, published by *Neugebauer* and by *Aaboe*, that threw light on a certain problem raised and left unsolved in ACT. It concerns the manner in which time intervals between like syzygies – either conjunctions or oppositions of Sun and Moon – were computed in the lunar theory according to System A.

The length of the true synodic month, say, from one conjunction to the next, is affected by the variation in both lunar and solar velocity. Babylonian lunar theory of both System A and System B separate these effects into two independent additive terms and consider the synodic month to be

$$1 \text{ month} = 29^{d} + G^{H} + J^{H} \text{ (see footnote 3)}$$

where G depends on lunar anomaly, and J on solar anomaly or rather on the Sun's longitude, for no distinction is drawn between sidereal and anomalistic year. System A has J depend directly on solar longitude in a manner that was under complete arithmetical control in ACT, and that makes astronomical sense as well. However, the derivation of G in this system raised difficulties, not of arithmetical, but of astronomical character.

Column G is derived from Column Φ, a zig-zag function of extrema

$M = 2;17,4,48,53,20^{H}$ and $m = 1;57,47,57,46,40^{H}$

and monthly difference

$$d = 0;2,45,55,33,20^{H/m}.$$

The period of Φ is the anomalistic month and Φ is, in fact, in precise phase with lunar velocity F. Column Φ is the second column of a standard lunar ephemeris according to System A, just following the one giving year and month. The sole purpose of Φ seemed to be to serve as a basis for the computation of G. The transformation of Φ into G is the subject of several procedure texts, a new variant of which we publish below as "ACT No. 207cc augmented." The scheme is summarised in Table 1 which in essence is a copy of the table given in ACT, p.60. It presents a selection of values of Φ – it matters whether such a value belongs to an ascending or descending branch of the zig-zag function, and this is indicated by the

3. $1^{d} = 6^{H} = 6,0$ (time) degrees. The large hour (H) is introduced for convenience in modern text editions; the Babylonian unit is the time degree (uš).

Table 1.

Φ		G	Interpol.
2;13,20↓	2;15,48, 8,53,20↑	2;40	0
2;13, 2,13,20↓	2;15,30,22,13,20↑	2;40,17,46,40	1
2;12,44,26,40↓	2;15,12,35,33,20↑	2;40,53,20	2
2;12,26,40↓	2;14,54,48,53,20↑	2;41,46,40	3
2;12, 8,53,20↓	2;14,37, 2,13,20↑	2;42,57,46,40	4
2;11,51, 6,40↓	2;14,19,15,33,20↑	2;44,26,40	5
2;11,33,20↓	2;14, 1,28,53,20↑	2;46,13,20	6
2;11,15,33,20↓	2;13,43,42,13,20↑	2;48,17,46,40	7
2;10,57,46,40↓	2;13,25,55,33,20↑	2;50,40	8
2;10,40↓	2;13, 8, 8,53,20↑	2;53,20	9
2;10,22,13,20↓	2;12,50,22,13,20↑	2;56, 5,55,33,20	9;20
⋮	⋮	⋮	⋮
1;58,31, 6,40↓	2; 0,59,15,33,20↑	4;46,42,57,46,40	9;20
1;58,13,20↓	2; 0,41,28,53,20↑	4;49,11, 6,40	8;20
1;57,55,33,20↓	2; 0,23,42,13,20↑	4;51,21,28,53,20	7;20
1;57,58, 8,53,20↑	2; 0, 5,55,33,20↑	4;53,14, 4,26,40	6;20
1;58,15,55,33,20↑	1;59,48, 8,53,20↑	4;54,48,53,20	5;20
1;58,33,42,13,20↑	1;59,30,22,13,20↑	4;56	4
	1;59,12,35,33,20↑	4;56,35,33,20	2
	1;58,54,48,53,20↑	4;56,35,33,20	0
	1;58,37, 2,13,20↑	4;56	2

Interpolation coefficients belong to the preceding intervals

small arrows – and the corresponding values of G. If a value of Φ is not in the table, the corresponding G-value is found by linear interpolation in Table 1. Thus the arithmetical structure of the Φ-G scheme was completely under control in ACT, but it was otherwise with its astronomical signifiance. It was not even known in what units Φ was measured – as indicated above, they turned out to be large hours.

The text that established this, and which brought Φ in connexion with the length of a Saros (an interval of 223 months) – we call it the Saros Text – was published by *Neugebauer* in 1957. The text is difficult and still not fully understood, but it enabled *Neugebauer* to perceive astronomical relations between Φ and G, and recognise that the difference between the length of 223 consecutive months and Φ should be constant. *B. L. van der Waerden* [1966] suggested that this constant should be a whole number of days (6585) and put forward the conjecture that the Φ-function when in actual use, and not just employed as an indicator of lunar anomaly, should be truncated above and below.

Beginning in 1968, *Aaboe* published a series of texts that were concerned with Column Φ in relation to other functions, measuring the length of various numbers of consecutive lunations. It was particularly Text E in *Aaboe* [1968] that offered an insight into a consistent methodology underlying the derivation of such intervals as G from Φ. Text E dealt in tabular form with Φ and a function Λ, attested and named already in ACT though then of unknown significance, but in such a manner that it enabled *Aaboe* to identify Φ beyond doubt, as well as Λ, and to give an astronomical justification of the table's structure.

It appeared that Λ played a role equivalent to that of G, but for intervals of twelve months. Thus the twelve-month interval, or "year", is

$$12 \text{ months} = 354^{d} + \Lambda^{H} + Y^{H}$$

where Λ depends on Φ, i. e. on lunar anomaly, and Y on longitude (this component was found later, see *Aaboe* [1969]; we may add that though Y is closely related to J, it is not precisely equal to the sum of the relevant twelve J-values but incorporates an additive constant so that it vanishes on most of the fast arc, like J; this constant is absorbed in the mean value of Λ). With the method underlying Text E, *Aaboe* could justify the Φ-G scheme (see *Aaboe* [1968] and, most recently, HAMA).

Prompted by various ephemerides, mostly eclipse texts, *Aaboe* [1971] proposed the existence of an analogous function W concerned with six-month intervals and restored a small fragment of an auxiliary table according to the methods for generating G and Λ from Φ, but failed to reach perfect agreement with the ephemerides. He took the total length of the six lunations preceding syzygy number n to be

$$6 \text{ months} \equiv W(n) + Z(n),$$

modulo an integral number of days, where $W(n)$ depends on lunar anomaly, and $Z(n)$ on solar anomaly (the integral number of days is either 176, 177, or 178). The term $Z(n)$ offers no difficulties, for it is simply the sum of $J(n)$ and its five monthly predecessors (see, e. g. *Aaboe* [1971]); but it is not so with $W(n)$. Disregarding for the moment the five-month intervals, it is this function we find in the eclipse text ACT No. 60, Column VIII, and *Neugebauer* assumed, very naturally, that is should be the sum of the six relevant G-values, but this assumption led to perfect agreement with the preserved values of the text only in very few instances. To be sure, the deviations were in general small – small enough to allow a secure identification of the astronomical meaning of this column – but they seemed to be of a systematic character. Thus it was clear to *Neugebauer* that his assumption could not offer a consistent explanation of the arithmetical structure of Column VIII of No. 60.

It turns out that all but a few of the entries in this column – still excepting the five month intervals – can be exactly derived by linear interpolation in a Φ-W table (Table 2) which is gathered in the following simple arithmetical fashion (most of the few exceptions appear to be consequences of very natural arithmetical mistakes).

Table 2.

	Φ	W	Interpol.
1.	2,13,20 ↓	15,5,11,6,40	
	2,13,2,13,20 ↓	6,34,48,53,20	28;42,30
	2,12,44,26,40 ↓	5,58,40	26;42,30
	2,12,26,40 ↓	5,51,21,14,4,26,40	24;40,50
5.	2,12,8,53,20 ↓	5,44,37,31,51,6,40	22;42,30
	2,11,51,6,40 ↓	5,38,29,22,57,46,40	20;42,30
	2,11,33,20 ↓	5,32,56,47,24,26,40	18;42,30
	2,11,15,33,20 ↓	5,27,59,45,11,6,40	16;42,30
	2,10,57,46,40 ↓	5,23,38,16,17,46,40	14;42,30
10.	2,10,40 ↓	5,19,52,20,44,26,40	12;42,30
	2,10,22,13,20 ↓	5,16,41,58,31,6,40	10;42,30
	2,10,4,26,40 ↓	5,14,7,9,37,46,40	8;42,30
	2,9,46,40 ↓	5,12,11,36,17,46,40	6;30
	2,9,28,53,20 ↓	5,10,47,54,4,26,40	4;42,30
15.	2,9,11,6,40 ↓	5,9,59,45,11,6,40	2;42,30
	2,8,53,20 ↓	5,9,47,9,37,46,40	0;42,30
	2,8,35,33,20 ↓	5,10,10,7,24,26,40	1;17,30
	2,8,17,46,40 ↓	5,11,8,38,31,6,40	3;17,30
	2,8 ↓	5,12,42,42,57,46,40	5;17,30
20.	2,7,42,13,20 ↓	5,14,50,22,13,20	7;10,50
	2,7,24,26,40 ↓	5,17,35,33,20	9;17,30
	2,7,6,40 ↓	5,20,58,1,28,53,20	11;23,20
	2,6,48,53,20 ↓	5,24,54,19,15,33,20	13;17,30
	2,6,31,6,40 ↓	5,29,26,10,22,13,20	15;17,30
25.	2,6,13,20 ↓	5,34,33,34,48,53,20	17;17,30
	2,5,55,33,20 ↓	5,40,16,32,35,33,20	19;17,30
	2,5,37,46,40 ↓	5,46,35,3,42,13,20	21;17,30
	2,5,20 ↓	5,53,29,8,8,53,20	23;17,30
	2,5,2,13,20 ↓	54,48,53,20	25;4,10
30.	2,4,44,26,40 ↓	9	27;17,30
	2,4,26,40 ↓	17,40,44,26,40	29;17,30
	2,4,8,53,20 ↓	26,51,51,6,40	31
	2,3,51,6,40 ↓	36,20,44,26,40	32
	2,3,33,20 ↓	46,7,24,26,40	33
35.	2,3,15,33,20 ↓	56,11,51,6,40	34
	2,2,57,46,40 ↓	1,6,34,4,26,40	35
	2,2,40 ↓	1,17,14,4,26,40	36
	2,2,22,13,20 ↓	1,28,11,51,6,40	37
	2,2,4,26,40 ↓	1,39,27,24,26,40	38
40.	2,1,46,40 ↓	1,51,0,44,26,40	39
	2,1,28,53,20 ↓	2,2,51,51,6,40	40
	2,1,11,6,40 ↓	2,15,0,44,26,40	41
	2,0,53,20 ↓	2,27,27,24,26,40	42
	2,0,35,33,20 ↓	2,40,11,51,6,40	43
45.	2,0,17,46,40 ↓	2,53,14,4,26,40	44
	2 ↓	3,6,34,4,26,40	45
	1,59,42,13,20 ↓	3,20,11,51,6,40	46
	1,59,24,26,40 ↓	3,34,5,25,55,33,20	46;53,20
	1,59,6,40 ↓	3,48,18,45,55,33,20	48
50.	1,58,48,53,20 ↓	4,2,49,52,35,33,20	49
	1,58,31,6,40 ↓	4,17,38,45,55,33,20	50
	1,58,13,20 ↓	4,32,27,39,15,33,20	50
	1,57,55,33,20 ↓	4,47,16,32,35,33,20	50
	1,57,58,8,53,20 ↑	5,2,5,25,55,33,20	50
55.	1,58,15,55,33,20 ↑	5,16,54,19,15,33,20	50
	1,58,33,42,13,20 ↑	5,31,37,17,2,13,20	49;40
	1,58,51,28,53,20 ↑	5,45,51,51,6,40	48;4,10
	1,59,9,15,33,20 ↑	5,59,48,8,53,20	47;2,30
	1,59,27,2,13,20 ↑	13,8,53,20	45;2,30
60.	1,59,44,48,53,20 ↑	25,54,4,26,40	43;2,30
	2,0,2,35,33,20 ↑	38,1,28,53,20	40;55
	2,0,20,22,13,20 ↑	49,32,13,20	38;51,15
	2,0,38,8,53,20 ↑	1,0,27,24,26,40	36;51,15
	2,0,55,55,33,20 ↑	1,10,47,2,13,20	34;51,15
65.	2,1,13,42,13,20 ↑	1,20,27,9,37,46,40	32;37,55
	2,1,31,28,53,20 ↑	1,29,25,40,44,26,40	30;17,30

