$$G_{\text{min.}} = \text{ca. } 0{,}50 \cdot G_{1,0} \quad \text{bei} \quad \left(\frac{h}{\lambda}\right)_{\text{min.}} = \text{ca. } 1\,,$$

in guter Übereinstimmung mit der hier entwickelten Theorie sind, und hierdurch auch ihre einwandfreie Erklärung erhalten.

Auch bei dieser Gelegenheit möchte ich der Direktion des Dänischen Carlsbergfonds meinen aufrichtigen Dank für gewährte Stütze aussprechen. —

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ON THE PSEUDOSCALAR INTERACTION IN β-DECAY

BY

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Assuming the β-decaying nucleon to move in a simple scalar potential, the Foldy-Wouthuysen transformation is used to obtain an equivalent non-relativistic form for the pseudoscalar interaction. The effect of the recoil of the nucleus has been taken into account. The observed shapes of allowed β-spectra permit an estimate of an upper limit to the pseudoscalar coupling constant, which is difficult to reconcile with the value derived from the analysis by PETSCHEK and MARSHAK of the Ra E spectrum. This conclusion is dependent on the assumption of a simple nuclear potential.

The pseudoscalar β-interaction differs from the other four types of interactions in not possessing any non-relativistic analogy. The matrix elements therefore may depend essentially on the nuclear forces acting upon the decaying particles, and great care is needed in the derivation of equivalent non-relativistic forms which permit a comparison with other β-decay interactions. The significance of the pseudoscalar interaction for the shape of certain forbidden spectra has recently been suggested[1)].

The contribution of the pseudoscalar interaction to β-transitions may in the plane wave approximation for the leptons be expanded as follows:

$$\langle f|\beta\gamma_5 L|i\rangle = \langle f|\beta\gamma_5|i\rangle L_0 + \frac{1}{6}\langle f|\beta\gamma_5 r^2|i\rangle(\Delta L)_0 \quad (1.1)$$

$$+\langle f|\beta\gamma_5 r_i|i\rangle(\nabla_i L)_0 \quad (1.2)$$

$$+\frac{1}{2}\langle f|\beta\gamma_5\left(r_i r_k - \frac{1}{3}\delta_{ik} r^2\right)|i\rangle(\nabla_i\nabla_k L)_0 \quad (1.3)$$

where $|i\rangle$ and $|f\rangle$ are the initial and final states of the transforming nucleus and L is the lepton covariant $\psi_e^*\beta\gamma_5\varphi_\nu$. The index 0 means that the functions will be evaluated at the position

1) A. G. PETSCHEK and R. E. MARSHAK, Phys. Rev. 85, 698 (1952).

of the nucleus. The operator Q, which transforms neutrons into protons, is omitted in the following. The first two terms (1.1) contribute to first forbidden $\Delta J = 0$ (yes) transitions. The third (1.2) contributes to allowed transitions $\Delta J = 0, \pm 1$ (no), whereas the fourth contributes to first forbidden $\Delta J = 0, \pm 1, \pm 2$ (yes) transitions.

An especially well suited method for estimating the nuclear matrix elements in (1) is provided by the Foldy-Wouthuysen transformation[2) 3)]. The Foldy-Wouthuysen transformation also proves very useful for the evaluation of other relativistic matrix elements which occur in β-theory. The approximations made by Ahrens and Feenberg[4)] in a similar evaluation stand out clearly when this method is used.

The Hamiltonian to be transformed is

$$H = H_0 + H_{\text{int}} + H_\beta + H_0^{\beta\nu} \tag{2}$$

where H_0 is the free particle Hamiltonian for the nucleons $\sum_{i=1}^{A} (-\vec{\alpha}\cdot\vec{p} - \beta M)_i$, H_{int} is the nucleon-nucleon interaction, H_β is the β-interaction $\sum g_5 \beta\gamma_5 L$, and $H_0^{\beta\nu}$ is the free lepton Hamiltonian. The usual nucleon-nucleon interactions do not contain odd operators, i. e., operators which mix large and small components.