Text D

	Φ	W	Interpol.
66.	2,1,31,28,53,20 ↑	1,29,25,40,44,26,40	
	2,1,49,15,33,20 ↑	1,38,1,58,31,6,40	29;2,30
	2,2,7,2,13,20 ↑	1,46,2,42,57,46,40	27;2,30
	2,2,24,48,53,20 ↑	1,53,27,54,4,26,40	25;2,30
70.	2,2,42,35,33,20 ↑	2,0,17,17,2,13,20	23;1,40
	2,3,0,22,13,20 ↑	2,6,28,1,28,53,20	20;51,15
	2,3,18,8,53,20 ↑	2,12,3,12,35,33,20	18;51,15
	2,3,35,55,33,20 ↑	2,17,2,50,22,13,20	16;51,15
	2,3,53,42,13,20 ↑	2,21,24,56,17,46,40	14;44,35
75.	2,4,11,28,53,20 ↑	2,25,3,27,24,26,40	12;17,30
	2,4,29,15,33,20 ↑	2,28,19,45,11,6,40	11;2,30
	2,4,47,2,13,20 ↑	2,31,0,29,37,46,40	9;2,30
	2,5,4,48,53,20 ↑	2,33,5,40,44,26,40	7;2,30
	2,5,22,35,33,20 ↑	2,34,37,2,13,20	5;8,20
80.	2,5,40,22,13,20 ↑	2,35,27,46,40	2;51,15
	2,5,58,8,53,20 ↑	2,35,42,57,46,40	0;51,15
	2,6,15,55,33,20 ↑	2,35,22,35,33,20	1;8,45
	2,6,33,42,13,20 ↑	2,34,26,40	3;8,45
	2,6,51,28,53,20 ↑	2,32,51,51,6,40	5;20
85.	2,7,9,15,33,20 ↑	2,30,34,48,53,20	7;42,30
	2,7,27,2,13,20 ↑	2,27,55,33,20	8;57,30
	2,7,44,48,53,20 ↑	2,24,40,44,26,40	10;57,30
	2,8,2,35,33,20 ↑	2,20,50,22,13,20	12;57,30
	2,8,20,22,13,20 ↑	2,16,22,13,20	15;5
90.	2,8,38,8,53,20 ↑	2,11,17,24,26,40	17;8,45
	2,8,55,55,33,20 ↑	2,5,37,2,13,20	19;8,45
	2,9,13,42,13,20 ↑	1,59,21,6,40	21;8,45
	2,9,31,28,53,20 ↑	1,52,25,40,44,26,40	23;22,5
	2,9,49,15,33,20 ↑	1,44,48,38,31,6,40	25;42,30
95.	2,10,7,2,13,20 ↑	1,36,49,22,57,46,40	26;57,30
	2,10,24,48,53,20 ↑	1,28,14,34,4,26,40	28;57,30
	2,10,42,35,33,20 ↑	1,19,4,11,51,6,40	30;57,30
	2,11,0,22,13,20 ↑	1,9,18,1,28,53,20	32;58,20
	2,11,18,8,53,20 ↑	58,53,12,35,33,20	35;8,45
100.	2,11,35,55,33,20 ↑	47,52,50,22,13,20	37;8,45
	2,11,53,42,13,20 ↑	36,16,54,48,53,20	39;8,45
	2,12,11,28,53,20 ↑	24,3,27,24,26,40	41;15,25
	2,12,29,15,33,20 ↑	11,6,25,11,6,40	43;42,30
	2,12,47,2,13,20 ↑	5,57,47,9,37,46,40	44;57,30
105.	2,13,4,48,53,20 ↑	5,43,52,20,44,26,40	46;57,30
	2,13,22,35,33,20 ↑	5,29,26,47,24,26,40	48;41,15
	2,13,40,22,13,20 ↑	5,14,42,57,46,40	49;42,55
	2,13,58,8,53,20 ↑	4,59,54,4,26,40	50
	2,14,15,55,33,20 ↑	4,45,5,11,6,40	50
110.	2,14,33,42,13,20 ↑	4,30,16,17,46,40	50
	2,14,51,28,53,20 ↑	4,15,27,24,26,40	50
	2,15,9,15,33,20 ↑	4,0,41,6,40	49;51,15
	2,15,27,2,13,20 ↑	3,46,12,35,33,20	48;51,15
	2,15,44,48,53,20 ↑	3,32,1,51,6,40	47;51,15
115.	2,16,2,35,33,20 ↑	3,18,11,43,42,13,20	46;41,40
	2,16,20,22,13,20 ↑	3,4,36,32,35,33,20	45;51,15
	2,16,38,8,53,20 ↑	2,51,19,8,8,53,20	44;51,15
	2,16,55,55,33,20 ↑	2,38,19,30,22,13,20	43;51,15
	2,16,55,55,33,20 ↓	2,25,37,39,15,33,20	42;51,15
120.	2,16,38,8,53,20 ↓	2,13,13,34,48,53,20	41;51,15
	2,16,20,22,13,20 ↓	2,1,7,17,2,13,20	40;51,15
	2,16,2,35,33,20 ↓	1,49,18,45,55,33,20	39;51,15
	2,15,44,48,53,20 ↓	1,37,48,1,28,53,20	38;51,15
	2,15,27,2,13,20 ↓	1,26,35,55,33,20	37;48,20
125.	2,15,9,15,33,20 ↓	1,15,40,44,26,40	36;51,15
	2,14,51,28,53,20 ↓	1,5,3,20	35;51,15
	2,14,33,42,13,20 ↓	54,43,42,13,20	34;51,15
	2,14,15,55,33,20 ↓	44,41,51,6,40	33;51,15
	2,13,58,8,53,20 ↓	34,57,46,40	32;51,15
130.	2,13,40,22,13,20 ↓	25,31,28,53,20	31;51,15
	2,13,22,35,33,20 ↓	16,22,57,46,40	30;51,15
	2,13,20 ↓	15,5,11,6,40	30

ACT No. 1005

First we take a sequence of Φ-values covering one wave of the Φ-function fairly densely. More specifically, we begin with the value 2,13,20↓ and proceed in steps of 17,46,40, reflecting as usual in Φ's extrema (see Table 2). The difference 17,46,40 has astronomical signifiance: it is the effective change in Φ over one Saros, or 223 months. This fact is of absolute essence in other tables involving Φ, but here any other small regular number might in principle have served as well. This particular choice of spacing between the Φ-values offers, however, the convenience that the first 56 entries in the Φ-column of Table 2 are also found in the Φ-G scheme (Table 1), and various other advantages as well. After 131 steps we are very nearly back to the initial value of Φ.

Next we associate with each of these 131 Φ-values a value of W which is the sum of six successive monthly G-values, the last of which corresponds to the relevant Φ-value.

As an example, consider the first entry in the table. The Φ-value is 2,13,20↓, and it and its five monthly prececessors have associated with them, according to Table 1, the following G-values:

$$\begin{array}{lcl}
\Phi_0 = 2;13,20\downarrow & \sim & G_0 = 2;40^{\mathrm{H}} \\
\Phi_{-1} = 2;16,5,55,33,20\downarrow & \sim & G_{-1} = 2;40 \\
\Phi_{-2} = 2;15,17,46,40\uparrow & \sim & G_{-2} = 2;40,42,57,46,40 \\
\Phi_{-3} = 2;12,31,51,6,40\uparrow & \sim & G_{-3} = 2;58,58,45,55,33,20 \\
\Phi_{-4} = 2;9,45,55,33,20\uparrow & \sim & G_{-4} = 3;24,47,24,26,40 \\
\Phi_{-5} = 2;7\uparrow & \sim & G_{-5} = \underline{3;50,36,\ 2,57,46,40} \\
\text{whose sum is:} & & \sum G = 18;15,\ 5,11,\ 6,40^{\mathrm{H}}
\end{array}$$

or, reduced modulo 6 in the first place, 15,5,11,6,40 which is the value of W entered opposite 2,13,20↓ in the table.

Thus the Φ-W table is constructed, and if we wish to find a value of W corresponding to a Φ-value that does not occur in the first column, we simply interpolate linearly. The relevant interpolation coefficients are given in the last column of Tables 2a and b (such a coefficient belongs to the interval preceding the line in which it is entered).

This scheme, motivated only by a desire for arithmetical convenience, serves to explain precisely several hitherto puzzling lunar texts where intervals of six months are of concern, as we shall see.

Intervals of six months are, of course, primarily of interest when one is dealing with eclipses, but here an occasional five-month interval will occur. It is a natural assumption that there was a table, similar to that for W, for five-month intervals, and *Aaboe* [1971] constructed such a function – he called it V – according to the same general principles that underlie the Φ-G and Φ-Λ schemes. In fact, it turns out that the length of a five month interval was derived by combined use of the tables for W and G. More precisely, to compute the length of the five months previous to syzygy number n one found first the six-month interval belonging to the *next* syzygy and then subtracted the length of the last month or, if we retain the notation V for the five month interval,

$$V(n) = W(n+1) - G(n+1).$$

2° Textual Evidence for Φ-W Scheme

There is direct textual evidence that the Babylonian astronomers possessed a scheme much like Table 2. First there is the small fragment BM 45930 (Text D in *Aaboe* [1971]) whose preserved surface corresponds to the outlined area between lines 36 and 47 of Table 2, except for isolated digits. *Aaboe* recognised that he was dealing with a function concerning the length of six-month intervals, named it W, and reconstructed the scheme according to the general methodology of functions related to Φ. His reconstruction fits the fragment perfectly, but it failed to reproduce exactly the entries in texts like No. 60. As fate would have it, his Text D is broken just so that it can equally well be accounted for by *Aaboe's* scheme as by Table 2, for it covers almost precisely the intersection of the two schemes. A few more preserved lines would surely have deviated from the values reconstructed by *Aaboe*, for not only is there close agreement between Table 2 and the ephemerides, but ACT No. 207 gives the endings of lines 15-26 of the scheme's W-values, and ACT No. 1005 actually preserves eight lines of the interpolation column of Table 2.

ACT No. 207 (BM 33593)
Transcription: Table 3.
The text is a fragment with only one side preserved. *Neugebauer* restored lines 8' to 14' as though they belonged to the Φ-G scheme, but could not make sense of the first seven lines. In fact, the text agrees perfectly with the endings of the W-values in lines 15 to 26 of Table 2. We think it unlikely that this fragment derives from the same tablet as Text D in *Aaboe* [1971]. The phrase *šá alla* [. . .] (whatever is beyond/below) indicates that an interpolation is called for. Thus we find, for example, in ACT No. 200b, Section 2 the formular "*šá alla x* dirig gam *y* DU-*ma* . . ." which means "whatever exceeds *x* multiply by *y* . . ." where interpolation is described. For the rest of the phrase see ACT No. 1005 below.

Table 3.

1.'	[5, 9,59,45,11],⌜6⌝,40	⌜šá⌝[al-la]	1.'~15.
	[5, 9,47, 9],⌜37⌝,46,40	šá [al-la]	
	[5,10,10],7,24,26,40	šá [al-la]	
	[5, 11]8,38,31, 6,40	šá [al-la]	
5.'	[5,12,4]2,42,57,46,40	[šá al-la]	5.'~19.
	[5,14]50,22,13,20	[šá al-la]	
	[5,17,35],33,20	[šá al-la]	
	[5,20,58],1,28,53,20	šá [al-la]	
	[5,24,54]19[15]33,20	šá a[l-la]	
10.'	[5,29,26,10,22],13,20	šá al-[la]	10.'~24.
	[5,34,33,34,4]8,53,20	šá al-l[a]	
	[5,40,16,32,35,33]20	šá al-l[a]	
	[5,46,35,3,42,13,20]	šá al-l[a]	

ACT No. 207

ACT No. 1005 (BM 34497)

This small fragment is published in transcription among the unidentified texts at the end of ACT, and in *Pinches's* copy as LBAT No. 156. It contains the outlined interpolation coefficients from line 121 to 128 of Table 2. What raises our identification of the text beyond any doubt is the presence of the peculiar value 37;48,20 in line 124, for it breaks an otherwise simple and natural pattern. In *Neugebauer's* transcription the numbers from Table 2 are preceded by initial 10's. However, an inspection of *Pinches's* copy shows that what *Neugebauer* read as 10 is almost certainly the sign GAM of two diagonal wedges and this is confirmed by collation. The second and third preserved lines end with remnants of signs that *Neugebauer* read as DU (rá) on which basis he surmised that the text gave a list of coefficients. The phrase "gam *x* DU" means "multiply by *x*" (cf. glossary of ACT and commentary to No. 207 above). In the fourth line is preserved what may be part of the last digit of a W-value.

Where Pinches read 32 in the last line, *Neugebauer* read 33 in agreement with Table 2. *Neugebauer's* reading is confirmed by collation.

3° Texts Employing W and V

We shall now turn to ephemerides containing the W-function. In our discussion we shall limit ourselves to the relevant columns; for the full extent of these texts we refer the reader to the original publications.

ACT No. 60 (BM 45688)
This large, well-preserved tablet from Babylon concerns lunar eclipse possibilities from S. E. 137 to 160 (=–174/3 to –151/0). It gives information about those members of a sequence of consecutive oppositions of the Moon where lunar latitude at a change of sign is smaller in absolute value. Curvature suggests that the right third of the tablet is broken off, so that it very likely contained all necessary columns up to and including at least Column M, the moment of syzygy, here opposition.

In Table 4 we have extracted Columns I, II, and VIII from this text, the column giving year and month, Column Φ, and a column with W and V. The lines are mostly six months apart. A dotted line indicates an occasional five-month interval and in the following line a value of V is in order in Column VIII.

Where the preserved numbers in Column VIII agree with the scheme in Table 2 – or with the rule for computing a five-month interval – we have simply supplied the missing digits of the recomputed values in square brackets. Where they disagree, we have added the correctly computed number in the last column of Table 4. For an explanation of most of these divergencies, see the critical apparatus below.

We have a particular affection for this text: from its Column B *Kugler* (BMR p. 55f) managed to discover for the first time the rules for computing lunar longitudes according to System A, a remarkable achievement since it contains longitudes, not month by month, but 5 or 6 months apart. *Strassmaier's* copy is published in BMR, Tafel XIII, and *Pinches's* as LBAT No. 50.

Critical Apparatus
With but three exceptions, each of the errors in Column VIII can be explained as the result of a simple misuse of Table 2. We include a discussion of these, for such errors – rare in the astronomical literature as whole – afford us an insight into the computational routines of the Babylonian astronomers. Thus we learn that when they entered their version of Table 2 with a given value of Φ, they invariably related it to the Φ-value in the *first* line of the interval to which it belongs. In three instances they erred by taking the interpolation coefficient from the line just before that of the correct one. In four cases the error consists in a shift of the interpolated part one sexagesimal place to the right or left of the correct position.
Obv. 1. 1: We cannot derive this value from Table 2. A recent collation shows clearly 20,52,5[0. . .], with 4[0] possible in the last place, but not 20,32,5[0. . .] as in ACT.

Obv. 1. 2:

We have for

$$\begin{array}{lrl}
2{,}17\ \text{VIII}: \Phi = 1;58,40,44,26,40\uparrow & & \\
\text{Table 2, l. 56: } \underline{1;58,33,42,13,20\uparrow} \sim & & 5;31,37,17,\ 2,13,20 \\
0;\ 0,\ 7,\ 2,13,20 \times 49;40 = & & \underline{0;\ 5,49,30,22,13,20} \\
& W = & 5;37,26,47,24,26,40
\end{array}$$

which agrees with the text's 5,37,26,47 [. . .]. However, 49;40 is the interpolation coefficient for the interval preceding l. 56. The correct coefficient, 48;4,10, is found in l. 57.

Obv. 1. 3:

If we use the interpolation coefficient 49;51,15 (l. 112) we obtain in a like manner

$$W = 3;54,50,17,7,46,40^{H}$$

(text: 3,54,50,17,⌜6⌝[. .]). The correct coefficient is 48;51,15 (l. 113).

Obv. 1. 4:

If we use the interpolation coefficient 37 (l. 38) we get

$$W = 1;30,56,17,46,40^{H}$$

(text: 1,30,56,17,4[0. . .]). The correct coefficient is 38 (l. 39).

Obv. 1. 11:

We obtain the text's $0;13,20,22,13,20^{H}$ if we enter Table 2 with

$$\Phi = 2;4,35,33,20\downarrow$$

instead of the correct

$$\Phi = 2;4,35,55,33,20\downarrow.$$

Obv. 1. 21:

At 2,26 VII we expect a value of V for the five-month interval. If we go one month ahead to month VIII we find the value of Φ to be

$$\Phi = 1;59,54,48,53,20\uparrow$$

to which Table 2 assigns $W = 0;32,43,14,26,40^{H}$. This is precisely the entry of the text, so the scribe forgot to subtract the appropriate value of G, namely

$$G = 4;54,13,20^{H}.$$

Four of the next five errors arise from a misalignment of the interpolated part. We shall only deal with the first of them in detail since the rest follow the same pattern.

Table 4.

	I	II	VIII	
Obv.	2,17 II	2,13,30,44,26,40↓	20,52,5[0///////]	20.34.22. 2,13,20
	VIII	1,58,40,44,26,40↑	5,37,26,47[///////]	5,37,15,33, 1,28,53,20
	2,18 I	2,15,16,17,46,40↑	3,54,50,17,⌜6⌝[//////]	3,54,57,19,21, 6,40
	VII	2, 2,17,46,40↓	1,30,56,17,4[0//////]	1,31, 0,44,26,40
	XII	2, 7, 7,46,40↑	4,19,14, . ,44,26,⌜40]	
	2,19 VI	2,10,26,17,46,40↓	5,17,25,36, 6,⌜40]	
	XII	2, 1,45,11, 6,40↑	1,36, 3,39,26,[40]	
	2,20 VI	2,15,48,53,20↓	1,40,26,19,10	
	XII	1,59,13,20↓	3,42,58,45,55,3[3,20]	
	2,21 V	2,12,58, 8,53,20↑	5,49, 5,24, 4,26[40]	
	XI	2, 4,35,55,33,20↓	13,20,22,1[0//////]	13, 9,31,17,46,40
	2,22 V	2, 7,35,33,20↑	2,26,22,12,24[26,40]	
	X	2,12,44,26,40↓	2,13,41,28,5[3,20]	
	2,23 III	1,59,27, 2,13,20↑	13, 8,53,⌜20]	
	IX	2,16, 2,35,33,20↑	3,18,11,43,42,13,[20]	
	2,24 III	2, 1,31,28,53,20↓	2, 1, 8, 8,53,20	
	IX	2,10,40↑	1,20,24,27,35,33,[20]	
	2,25 III	2, 6,54, 4,26,40↓	5,23,45,24, 4,26,40	
	IX	2, 5,17,24,26,40↑	2,34,10,23,27,24,26,40	
	2,26 II	2,12,16,40↓	5,47,34,9, 4,26,40	
	VIII	1,58,27, 2,13,20↓	32,43,14,26,40	1,38,29,54,26,40
	2,27 I	2,13,44,26,40↑	5,14,39,34, 4,26,40	5,11,19,15,33,20
	VIII	2, 3,49,37,46,40↓	37, 9,37,46,40	
	2,28 I	2, 8,21,51, 6,40↑	2,15,56,49,15,33,20	
	VII	2, 9,12,13,20↓	5,10, .2,45,44,26,40]	
	XII$_2$	2, 2,59,15,33,20↑	2, . ,23, 4,36[////]	2, 6, 4,51,12,13,20
	2,29 VI	2,14,34,48,53,20↓	55,22,25,5[0]	
	XI	2, . ,45,11, 6,40↓	5,[53,17, 2,13,20]	
Rev.	2,30 V	2,11,26,17,46,40↑	5[3]5[0,32,24,26,40]	
	XI	2, 6, 7,46,40↓	5,36,20,4[5,22,13,20]	
	2,31 V	2, 6, 3,42,13,20↑	2,3[0].4,2[0////////]	2,35,36,35,50
	X	2,11,30,22,13,20↓	5,32,[7,17, 2,13,20]	
	2,32 IV	2, . ,41, 6,40↑	1, 2,1[0,40,44,26,40]	
	X	2,16,52,57,46,40↓	2,25,35, 1[0//////]	2,23,33,38,31, 6,40
	2,33 IV	2, . ,17,24,26,40↓	2,53,30,44[26,40]	
	IX	2, 9, 8, 8,53,20↑	3,31,10,16,3[0,44,26,40]	
	2,34 II	2, 8,25,55,33,20↓	5,41,51,58,2[0////////]	5,10,41,49,15,33,20
	VIII	2, 3,45,33,20↑	2,19,24,48,[34,46,53,20]	
	2,35 II	2,13,48,31, 6,40↓	29,51, 2, 2,13,[20]	
	VIII	1,58,22,57,46,40↑	5,22,43,49,37,46,[40]	
	2,36 II	2,14,58,31, 6,40↑	4, 9,37,34,54,[26,40]	4, 9,36,34,54,26,40
	VIII	2, 2,35,33,20↓	1,19,58,31,[6,40]	
	XII$_2$	2, 6,50↑	3,24,10,4[6//////]	4,24,10,40,44,26,40
	2,37 VI	2,10,44, 4,26,40↓	5,20,44,[7,13,20]	
	XII	2, 1,27,24,26,40↑	1,27,22,[16, 6,40]	
	2,38 VI	2,16, 6,40↓	⌞2,34⌟[//////////////]	1,52, 1, 8, 3,20
	XII	1,59,31, 6,40↓	[//////////////////]	
	2,39 VI	2,12,40,22,13,20↑	[//////////////////]	
	XII	2, 4,53,42,13,20↓	[//////////////////]	
	2,40 V	2, 7,17,46,40↑	[//////////////////]	

From ACT No. 60

Obv. l. 22:

The computation should have proceeded thus:

$$
\begin{array}{lll}
2{,}27\text{ I:} & \Phi = 2;13,44,26,40\uparrow & \\
\text{Table 2, l. 107:} & \underline{2;13,40,22,13,20\uparrow} & \sim 5;14,42,57,46,40 \\
 & 0;\ 0,\ 4,\ 4,26,40 \times 50 & = \underline{0;\ 3,23,42,13,20} \\
\text{which yields} & & W = 5;11,19,15,33,20^{H}.
\end{array}
$$

If, however, the interpolated part by mistake is moved one place to the right we obtain

$$
\begin{array}{r}
5;14,42,57,46,40 \\
\underline{-0;\ 0,\ 3,23,42,13,20} \\
5;14,39,34,\ 4,26,40.
\end{array}
$$

which is precisely the value in the text.

Obv. l. 26:

If the interpolated part is shifted one place to the right we get

$$W = 2;0,23,4,36,23,20^{H}$$

(text: 2,.,23,4,3ṣ[. . .]).

Rev. l. 3:

The reading given in Table 4 is not at all secure. However, we are fairly certain that neither the correct value, nor the results of natural mistakes can be brought in agreement with the preserved traces.

Rev. l. 6:

Shifting the interpolated part one place to the right we get

$$W = 2;25,35,35,14,48,53,20^{H}$$

(text: 2,25,35,1[0 . . .] ending with one corner wedge).

Rev. l. 9:

If we move the interpolated part one place to the left we get

$$W = 5;41,51,58,31,6,40^{H}$$

(text: 5,41,51,58,2[0 . . .] ending with two corner wedges).

Rev. l. 13:

The text has 37 for 36 in the third place.

Rev. l. 15:

The text has 3 for 4 in the first place.

Rev. l. 18:

Only traces of the first two digits remain. The best reading is 2,34 [. . .], but it is not very secure. However, they clearly do not agree with the correct value of

$$W = 1;52,1,8,3,20^{H}.$$

ACT No. 61a (BM 77238)

This fragmentary text, also from Babylon, concerns solar eclipse possibilities, but it is in other respects quite like No. 60. However, it contains further columns with corrections for solar anomaly and variable length of daylight while such columns were broken away from No. 60. In Table 5, the V-W column of No. 61a is presented in the manner of Table 4. The first line contains a V-value, computed as described above, though proper consideration of latitude would have placed the five-month interval one line earlier. Except for the remnants of line 2 the preserved parts are in agreement with the W-scheme in Table 2.