Treating $H_\beta + H_0^{\beta\nu}$ as a perturbation it is easy (cf. ref. 2) to construct a unitary transformation e^S which transforms $H_0 + H_{\text{int}}$ into a Schrödinger Hamiltonian:

$$S = \frac{\beta}{2M}\sum \vec{\alpha}\cdot\vec{P} + \frac{1}{4M^2}\left[\sum \vec{\alpha}\cdot\vec{P},\ H_{\text{int}}\right] - \frac{\beta}{6M^3}\left(\sum \vec{\alpha}\cdot\vec{P}\right)^3. \tag{3}$$

If we take H_{int} simply to be a scalar central potential βV we get the non-relativistic equivalent of the pseudoscalar matrix element:

2) L. L. Foldy and S. A. Wouthuysen, Phys. Rev. **78**, 29 (1950). This method was first applied to β-decay by Herbst and Bushkovitch, Phys. Rev. **91**, 442 (1953).

3) Previous attempts have been made by T. Ahrens, E. Feenberg and H. Primakoff, Phys. Rev. **87**, 663 (1953) and T. Ahrens, Phys. Rev. **90**, 974 (1953) to express the relativistic matrix element $\langle f|\beta\gamma_5 L|i\rangle$ in terms of the non-relativistic matrix elements. The results are essentially different from those obtained here, and it also appears that some of the approximations involved cannot be justified.

4) T. Ahrens and E. Feenberg, Phys. Rev. **86**, 64 (1952).

$$\left.\begin{aligned} \langle f|\beta\gamma_5 L|i\rangle \approx \frac{i}{2M}\langle f|\vec{\sigma}\cdot\vec{\nabla} L|i\rangle + \frac{i}{4M^2}\langle f|L\vec{\sigma}\cdot\vec{r}\,\frac{1}{r}\frac{dV}{dr}|i\rangle \\ - \frac{i}{4M^2}\langle f|(E_f - E_i - V_f + V_i) L\vec{\sigma}\cdot\vec{\nabla}|i\rangle. \end{aligned}\right\} \tag{4}$$

Here we have omitted terms of the order $\frac{V}{M^2}$ in the first term. Also the last term having the same selection rules as the second term will be neglected in the following.

If one transforms the total Hamiltonian H into a Schrödinger Hamiltonian one will get an additional term from $H_0^{\beta\nu}$. However, this term vanishes since it is of the form[5)]

$$\langle f|[H, [\sum \vec{\alpha}\cdot\vec{p},\ \sum g_5\beta\gamma_5 L]]|i\rangle. \tag{5}$$

Both methods are thus identical, and this result can be used to show that one need not care about the operator Q. The formula (4) applies as well to the scalar potential βV as to the static part of the vector potential V.

In the plane wave approximation (4) becomes

$$\langle f|\beta\gamma_5 L|i\rangle \approx$$

$$\frac{i}{4M^2}\langle f|\vec{\sigma}\cdot\vec{r}\,\frac{1}{r}\frac{dV}{dr}|i\rangle (L)_0 + \frac{i}{6M}\langle f|\vec{\sigma}\cdot\vec{r}|i\rangle(\Delta L)_0 \tag{6.1}$$

$$+ \frac{i}{2M}\langle f|\sigma_i|i\rangle(\nabla_i L)_0 + \frac{i}{4M^2}\langle f|\vec{\sigma}\cdot\vec{r}\,\frac{1}{r}\frac{dV}{dr} r_i|i\rangle(\nabla_i L)_0 \tag{6.2}$$

$$+ \frac{i}{4M}\langle f|\sigma_i r_k + \sigma_k r_i - \frac{2}{3}\delta_{ik}\vec{\sigma}\cdot\vec{r}|i\rangle(\nabla_i\nabla_k L)_0 \tag{6.3}$$

where the terms are ordered according to selection rules, the second term in (6.1) being of the order of magnitude of the (6.3) term. In (6.3) only the major term is included.

The terms (6.1) correspond to the first terms (1.1) in (1). The terms (6.2) correspond to (1.2), and so on. This correspondence can also be seen directly by evaluating the matrix elements using single particle Dirac wave functions for the nucleons.

The relative order of magnitude of the terms (6.1) can also

5) This was pointed out to us by L. L. Foldy.

be found in this simple case. The result for a square well potential is

$$\frac{\frac{i}{4M^2}\langle f|\vec{\sigma}\cdot\vec{r}\,\frac{1}{r}\frac{dV}{dr}|i\rangle}{\frac{i}{6M}\langle f|\vec{\sigma}\cdot\vec{r}\,|i\rangle} \approx \frac{\langle f|\beta\gamma_5|i\rangle}{\frac{i}{6M}\langle f|\vec{\sigma}\cdot\vec{r}\,|i\rangle} \approx E^2 \tag{7}$$

where E is the kinetic energy of the decaying nucleon. This means that for low Z the ratio of the first term to the second in (6.1) is of the order of $E^2/(\vec{p}+\vec{q})^2$. For heavy nuclei, however, this ratio will tend towards $E^2/(\alpha Z/2\varrho)^2$, that is the terms will be of nearly the same order of magnitude. By ϱ we denote the nuclear radius, $\vec{p}$ is the electron and $\vec{q}$ the neutrino momentum.