Critical Apparatus

l. 2: If we enter Table 2 with

$$\Phi = 2;15,40,17,46,40\uparrow$$

instead of the correct value

$$\Phi = 2;15,41,17,46,40\uparrow$$

we obtain $W = 3;35,38,4,54,26,40^{H}$

(text: [. . .] 5,38,4,54,26,40).

Table 5.

	[T]	[Φ]	I	
1.	3,11* II	1,59, 5,44,26,40↑	[3, 0,3]3,14,54,26,40	(5 mo.)
	VIII	2,15,41,17,46,40↑	[////////]35,38, 4,54,26,40	3,34,50,13,39,26,40
	3,12 I	2, 1,52,46,40↓	[1,4]7, 2,24,26,40	
	VII	2,10,18,42,13,20↑	[1,3]1,11,32, 7,46,40	
5.	3,13 I	2, 7,15,22,13,20↓	[1,19],18,53,57, 2,13,20	
	VII	2, 4,56, 6,40↑	[2,32],4,23,[25,33,20]	

From ACT No. 61a

ACT No. 55 (BM 46015)

This text was first published in ACT. *Neugebauer* realised that an advance of one line in the text very likely corresponded to a 12-month interval, at least in most cases. However, he recognised the entry in Obv. Col. II, l.4 as the G-value belonging to the conjunction at the end of S.E. 3,2 III. With his newly gained knowledge of the family of functions related to Φ, *Aaboe* [1971] identified most of the entries of the text's second column as Λ-values, confirmed *Neugebauer's* dating, and restored the text. He further identified the entry in Obv. II, 1.3 as a W-value, but could not reproduce it precisely. He realised that the text broke an interval of one Saros (223 months) into 18 twelve-month intervals, one six-month, and one one-month interval, or

$$223 \text{ months} = (18\cdot 12 + 1\cdot 6 + 1\cdot 1) \text{ months}.$$

The later columns contain corrections for solar anomaly, and for seasonal change in length of daylight, and finally the corrected values of Λ, W, and G.

For the W-value in Obv. II, l.3 *Aaboe* read with *Neugebauer*

$$W = 1,.,6,55,48,20 \text{ tab.}$$

The Φ-value given by *Aaboe* for this line (S.E. 3,2 II) is

$$\Phi = 2;0,37,35,33,20\uparrow$$

to which Table 2 assigns

$$W = 1;0,6,55,\mathit{58},20^{H}.$$

Thus the last bit of ACT No. 55 is explained, for the next to last digit is poorly preserved and can be read 58 equally well as 48 (confirmed by recent collation).

ACT No. 54 (BM 35231+35355)

This text, composed of two small rejoined fragments, is published in transcription as ACT No. 54 and, in *Pinches's* copy, as LBAT Nos. 47 and 48. *Neugebauer* recognised that it is part of an eclipse text, and identified the character of the columns. The text, it now turns out, does indeed deal with eclipse possibilities – solar eclipses – from S.E. 235 to 241, as far as it is preserved. In Table 6 (p. 19) we have reconstructed the relevant columns of the text, numbering them so that Column I is the first of which any trace is preserved (this is little enough: merely traces of a final lal).

Critical Apparatus

II,8': only traces of tops of signs remain.

III,8': the last two digits are nearly destroyed.

III,10': 3,1*9*,*9* should be 3,1*8*,*47*; this error affects the rest of Column M.

III,14': [ ],39,21 – reading uncertain, but traces are not consistent with the correct value of 3,51,37.

IV,14': leading digit – all preserved – could also be 5.

Comments

We have included in Table 6 only those columns that are relevant for the restoration of the fragment. We shall here merely identify them briefly and refer to ACT and later publications for further explanations:

Columns T_1 and Φ_1 we have already met. The subscript$_1$ means, in the convention of ACT, that we are dealing with conjunctions rather than oppositions of Sun and Moon.

Column B_1 gives the common longitude of Sun and Moon at conjunction. It is computed on the basis of a step-function which, in modern terms, can be looked upon as giving solar velocity in degrees per month, thus:

from Virgo 13° to Pisces 27°: $w_1 = 30^{\circ/m}$
from Pisces 27° to Virgo 13°: $w_2 = 28;7,30^{\circ/m}$.

Column C_1 gives length of daylight in large hours as a function of solar longitude. For the scheme for converting B_1 into C_1 see ACT.
Column E_1 gives lunar latitude at conjunction in še (barley corn) where

$$1 \text{ še} = 0;0,50^\circ.$$

These values are designated

lal lal (positive, increasing)	u u (negative, decreasing)
lal u (positive, decreasing)	u lal (negative, increasing)

with a terminology peculiar to Column E (elsewhere lal means negative, as in Columns I and II, or decreasing, while tab means positive or increasing). For the structure of Column E see ACT, HAMA, and *Aaboe-Henderson* [1975]. It is Column E that guides the selection of the conjunctions included in the text: they are the ones at which E at a sign change has smaller absolute value, as already mentioned.

Column [0] contains the values of V or W, whichever is appropriate, computed from Table 2. V is found in lines 7' and 14', just after 5-month intervals.

Column I contains corrections due to solar anomaly. It is based on Column J which complements G to form the time interval, less 29 days, from one to the next like syzygy. Column J vanishes on most of the fast arc and has the value

$$J = -0;57,3,45^{H}$$

on most of the slow, with occasional transitional, always negative, values (see ACT p. 61). We find here in Column I the sum of the five or six appropriate J-values; where it is six the entry is the function called Z above and in *Aaboe* [1971]. We might point out that the last line should be blank, for the longitude and its four predecessors all belong to the fast arc (we are dealing with a five-month interval).

In Column II, the first where numbers are preserved, we find corrections C' for variation of length of daylight. We have simply, and with sign,

$$C'(n) = \tfrac{1}{2}(C(n-1) - C(n)).$$

The next column is the equivalent of Column K in the ephemerides and gives the sum of the entries in the preceding three columns so that for a six-month interval we have

$$\tilde{K}(n) = W(n) + Z(n) + C'(n),$$

reduced modulo 6^H to the smallest positive value. The entries are abbreviated to three digits.

The last preserved column, Column M, gives the moment of conjunctions, measured in large hours before sunset (šú). It is derived by continued subtraction from an initial value of the entries in Column $\tilde{K}$. A comparison with modern tables shows that though the exlipse possibilities are correctly identified, the moment of conjunction given in Column M is consistently 10 hours wrong.

To the right of Column M in Table 6 we have added values of $\tilde{K}$ computed to four places for comparison with the text's Column III. The deviations form no recognizable pattern.

4° Other Lunar Texts of System A

ACT No. 207cc augmented.

Contents: Procedure text for converting Φ into G.

Photograph and Transcription: pp. 20-23.

Description of Text:

BM 36438 (=ACT No. 207cc) is joined by B.M. 37012, 37026, 37274, and 37319 to form the lower part of a tablet with right, left, and bottom edges preserved.

Translation:

With but two exceptions the text adheres to a formular – the text employs two close variants – which freely translated says:

opposite a put b; whatever exceeds a, until c, multiply by 3,22,30; multiply the result by d and add (subtract) the result to (from) b and put the result down.

Here *a* stands for a Φ-value in the standard table for converting Φ into G, *b* is the corresponding G-value, *c* is the neighbouring Φ-value to *a*, and *d* is the change in G over the interval from *a* to *b*.

The two exceptions (Obv. lines 6' and 7', and lines 12' and 13') both concern intervals in which G stays constant. Here the statement is:

Opposite a put b; until c add or subtract nothing.

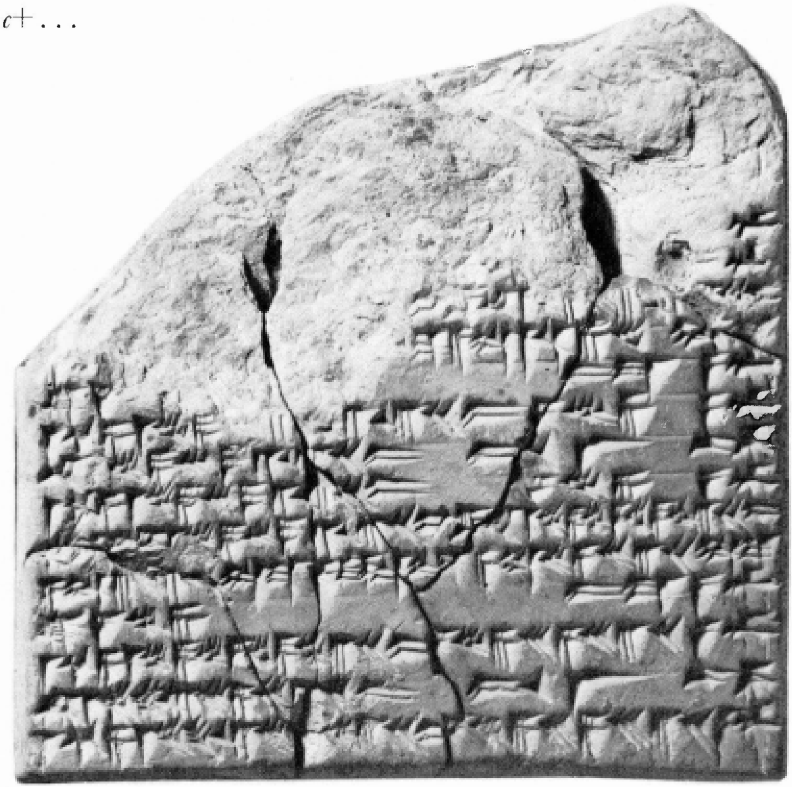
Critical Apparatus:

Reverse line 9': the text has as the second factor 1,34,48,*8*,53,20 for 1,34,48, 53,20.

Table 6.
(pp. 16-18).
(ACT No. 54).

[-V(T₁)]	[-IV(Φ₁)]	[-III(B₁)]	[-II(C₁)]	[-I(E₁)]	[0(V₁,W₁)]	I(ΣJ₁)	II(C′₁)	III(K̃₁)	IV(M₁)	
235* X	2,12,27,13,20↓	23,12 ♑	2,25,45,36	51,17,18 u lal	5,51,34,56,45,33,20	[1,34,43,25,30 lal]	[34]⌜1,57⌝tab	4,50,54	destroyed	4,50,53,28
236 III	1,59,44,15,33,20↑	15,56,15 ♋	3,35,12,30	34,14,54 lal u	25,30,9,43,20	[3,41, 1,19,30 lal]	[3]4,43,27 lal	2,9,46		2,9,45,23
IX	2,16,19,48,53,20↑	12,8 ♑	2,24,17,4	10,27,30 lal lal	3,5,2,1,3,53,20	[1,55,46,24,30 lal]	35,27,43 tab	1,44,44		1,44,43,20
237* III	2,1,14,15,33,20↓	5,33,45 ♋	3,35,24,30	33,1,54 u u	2,12,51,40	[3,19,58,20,30 lal]	35,33,43 lal	4,17,20		4,17,19,37
IX	2,10,57,13,20↑	1,4 ♑	2,25,11,28	1,12,12,18 lal lal	1,11,1,49,34,4,26,40	[2,16,49,23,30 lal]	35,6,31 tab	5,29,19		5,29,18,57
238 II	2,6,36,51,6,40↓	25,11,15 ♊	3,34,1,30	1,40,18,42 u u	5,27,58,23,14,26,40	[2,58,55,21,30 lal]	34,25,1 lal	1,55,38		1,54,38,1
VIII	2,2,48,42,13,20↑	20 ♏	2,36	1,58,34,18 u lal	4,31,18,33,11,40 (V)	[2,37,52,22,30 lal	29,.,45 tab	2,22,26		2,22,26,56
239 I	2,14,45,22,13,20↓	16,41,15 ♉	3,22,40,30	1,9,55,54 lal u	1,1,30,20,8,20	[1,40,48,37,30 lal]	⌞23,20,15 lal⌟	4⌞57,22⌟	⌜5⌝[9,59 šú]	4,57,21,28
VII	1,58,9,48,53,20↓	8,56 ♏	2,40,42,40	56,49,30 u lal	4,35,23,34,48,53,20	[3,55,59,6,30 lal]	[20,58,55 tab]	1,.,22	4,9⌜37⌝[šú]	1,0,23,23
240* I	2,14,1,40↑	6,18,45 ♉	3,17,32,30	2,39,6 lal u	4,56,58,8,53,20	[1,19,45,38,30 lal]	[18,24,55 lal]	3,19,9	.,50,28 [šú]	3,18,47,35 !
VII	2,3,32,24,26,40↓	27,52 ♎	2,48,5,20	4,55,18 lal lal	46,38,53,20	[4,17,2,5,30 lal]	[14,43,35 tab]	[2],44,19	4,6,8 [šú]	2,44,20,23
XII₂	2,8,39,4,26,40↑	25,56,15 ♈	3,10,37,30	1,4,37,42 u u	2,10,59,40,47,13,20	[58,42,39,30 lal]	[11,16,5 lal]	1,1,1	3,5,7 [šú]	1,1,0,56
241 VI	2,8,55↓	16,48 ♎	2,55,28	1,6,40,6 lal lal	5,9,48,20,27,46,40	[4,38,5,4,30 lal]	[7,34,45 tab]	39,18	2,2[5,49 šú]	39,18,1
XI	2,0,30,33,20↑	16,48 ♓	2,44,32	1,50,42,54 lal u	3,46,9,33,50,33,20 (V)	[ ]	[5,28 tab]	[ ]39,21	4[ ]	3,51,37,34 !

ACT No. 207 c+ . . .
Obv.



Commentary:

This text confirms the standard rules for converting Φ into G by linear interpolation in the Φ-G table (Table 1). The only feature of interest is the manner in which the interpolation is handled. Each interpolation involves a multiplication of the relevant increment of Φ, not by the usual single interpolation coefficient, but by two factors. The first is always

$$\frac{1}{17,46,40} = 3,22,30,$$

where 17,46,40 is the difference $\triangle\Phi$ between two consecutive listed values of Φ, and the other factor, varying from instance to instance, is $\triangle G$, the difference between the two relevant tabulated values of G. Trivially,

$$3,22,30 \cdot \triangle G$$

Transcription:

Obverse

−2' [*ana tar-ṣa* 1,57,58,8,53,20 tab 4,53,14,4,26,40 gar-*an*]
−1' [*mim-ma šá al* 1,57,58,8,53,20 tab dir en 1,58,15,55,33,20 tab]
0 [a-rá 3,22,30 DU-*ma šá* DUL+ DU-*ka* a-rá 1,34,48,53,20 DU-*ma*]
1' [ki 4,53,14,4,26,40 tab-*ma* gar-]*an*
2' [*ana tar-ṣa* 1,58,15,55,33,20 tab 4,54,48,53,20] gar-*an*
3' [*mim-ma šá al* 1,58,15,55,33,20] tab dir en 1,58,33,42,13,20 tab
4' a-rá[3,22,30 DU-*ma šá* DUL+ DU-]*ka* a-rá 1,11,6,40 DU-*ma*
5' ki 4,[54,48,53,20] tab-*ma* gar-*an*
6' *ana tar-sa* 1,58,33,42,13,20 tab 4,56 gar-*an*
7' en 1,58,37,2,13,20 tab tab *u* lal nu tuk
8' *ana tar-ṣa* 1,58,37,2,13,20 tab 4,56 gar-*an*
9' *mim-ma šá al* 1,58,37,2,13,20 tab dir en 1,58,54,48,53,20 tab
10' a-rá 3,22,[30] DU-*ma šá* DUL+ DU-*ka* a-rá 35,33,20 DU-*ma*
11' ki 4,56 tab-*ma* gar-*an*
12' *ana tar-ṣa* 1,58,54,48,53,20 tab 4,56,35,33,20 gar-*an*
13' en 1,59,12,35,33,20 tab tab *u* lal nu tuk
14' *ana tar-ṣa* 1,59,12,3[5],33,20 tab 4,56,35,33,20 gar-*an*

---

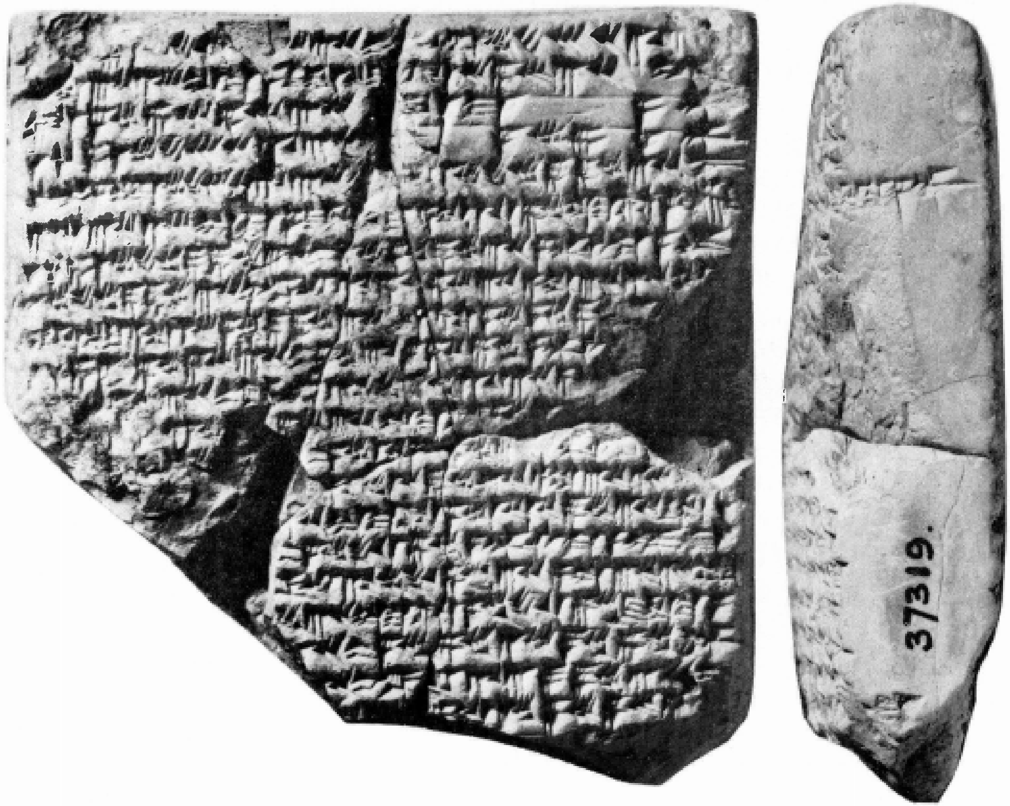
is the standard interpolation coefficient. For the linear stretch of G we are told (Rev. lines 20-21) to apply the two factors

$$3,22,30 \cdot 2,45,55,33,20$$

which product, indeed, comes out to be 9,20 as it should. The second factor is readily recognized as $d(\Phi)$, the monthly difference in Φ, but it is also, as we have learnt, the Sarosly difference in G, and so corresponds to 17,46,40, the Sarosly difference in Φ.

The preserved part of the text happens to begin with the smallest Φ-value on an ascending branch, takes us through the maximum of G, and ends with one of the two linear stretches of G. It is our guess that about half of the original tablet is lost so that only the middle of the text is preserved.

ACT No. 207 c+ . . .
Rev.



As an illustration of the perverse situations that may arise – and often do – when one is forced to use fragmentary material we can point to *Neugebauer's* reconstruction of the text on the basis of BM 36438 only. This fragment is the one at the lower right corner of the obverse and the upper right corner of the reverse (see the photograph), and whenever an interpolation coefficient is expected, either 3,22,30 or $\triangle G$ is preserved, but not both. *Neugebauer* identified 3,22,30 as the reciprocal of $\triangle\Phi$, recognized the values of $\triangle G$, and drew the conclusion, entirely reasonable in view of the evidence, that the interpolation scheme was wrong.

Reverse

1. *mim*-[*ma šá*] *al* 1,59,12,[35,]33,20 tab dir en 1,59,30,22,13,[20 tab]
2. a-rá 3,22,30 DU-*ma šá* DUL+ DU-*ka* a-rá 35, 33, 20 DU-[*ma*]
3. ta 4,56,35,33,20 [DU]L+ DU-*ma* gar-*an*
4. *ana tar-ṣa* 1,59,30,22,13,20 [ta]b 4,56 gar-*an*
5. *mim-ma šá al* 1,59,30,22,13,20 t[ab dir] en 1,59,48,8,53,20 tab
6. a-rá 3,22,30 DU-*ma šá* DUL+ DU-*ka* a-rá 1,11,6,40 DU-*ma* ta 4,56 DUL+ DU-*ma* gar-*an*]
7. *ana tar-ṣa* 1,59,48,8,53,20 tab 4,54,48,53,20 gar-*an mim-ma šá al*
8. 1,59,48,8,53,20 tab dir en 2,.,5,55,33,20 a-rá 3,22,30 DU-*ma*
9. *šá* DUL+ DU-*ka* a-rá 1,34,48,{8,}53,20 DU- <*ma*> ta 4,54,48,53,20 [DUL+ DU-*ma* gar-*an*]
10. *ana tar-ṣa* 2,.,5,55,33,20 tab 4,53,14,4,26,40 gar-*an mim*-[*ma šá al*]
11. [2,0,5,55]33,20 tab dir en 2,.,23,42,13,20 a-rá [3,22,30 DU-*ma šá*]
12. [DUL+ DU-*ka* a-rá 1,52,35,]33,20 DU-*ma* ta [4,53,14,4,26,40 DUL+ DU-*ma* gar-*an*]
13. [*ana tar-ṣa* 2,0,23,42,13,20] tab 4,51,21,28,53,20 gar-*an mim-ma* [*šá al*]
14. [2,0,23,42,13,20 tab dir en] 2,.,41,28,53,20 a-rá 3,22,30 DU [-*ma*]
15. [*šá* DUL+ DU-*ka* a-rá 2,10,22,13,]20 DU-*ma* ta 4,51,21,28,53,20 DUL+ DU- <*ma*> gar-*an*
16. [*ana tar-ṣa* 2,0,41,28,53,20] tab 4,49,11,6,40 gar-*an mim-ma šá al-la*
17. [2,0,41,28,53,20 tab] dir en 2,.,59,15,33,20 a-rá 3,22,30 DU-*ma*
18. [*šá* DUL+ DU-*ka* a-rá 2,28,8,]53,20 DU-*ma* ta 4,49,11,6,40 DUL+ DU-*ma* gar <-an>
19. [*ana tar-ṣa* 2,0,59,15,33,] 20 tab 4,46,42,57,46,40 gar-*an*
20. [*mim-ma šá al* 2,0,59,15,33,] 20 tab dir en 2,13,8,8,53,20 tab
21. [a-rá 3,22,30 DU-*ma šá* DUL+ DU-*ka*] a-rá 2,45,55,33,20
22. [DU-*ma* ta 4,46,42,57,46,40 DUL+ DU-*ma* gar-*an*]

BM 40094, Reverse, + *ACT No. 128*
Contents:]K_1, M_1, Λ_1, Y_1, $C_1^?$, $\tilde{K}_1$, M_1, P_3 for Philip Arrhidaeus 6,VI to 7,XII (= S.E. –6,VI to –5,XII).
Transcription: Table 7, p. 27.
Photograph: p. 25.

Description of Text

The text ACT No. 128 (BM 45662) is a fragment with only one side, and part of lower and right edge preserved. It now joins the reverse of BM 40094 so that the mended break runs through Columns V and VI of the transcription. Part of Column VIII (P_3) is on the right edge (not shown in the photograph). Upper and lower edges of BM 40094 are preserved, but parts of its surface, particularly the first five lines of Reverse, Column III, are rather badly eroded. We believe that the present text is a copy of an ill-preserved exemplar, for it contains an unusual number of isolated errors without consequence readily committed when one copies a poor text (e.g., 8 for 5, and 5 for 8).

We have not reproduced a copy of the obverse of BM 40094, for the join adds nothing to this side, yet we have retained and extended the numbering of columns from *Aaboe* [1969] though Column I (K) is not preserved on the reverse of this fragment. For the contents of the obverse we refer to *Aaboe's* publication with corrections given below.

The scribe's ductus is such that it is often difficult to distinguish between his "tab," "20," and "." where "." denotes the separation mark of two diagonal wedges used for zero.

Critical Apparatus (in part excerpted from *Aaboe* [1969]).

Rev. II,7: [2,1,] 1 is also possible.

Rev. II,8: [4],57,*44* (or *45*) should be 4,57,46 (cf. VIII,8).

Rev. II,14: 1,*49*,*5* should be 1,*39*,*35* (cf. VII,14), isolated error.

Rev. III,7: 1,33,25,53,5*8*, . . . should be 1,33,25,53,5*6*,6,40, an error without consequence.

Rev. III,8: *1*,23,36,45,47,13,20 should be *2*,23,36,45,47,13,20, isolated error.

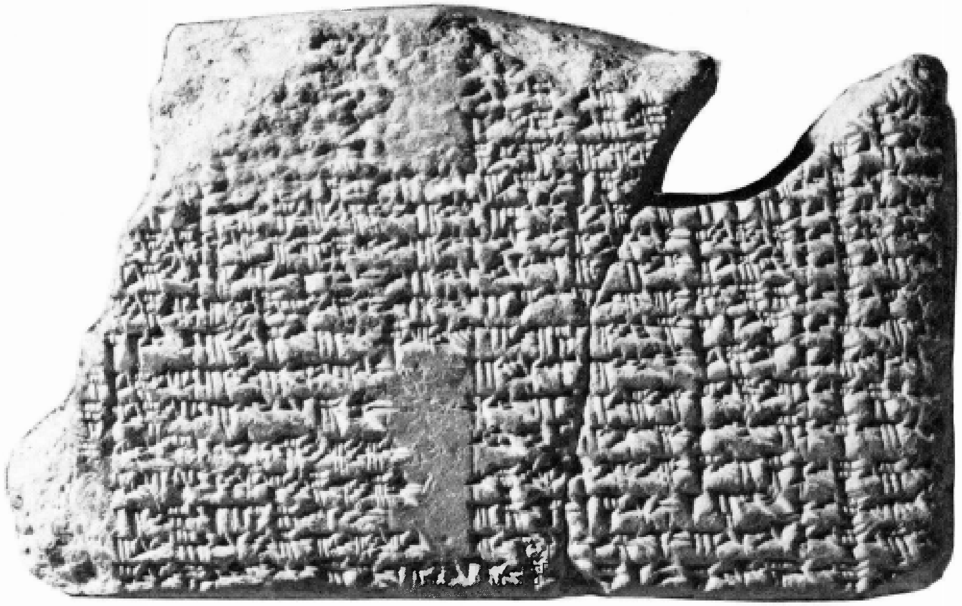
Rev. III,13: 3,29,11,*6*,40 should be 3,29,11,40, error without consequence.

Rev. III,18: the Λ-value should be denoted lal, isolated error.

Col.IV: Except in the first six lines of the obverse, the values are denoted lal instead of tab, perhaps in imitation of Col.J.

Rev. line 5: Col. IV,5 is empty; the Y-value should be 8,14,32,30 tab. In Col. V,5 the text has 8,12,35,30 tab; this should be 3,35,55 tab. In Col. VI,5 we read 3,36,45 with the final 5 damaged; this should be 0;7,29 tab. It is likely that the scribe copied from a text which, like his copy, occasionally ran the columns together, and in which line 5 of the reverse was damaged; and that he copied what he saw in

BM 40094, Rev.+ACT No. 128



the correct line, but shifted one column to the right. The join adds no new information about this line.

Rev. V,5: see note to Rev. line 5.

Rev. V,16: *3*,6,24 should be *2*,6,24, isolated error.

Rev. VI,3: reading uncertain.

Rev. VI,5: see note to Rev. line 5.

Rev. VI,10: the best, but far from secure reading is 4,12,23; the correct value 4,6,13 is not consistent with the traces.

The following errors in *Aaboe's* transcription and restoration of the obverse should be noted:

Obv. IV,5,6: these two lines should not be blank, but should both give 21,2,59 tab.

Obv. V,5: 1,[19,30 lal] should be 1,[16 lal].

Obv. V,9: 1,53,4[0 lal] should be 1,53,3[6 lal].

Obv. VI,5: [3,31,49 tab] should be [3,52,56 tab].

Obv. VI,6: [2,45,38 tab] should be [3,6,41 tab].

Commentary

When BM 40094 was first published it presented several singular features: it was the oldest datable lunar ephemeris, and it was the only lunar ephemeris that month by month gave the function Λ and its corrections, Y and $\tilde{c}$', for solar anomaly and change of length of daylight. Only here was found the column $\tilde{K}$, the sum of Λ, Y, and $\tilde{c}$', which represents the length of a normal year, except for an integral number of days, and adjusted to a variable sunset epoch. To these unique traits we now add one more: our text is the only lunar ephemeris that contains two nearly identical, but differently computed columns M (giving the moment of conjunction in large hours before sunset).

ACT No. 128 complements Column $\tilde{K}$ of BM 40094 and adds the second Column M and a Column P_3 which gives date of the last visibility of the Moon and the time interval from moonrise to sunrise on that morning. The inclusion of Column P_3 and the absence of a Column P_1 (first visibility) raise a problem we shall return to briefly below.

We are convinced that Column VII, the second Column M, is computed from Column VI ($\tilde{K}$) and an initial value in 12-line steps thus:

$$M(n) = M(n\text{-}12) - \tilde{K}(n).$$

The text provides only one instance where this can be checked:

$$\begin{array}{lll} \text{VII,6 : } M(6) & = & [5,26,3]6 \text{ šú} \\ \underline{-\text{VI,18:} -\tilde{K}(18)} & \underline{=} & \underline{\quad 5,\ 1} \\ \text{VII,18: } M(18) & = & 5,21,35 \text{ šú} \end{array}$$

where, alas, only the last digit of VII,6 is preserved.

Aaboe surmised in 1969 that a main purpose of Λ and its corrections is to provide a much needed control for Column M – a conglomerate of quite unrelated parts which is difficult to check with rules of the common kind – and this appears confirmed by the join.

Finally, to find the dates in Column VIII (P_3) and, for that matter, in Column II (M) – dates are omitted in the second Column M – we must know the character, full or hollow, of the relevant month, and this we learn from Column P_1, which is not in our text. Thus our text is not self-contained, and this information must have been drawn from elsewhere. It is at first sight welcome to have our evidence of the poorly understod visibility columns increased by the entries in Column P_3. This much seems clear, however, that it was very poorly computed – in lines 12, 15, and 16 even the date is wrong – so it looks at present as if this join offers little hope of advancing our understanding of how the visibility columns were constructed, and we shall here refrain from any further analysis of Column VIII.

[T_1]	[Φ_1]	[B_1]	[C_1]	[F_1]	[G_1]	[J_1]	[C'_1]	[$I(K_1)$]
Rev. S.E. -6 VII	2,9,31,40	20;16 ♏	2,35,53,36	14,2,12,11,15	3,3,57,46,40		+ 8,37,52	3,12,36
VIII	2,6,45,44,26,40	20;16 ♐	2,26,37,52	13,20,12,11,15	3,29,46,25,11,6,40		+ 4,37,52	3,34,24
IX	2,3,59,48,53,20	20;16 ♑	2,25,22,8	12,38,12,11,15	3,55,35,3,42,13,20		+ 37,52	3,56,13
X	2,1,13,53,20	20;16 ♒	2,32,6,24	11,56,12,11,15	4,21,23,42,13,20		- 3,22,8	4,18,2
XI	1,58,27,57,46,40	20;16 ♓	2,46,50,40	11,14,12,11,15	4,47,9,11,51,6,40		- 7,22,8	4,39,47
XII	1,59,53,53,20	18;48,45 ♈	3,5,52,30	11,35,57,11,15	4,54,18,16,17,46,40	- 44,15,18,30	- 9,30,55	4,0,32
-5 I	2,2,39,48,53,20	16;56,15 ♉	3,22,46,30	12,17,57,11,15	4,31,4,26,40	- 57,3,45	- 8,27	3,25,34
II	2,5,25,44,26,40	15;3,45 ♊	3,32,40,30	12,59,57,11,15	4,5,15,48,8,53,20	- 57,3,45	- 4,57	3,3,15
III	2,8,11,40	13;11,15 ♋	3,35,34,30	13,41,57,11,15	3,39,27,9,37,46,40	- 57,3,45	- 1,27	2,40,56
IV	2,10,57,35,33,20	11;18,45 ♌	3,31,28,30	14,23,57,11,15	3,13,38,31,6,40	- 57,3,45	+ 2,3	2,18,38
V	2,13,43,31,6,40	9;26,15 ♍	3,20,13,30	15,5,57,11,15	2,48,19,15,33,20	- 57,3,45	+ 5,37,30	1,56,53
VI	2,16,29,26,40	9;12 ♎	3,0,32	15,47,57,11,15	2,40	- 7,13,40,30	+ 9,50,45	2,42,37
VII	2,14,54,15,33,20	9;12 ♏	2,40,32	15,23,51,33,45	2,40		+ 10	2,50
VIII	2,12,8,20	9;12 ♐	2,28,19,12	14,41,51,33,45	2,43,0,33,20		+ 6,6,24	2,49,7
IX	2,9,22,24,26,40	9;12 ♑	2,24,6,24	13,59,51,33,45	3,5,24,11,51,6,40		+ 2,6,24	3,7,31
X	2,6,36,28,53,20	9;12 ♒	2,27,53,36	13,17,51,33,45	3,31,12,50,22,13,20		- 1,53,36	3,29,19
XI	2,3,50,33,20	9;12 ♓	2,39,40,48	12,35,51,33,45	3,57,1,28,53,20		- 5,53,36	3,51,8
XII	2,1,4,37,46,40	8;26,15 ♈	2,58,57,30	11,53,51,33,45	4,22,50,7,24,26,40	- 23,12,19,30	- 9,38,21	3,49,59

II(M_1)	III(Λ_1)	IV(Y_1)	V($\tilde{C}'_1$)	VI($\tilde{K}_1$)	VII(M_1)	VIII(P_3)
Rev. [1,55,33 šú]	[1,55,2,59,23,53,20		[2,12,48 lal]	[1,52,50 tab]	[1,55,35 šú]	[.. .. .. ..]
[4,21,9 šú]	1,4,52,7,32,46,40		44,16 lal	[1,4,8 tab]	[4,21,10 šú]	[..]
[24,56 šú]	14,41,15,41,40 tab		44,16 tab	15,26 [tab]	[24,58 šú]	[..]2,3[0 kur]
[2,6,54 šú]	19,1,17,46,40 lal		2,12,48 tab	16,48 la[l]	[2,7 šú]	[27] 27,50 kur 28 2[0 kur]
[3,27,7 šú]	4,21,•,52,46,40 lal	⟨8,14,32,30⟩	8,12,35,30 tab	3,36,55,[ ]	[3,27,12 šú]	27 15,20 kur
[5,26,35 šú	43,15,2,5 tab	21,2,59 lal	3,27,30 tab	1,7,46 tab]	[5,26,3]6 šú	28 14,30 kur
[2,1,]2 šú	1,33,25,53,58, ...	21,2,59 lal	2,4,30 tab	1,56,33 tab	2,1,3 šú	27 20,40 kur 28 2[0 kur]
[4],57,44 šú	1,23,36,45,47,13,20 tab	21,2,59 lal	41,30 tab	2,45,21 tab	4,57,48 šú	27 16,40 kur
[ ]2,16,50 šú	3,13,31,40 tab	21,2,59 lal	41,30 lal	3,33,52 tab	2,16,49 šú	27 23 kur
[ ] 5,58,12 šú	3,47,14,4,26,40 tab	21,2,59 lal	2,4,30 lal	4,12,23 tab	5,58,6 šú	27 14,30 kur
[28 4,1,19 šú	3,55,33,20 tab	7,13,40,30 lal	3,32,5 lal	3,59,15 tab	4,1,17 šú	27 23,40 kur
[2]8 1,18,42 šú	3,53,10,22,13,20 tab		3,41,20 lal	3,49,29 tab	1,18,41 šú	26 31,40 kur 27 12,1[0 kur]
[28 4,28,42 šú	3,29,11,6,40 tab		2,19,12 lal	3,26,52 tab	4,28,43 šú	27 18,10 kur
[28 1,49,5 šú	2,42,25,50,13,53,20 tab		50,40 lal	2,41,35 tab	1,39,35 šú	27 23,10 kur
[2]8 4,32,4 šú	1,52,14,58,22,46,40 tab		37,52 tab	1,52,53 tab	4,32,5 šú	27 16,30 kur
[ ] 1,2,46 šú	1,2,4,6,31,40 tab		3,6,24 tab	1,4,11 tab	1,2,49 šú	27 22,20 kur 28 12,1[0 kur]
[2]9 3,11,36 šú	11,56,23,33,53,20		3,34,56 tab	15,31 tab	3,11,41 šú	27 16,50 kur
[2]9 5,21,38 šú	19,29,26,40⟨lal⟩	21,2,59 lal	3,27,30 tab	5,1 tab	5,21,35 šú	28 1[..]0 kur

BM 40094, Rev. + ACT No. 128

Table 7.

Neugebauer assigned tentatively ACT No. 128 to System B, in error, it now appears. The absence of dates in its Column M, and of the usual difference column (K), were indeed puzzling out of the context provided by the join with BM 40094.

ACT No. 3b and BM 37375 (80-6-17,1132)
Contents:
Obverse: Cols. C_1,K_1,M_1,P_1,P_3; Reverse: Cols. $P_{2,1},P_{2,2},P_{2,3},P_{2,4}$ for part of S.E. 2,22.
Transcription: Table 8.
Description of Text:

Though too much clay is missing for a physical join, there can be no doubt that these two fragments derive from the same tablet. Clay, hand, size of writing, and shape and dimensions of the lower edge all agree. Further, both fragments have an intercalary month XII_2 in the last line of the obverse, and the moments of last visibility in BM 37375 all precede very nicely the corresponding moments of conjunction in No. 3b. The missing sliver contained most of the column of first visibility on its obverse, and the middle section of the me-column on its reverse.

It is very fortunate that No. 3b could be dated, for without a date BM 37375 would be quite uninteresting. As it is now, it adds new evidence about the difficult and imperfectly understood visibility columns. For a restoration of most of the missing columns we refer to the partial duplicate ACT No. 3a.

Critical Apparatus.

Obv. line 1': illegible traces of a second choice remain on the edge; the traces following the 11 in Col. V are conformal with 27 11,33 kur[as well as 27 11 kur 26[.

Rev. IV,4: 20 gar could be read 24.

ACT No. 53 (BM 34083 (Sp. 181))
Contents: Columns T_1,B_1, and Ψ_1 for solar eclipse possibilities for at least 216 years, including S.E. 137 to 343.
Arrangement: The text turns from obv. to rev. about the right edge, like a page of a book, which is unusual for Babylon.
Copy: LBAT No. 49.
Transcription: Table 9 (one side).
Description of Text:

Neugebauer restored the text in ACT under the assumption that it dealt with solar eclipses on its "obverse" and lunar eclipses on its "reverse" which is badly

Table 8.

			I		II	III				IV	V				Edge	
Obv. 1.'	2,22	VIII	[7, . ,48	tab]	[5 . ,20]	[apin	////	2,17,11	šú]	[clay missing]	[ap]in	27	11	⌜kur⌝⌜26⌝	[V///////////]	1.'
		IX	[3, . ,48	tab]	[4,51,38]	[gan	////	3,25,3]3	šú	[ ]	[ga]n	26	19,20	kur		
		X	[59,12	lal]	[4,2]2,4	ab	29	5,3,29	šú	[ ]	[a]b	27	17,20	kur		
		XI	[4,59,1]2	lal	3,52,16	ziz	28	1,11,13	šú	[ ]	[z]iz	27	13,40	kur		
5.'		XII	[8,52,2]7	lal	3,12,18	še	29	3,58,54	šú	[ ]50	še	28	11,20	kur	27 20,20 kur	5.'
		XII₂	[9,22,3]0	lal	1,59,12	dirig	28	1,59,42	šú	[ ]	dirig	27	16,30	kur		

			I			II			III		IV		
Rev. 1.	2,22	I	[1]4	1,50	šú	15	4,50	na	1[4 clay missing //]50	me	15	12,20	ge₆
		II	[14]	6,40	šú	15	2	na	1[4 //]9	me	15	3,20	ge₆
		III	[		šú	15	9	na	1[4 //]4	me	16	7,30	ge₆
		IV	[		šú]	[	2,1]0	na	[//]50	me	15	20 gar	ge₆
5.		V	[		šú]	[		na	[ ]3,40	me	15	2,10	ge₆
		VI	[		šú]	[		na]	[///]	⌜me⌝	16	4,20	ge₆

← ACT No. 3b → ← B.M. 37375 →

Table 9.

	[-I (T_1)]	[0 (B_1)]	I (Υ_1)		II (T_1)	III (B_1)	[IV (Υ_1)]	
1.'	2,18 XII	15,26,15 ♈	[20,5,49]	ḫab	dir	[25,33,45 ♈]	19,2,51	1.'
	2,19 VI	5,36 ♎	[19,30,1],3	ḫab	2,37 ⌜kin⌝	[16,24 ♎]	19,21,15	
	XII	5,3,45	[31,1]8,37	ḫab	še	15,[11,15 ♈]	30,15,39	
	2,20 VI	24,32 ♍	[29,4]7,41	ḫab	2,38 kin	5,[20 ♎]	29,38,43	
5.'	XI 5 áb	24,32 ♒	[13]51	ḫab	ziz 7 áb	5,[20 ♓]	4,53	5.'
	2,21 IV	15,18,45 ♌	[2]7,35	ḫab	2,39 kin	24,[16 ♍]	39,56,11	
	X	13,28 ♒	[10,13,1]9	ḫab	ab	24,[16 ♒]	10,22,21	
	2,22 IV	4,56,15 ♌	[11,40],23	ḫab	2,40 sig	15,[3,45 ♌]	10,37,25	
	X	2,24 ♒	[20,48,47]	ḫab	gan	13,[12 ♒]	20,39,49	
10.'	2,23 III	24,33,45 ♋	[22,5]3,11	ḫab	2,41 sig	4,[41,15 ♌]	21,50,13	10.'
	IX	21,20 ♉	[31],6,15	ḫab	gan	[2,8 ♒]	30,57,17	
	2,24 III	14,11,15 ♋	[34,5,5]9	ḫab	2,42 gu₄	[24,18,45 ♋]	33,3,1	
	IX	10,16 ♉	[41,23,43]	ḫab	apin	[21,4 ♉]	41,14,45	
	2,25 II 5 áb	5,41,15 ♊	[5,43,33	ḫab]	[2,4]3,[bar 5 áb]	[15,48,45 ♊]	4,40,35	

ACT No. 53 "Rev."

preserved. We shall suggest that the identification of obverse and reverse is wrong and that Side A, as we rename what *Neugebauer* called "reverse," presents solar eclipses preceding in time those given on Side B (ACT's "obverse"); *Neugebauer's* assumption that Side A concerned lunar eclipses led to severe difficulties. Incidentally, *Pinches's* original copy sheet agrees with our identification of obverse and reverse.[4]

We have nothing to change in *Neugebauer's* treatment of Side B; in fact, we shall use the peculiar rules he extracted from this side in our proposed restoration of Side A. Further, we shall rely heavily on *Pinches's* hand copy (LBAT No. 49) of this side.

The text was first noticed by *Strassmaier* (in June 1891, according to his notebook) and he says about the "reverse," as he also called our Side A: "*Rückseite sind nur Spuren von 3 Kolumnen so:* [copy of three lines follow] *u.s.w.; es ist nichts sicher, zu sehr beschädigt.*" In Col. II, 5' he read "zíz 5 áb," a reading *Neugebauer* followed, though *Pinches* here read "zíz 7 áb." For the year numbers in lines 2' and 4' *Strassmaier* and *Pinches* both give 2,37 and 2,38, while *Neugebauer* has 5,37 and 5,38. The photograph used by *Neugebauer*, and taken about 1950, supports the alternate readings equally well or ill and shows a text so damaged that we doubt that a collation would be of much help – in fact, we had not yet grown interested in the problems raised by this text when *Aaboe* collated some of the other tablets discussed here.

We mention these things to point to the fragility of the evidence. On the other hand, we have good reasons to trust the acuity of the remarkable *T. G. Pinches* when he had a difficult text in hand.

Critical Apparatus

The numbers given in Columns I and III of Table 9 are computed by us; any deviation from *Pinches's* copy is noted below.

I, 3': [31,1]8,37: *Pinches* read 33 in the last place.
II, 5': 7áb: so *Pinches; Strassmaier* and *Neugebauer* read 5 áb (see above).
II, 6': kin: *Pinches* gives a damaged dir; *Neugebauer* read šu.
II, 7': ab should be zíz. This error affects all subsequent dates in the text, including those of Side B; henceforth each month name is one too early.
III, 3': 15,[11,15]: *Pinches* read 13 in the first place, with bottom of the 3 damaged.

4. We are indebted to Professor *Neugebauer* for having placed at our disposal his photographs of the relevant ACT texts as well as copies of parts of *Strassmaier's* notebooks, with his transcription of *Strassmaier's* German shorthand, and of *Pinches's* original copy sheets.

III, 5': 5,[20]: *Pinches* read a damaged 8 in the first place.
III, 6: 24,[16]: *Pinches* read 26 in the first place with the top of the 6 damaged.
III, 7': 24,[16]: *Pinches* read 25 with damaged 5.

Commentary

We shall first briefly introduce the "eclipse magnitude" Ψ' which plays an essential role in this text. It is a simple transform of lunar latitude, E, usually defined only at syzygies where exlipse possibilities are announced, and computed according to the rule

$$\Psi' = 17;24 \begin{cases} +\frac{E}{6} \text{ near the ascending node} \\ -\frac{E}{6} \text{ near the descending node.} \end{cases}$$

Ψ' is measured in fingers (šu-si) while E's units are barley corn (še) – in a four-digit value of E the first two indicate the integral number of še. Closely related to Ψ' is Ψ'' which is defined for *all* syzygies. Ψ'' is a zig-zag function of extrema

$$M = -m = 2,0\ ,$$

but of differences that depend on longitude (for details see ACT, *Aaboe-Henderson* [1975], or HAMA). Near the nodes Ψ' and Ψ'' agree but for sign:

$$\Psi' = \begin{cases} \Psi'' \text{ near ascending node} \\ -\Psi'' \text{ near descending node.} \end{cases}$$

We can now state the results of *Neugebauer's* analysis of Side B. He proposed that it gave year and month, longitude, and Ψ' for solar eclipse possibilities, arranged in columns of 38 lines (corresponding to one Saros) each, selected according to an unusual principle, but with an error of one month in the dates.

Normally we find, as mentioned, eclipse possibilities selected on the basis of Column E, lunar latitude: they are the syzygies at which lunar latitude, when listed monthly, near a sign change has smaller absolute value. Here, however, the rule is that one selects the syzygy at which Ψ' for the first time becomes positive or, what is the same thing, the syzygy just after Ψ'' has changed sign. The E-rule and the Ψ'-rule, as we call them, agree often, but not always.

We have reconstructed Side A on the basis of the Ψ'-rule. This rule implies an occasional, but very rare interval of 7 months, flanked by two 5-month intervals, between consecutive "eclipse possibilities" (which the more correct E-rule also does, but at different syzygies). The last time this happens during the historically relevant period is at the conjunctions S.E. 138 VI, 138 XI, 139 VI, and 139 XI. These are covered in our text, and *Pinches* read 7 áb (7 months) where we would normally expect 5 áb. It also appears that it is in this part of the text that the error in month names begins. The month names še (XII), kin (VI), zíz (XI) in lines 3', 4', and 5' are clear and correct. On line 6' *Pinches* copies a damaged dir (XII_2) which makes no sense, but here one should probably read kin (VI) and thus obtain the 7-month interval. In line 7' we expect zíz (XI), but find ab (X), and from there onward all months are one too early. If *Neugebauer's* reading of šu (IV) in line 6' is followed we would have two consecutive 5-month intervals followed by 6-month intervals, which is also a possible reconstruction.

That the Ψ'-rule is employed is shown by line 6' where we find 2,39 kin (VI) and line 13' where we find apin (VIII). In both cases the E-rule would select the previous conjunction (we are allowing for the systematic error in the latter instance).

The Ψ'-rule makes little astronomical sense as a guide for choosing eclipse possibilities. It has in practice the effect of advancing the nodes by 2;17° on the average, which is gross compared with the error in Column E (about $\frac{1}{2}^{\circ}$ in S.E. 200; see *Aaboe-Henderson* [1975]). It may, however, be we who are wrong when we identify our tablet as an eclipse text, for it could equally well be an auxiliary table simply presenting a rational selection of corresponding values of T, B, and Ψ' for a purpose we cannot quite see. Indeed, the term ḫab can mean "eclipse magnitude" or "positive value of Ψ'" as well as "eclipse warning" (see Excursus: The Term ḫab-rat, ACT p. 197).

ACT No. 207f (BM 42685)
Contents: Column Φ_1 tabulated at 12-month intervals, very likely beginning S.E. 200, III.
Transcription: Table 10.
Photograph: ACT Plate 243.

Critical Apparatus

line 3': the 5 is fairly clear, and 0 is certainly excluded, so the numbers are values of Φ_1, not Φ_2.

line 11': *Neugebauer* read 42,14. The 42 could be 52 in which case all dates are advanced 265 months.

Commentary

The preserved endings agree with the endings in Column Φ_1 in ACT No. 55, Reverse, lines 8'-18' (restored until line 10' in *Aaboe* [1971]). Dating is unique if 42 in line 11' is taken seriously, otherwise see Critical Apparatus.

Table 10.

[T_1]		Φ_1	
3,20	III	[2, 3, 5],44,26,[40	tab]
3,21	III	[1,57,5]2,46,40	[lal]
3,22*	III	[2, 3, 1]5,22,13,20	lal
3,23	II	[2, 8,37],57,46,40	lal
3,24	II	[2,14, 0],33,20	lal
3,25*	II	[2,14,46],28,53,20	tab
3,26	I	[2, 9,23]53,20	tab
3,27	I	[2, 4, 1]17,46,40	tab
3,28**	I	[1,58,38]42,13,20	tab
	XII	[2, 2,19],48,5[3,20	lal]
3,29	XII	[2, 7],42,24[26,40	lal]

ACT No. 207f

ACT Nos. 22 and 23

We finally present in Tables 11 and 12 reconstructions of ACT Nos. 22 and 23 which were dated from preserved or reconstructed sections of Column K. Incidentally, ACT No. 20 was similarly dated and reconstructed, and published in *Aaboe* [1974].[5] Our dating of No. 22 – unique within the relevant period like the other datings – confirms *Neugebauer's* remark (ACT p. 104) that its Column K_2 is a close parallel to that in No. 15.

Appendix

BM 37021 (80-6-17,765)

Just after our manuscript was submitted we managed to date the text BM 37021. Since it presents two points of particular interest, we shall briefly describe it and its contents here, while the text will be published fully later in another context.

BM 37021 is from the Babylon archive and is a fragment measuring 7.5 cm by 14 cm. Upper and lower, but no other edges are preserved. When the tablet was first seen by Aaboe in 1963 it was quite illegible, but after it was baked and cleaned a few signs could be made out, though its surface remains in a wretched state. The upper edge carries a colophon stating that the text was copied from a wax tablet.

The text has 10 lines to a side and presents in pairs of columns, that continue

5. Nos. 20 and 23 were first dated in 1972 with the kind help of Mr. *C. Anagnostakis*.

from the Obverse across the lower edge to the Reverse, corresponding values of K_1 and M_1 for a long sequence of unidentified months. Indeed, of Column M_1 only the *hours* are given, denoted šú (before sunset). The term BAL indicates when an extra day has been included as 6^H in Column M_1, a useful warning when one computes the dates of Column M_1 (for which Column P_1 is needed). BAL occurs in the lines immediately after the borrowing of a whole day.

Of Column I remain only the endings of M_1, mostly šú and šú BAL, but we could date the three values of K_1 preserved in Column II, lines 2, 3, and 4 uniquely to S.E. 46, months X, XI, and XII with our computer-generated tables. The remaining traces of K_1-values are, with one exception, consistent with our re-computation, and the dating is secure.

Column I, line 1, would then correspond to S.E. 45* month II, and we believe this to be the beginning of the text. There is evidence of entries for 60 months, and the tablet's curvature suggests that when unbroken it covered 100 months, or some 8 years.

Two features of the text are noteworthy: its early date and the fact that it is an auxiliary table of System A giving selected columns of monthly values, very likely for the purpose of constructing ephemerides. Such auxiliary tables of System B from Uruk abound, but so far the only text of this sort of System A was ACT No. 70. This text from Babylon is also the oldest dated lunar text in ACT (S.E. 49 to 60), but otherwise the lunar texts of System A are on the whole younger than those of System B. We do not know how much of this apparent trend is due to accidents of the spade, scientific and otherwise, but the early dates of the contents of BM 40094 + ACT No. 128 (see above) and of the present text will surely have to be taken into account in attempts at reconstructing the relationship between the lunar Systems A and B.

Table 11.

	[T_1]	[B_1]	[C_1]	[E_1]	[F_1]	[G_1]	[J_1]	[C_1']	[K_1]	I (M_1)	II and Edge (P_1 & P_3)
Obv.	209 XII	10,3,45 ♈	3,0,2,30	- 5,47,39,57	13,34,32,48,45	3,20,57,31,51,6,40	- 26,30,8,30	- 9,42,35	2,44,44	[ ] šú]	[bar]13 kur
	210 I	8,11,15 ♉	3,18,47,30	- 3,48,54,15	12,52,32,48,45	3,46,46,10,22,13,20	- 57,3,45	- 9,22,30	2,40,20	[ ] šú]	[gu₄]be 27 17,1[0 ku]r
	II	6,18,45 ♊	3,30,31,30	- 1,16,17,6	12,10,32,48,45	4,12,34,48,53,20	- 57,3,45	- 5,52	3,9,39	[ ] šú]	[sig] 16,16 26 19,30 be
	III	4,26,15 ♋	3,35,15,30	+ 2,32,37,9	11,28,32,48,45	4,38,23,27,24,26,40	- 57,3,45	- 2,22	3,38,57	[ ] šú]	[šu]be 27 16,40 kur
	IV	2,33,45 ♌	3,32,59,30	+ 4,31,22,51	11,21,36,33,45	4,56,35,33,20	- 57,3,45	+ 1,8	4,0,39	[ ] šú	i[zi] 1 13[ ] 28 12 [k]ur 9 be 27 24
	V	41,15 ♍	3,23,43,30	+ 6,30,8,33	12,3,36,33,45	4,39,53,20	- 57,3,45	+ 4,38	3,47,27	[ ] šú	ki[n] 1 17 28 22 kur 7 be 20 be
	VI	29,52 ♍	3,6,45,20	+ 5,50,52,45	12,45,36,33,45	4,14,4,41,28,53,20	- 24,58,50,30	+ 8,29,5	3,57,34	[ ] šú	[du₆ 1] 16,40 tab ina 30-šú 10 uš 9 be 27 15
	VII	29,52 ♎	2,46,45,20	+ 3,44,37,3	13,27,36,33,45	3,48,16,2,57,46,40		+ 10	3,58,16	[ ]6,9 šú	[apin 3]0 1[ ] 27 26 kur [b]e
	VIII	29,52 ♏	2,32,3,12	+ 52,42,42	14,9,36,33,45	3,22,27,24,26,40		+ 7,21,4	3,29,48	[ ]6,21 šú	gan 1 17 [
	IX	29,52 ♐	2,25,21,4	- 2,51,54,21	14,51,36,33,45	2,56,38,45,55,33,20		+ 3,21,4	2,59,59	[,22 šú	ab 30 12[
	X	29,52 ♑	2,26,38,56	- 4,58,10,3	15,33,36,33,45	2,40,15,22,13,20		- 38,56	2,39,36	[šú]	[zíz 1

	[T_2]	[B_2]	[C_2]	[E_2]	[F_2]	[G_2]	[J_2]	[C_2]	I(K_2)	II (M_2)
Rev.	210 IV	18,30 ♑	3,34,52	- 3,32	15,53,26,15	2,40	- 57,3,45	- 44,30	[1,42,11]	[šu … 4,42,19 du]
	V	16,37,30 ♒	3,29,21	- 5,30,45,42	15,18,22,30	2,40	- 57,3,45	+ 2,45,30	[1,45,41]	[izi 1 28,0 d]u
	VI	14,52 ♓	3,16,45,20	- 6,54,0,36	14,36,22,30	2,44,53,20	- 53,30,43	+ 6,17,50	[1,57,40]	[kin 1 2,25 4]0 du
	VII	14,52 ♈	2,56,45,20	- 4,47,44,54	13,54,22,30	3,8,46,25,11,6,40		+ 10	[3,18,46]	[du₆ 1 5,4]4,26 du
	VIII	14,52 ♉	2,38,3,12	- 2,41,29,12	13,12,22,30	3,34,35,3,42,13,20		+ 9,21,4	[3,43,56]	[apin 30 14]3,28,22 du
	IX	14,52 ♊	2,27,21,4	+ 1,13,33	12,30,22,30	4,0,23,42,13,20		+ 5,21,4	[4,5,44]	[gan 1 14 1,34,6 du
	X	14,52 ♋	2,24,38,56	+ 3,55,2,12	11,48,22,30	4,26,12,20,44,26,40		+ 1,21,4	[4,27,33]	[a]b 30 15 .,1,39 du
	XI	14,52 ♌	2,29,56,48	+ 6,1,17,54	11,6,22,30	4,51,10,37,2,13,20		- 2,38,56	[4,48,3]2	zíz 1 14 4,50,11 du
	XII	14,52 ♍	2,43,14,40	+ 6,16,26,24	11,43,46,52,30	4,51,13,20		- 6,38,56	[4,44,3]5	še 30 15 3,34,46 du
	XII₂	13,45 ♎	3,2,30	+ 4,14,38,42	12,25,46,52,30	4,26,15,48,8,53,20	- 33,59,2	- 9,37,40	[3,42,3]9	dirig 30 16 1,17,25 du
	211 I	11,52,30 ♏	3,20,45	+ 2,7,46	13,7,46,52,30	4,0,27,9,37,46,40	- 57,3,45	- 9,7,30	[2,54,1]6	bar 1 15 4,11,41 du

ACT No. 22

Table 12.

[T_1]	[B_1]	[C_1]	[E_1]	[F_1]	[G_1]	[J_1]	I (C_1')	II (K_1)	III (M_1)	IV (P_1)
152 III	15,33,45 ♋	3,35,15,30	- 1,28,59,30	15,34,4,41,15	2,40	- 57,3,45	1,8 lal]	[1,41,48]	[sig … 2,56,28 šú]	[šu …]
IV	13,41,15 ♌	3,30,31,30	+ 2,26,15,57	14,52,4,41,15	2,40,44,48,53,20	- 57,3,45	2,22 tab]	[1,46,3]	[šu …]1,10,25 [š]ú	iz[i] 10,1[0 …]
V	11,48,45 ♍	3,18,47,30	+ 4,25,1,39	14,10,4,41,15	2,59,7,24,26,40	- 57,3,45	5,52 tab]	[2,7],55	izi 29 5,2,30 šú	kin [1] 10,2[0 …]
VI	11,44 ♎	2,58,50,40	+ 6,30,58,21	13,28,4,41,15	3,24,56,2,57,46,40	- 2,24,33,30	9,58,25 tab]	[3]32,30	kin 28 1,30 šú	du₆ [30] 13[ ]
VII	11,44 ♏	2,39,18,24	+ 5,46,45,57	12,46,4,41,15	3,50,44,41,28,53,20		9,46,8 tab]	4,-,30	du₆ 29 3,29,30 šú	apin 1 16[ ]
VIII	11,44 ♐	2,27,46,8	+ 3,40,30,15	12,4,4,41,15	4,16,33,20		5,46,8 tab]	4,22,19	apin 29 5,7,11 šú	g[an] 1 18..[ ]
IX	11,44 ♑	2,24,13,52	+ 44,29,6	11,22,4,41,15	4,42,21,58,31,6,40		1,46,8] tab	4,44,7	gan 28 .,23,4 šú	[ab 1] 29,1[0]
X	11,44 ♒	2,28,41,36	- 2,56,1,9	11,28,4,41,15	4,56,15,11,6,40		2,13,52 [l]al	4,54,1	ab 28 1,29,3 šú	[zí]z 1 22[ ]
XI	11,44 ♓	2,41,9,20	- 5,2,16,51	12,10,4,41,15	4,35,54,48,53,20		6,13,52 lal]	[4,29,41]	z[íz .. 2,59,22 š]ú	[še]
XII	10,48,45 ♈	3,0,32,30	- 7,4,51,33	12,52,4,41,15	4,10,6,10,22,13,20	- 28,1,26,30	9,41,35 lal]	[3,32,23]	[še .. 5,26,59 šú]	[ ]

ACT No. 23

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