As regards the absolute magnitude of the matrix elements we find for the square well potential

$$\langle f|\beta\gamma_5|i\rangle \approx \frac{E^2}{\varrho M^2} \tag{8}$$

which for heavy elements is of the order of magnitude 10^{-4}. The smallness of this matrix element reveals the peculiarity of the pseudoscalar coupling. Furthermore, for the ratio of the two terms in (6.2) we find

$$\frac{\frac{1}{2M}\langle f|\vec{\sigma}\cdot\vec{r}\,\frac{1}{r}\frac{dV}{dr}r_i|i\rangle}{\langle f|\sigma_i|i\rangle} \approx \frac{2E}{M}. \tag{9}$$

The term $\langle f|\vec{\sigma}\cdot\vec{r}\,\frac{1}{r}\frac{dV}{dr}\left(r_i r_k - \frac{1}{3}\delta_{ik}r^2\right)|i\rangle(\nabla_i\nabla_k L)_0$ is omitted in (6.3), because of the smallness of this ratio.

These considerations are limited to simple potentials. RUDERMAN[6] has shown that in the case of pseudoscalar meson theory with pseudoscalar coupling $\langle f|\beta\gamma_5|i\rangle$ may be large.

For the neutron decay the series expansion in (1) and (6) is not appropriate. The β-spectrum of the neutron[7] may be calculated from either (1) or (6) by using plane waves for all particles. The inclusion of the momentum difference between the neutron and the recoiling proton is essential to get a non vanishing matrix element.

6) M. RUDERMAN, Phys. Rev. 89, 1227 (1953).
7) L. MICHEL, Proc. Phys. Soc. 63A, 514 (1950).

Thus one might expect that recoil effects add essentially to the contribution to allowed transitions from the pseudoscalar interaction. This is, however, not the case as can be seen in the following way. To calculate the recoil effect the true final state, $\langle f|$, must be represented in the rest system of the daughter nucleus, where the state is represented by $\langle f'|$:

$$\langle f| \approx \langle f'|\left(1 - iM\vec{v}_R\sum_{i=1}^{A}\vec{r}^{(i)}\right). \tag{10}$$

Here the recoil momentum is $MA\vec{v}_R$ and the centre of mass coordinate is $\frac{1}{A}\sum_{i=1}^{A}\vec{r}^{(i)}$. After the substitution of (10) into (6), operators acting on two different particles have been neglected. Conservation of momentum requires that

$$\vec{v}_R = -\frac{1}{MA}(\vec{p}+\vec{q}) \tag{11}$$

and the result of operation with $\vec{v}_R$ on the lepton covariant L is

$$\vec{v}_R L = -\frac{i}{MA}\vec{\nabla}L. \tag{12}$$

Thus we obtain

$$\langle f|\beta\gamma_5 L|i\rangle \approx$$

$$\frac{i}{4M^2}\langle f'|\vec{\sigma}\cdot\vec{r}\,\frac{1}{r}\frac{dV}{dr}|i\rangle(L)_0 + \frac{i}{6M}\left(1-\frac{1}{A}\right)\langle f'|\vec{\sigma}\cdot\vec{r}\,|i\rangle(\varDelta L)_0 \tag{13.1}$$

$$+\frac{i}{2M}\langle f'|\sigma_i|i\rangle(\nabla_i L)_0 + \frac{i}{4M^2}\left(1-\frac{1}{A}\right)\langle f'|\vec{\sigma}\cdot\vec{r}\,\frac{1}{r}\frac{dV}{dr}r_i|i\rangle(\nabla_i L)_0 \tag{13.2}$$

$$+\frac{i}{4M}\left(1-\frac{1}{A}\right)\langle f'|\sigma_i r_k + \sigma_k r_i - \frac{2}{3}\delta_{ik}\vec{\sigma}\cdot\vec{r}\,|i\rangle(\nabla_i\nabla_k L)_0. \tag{13.3}$$

This may also be seen directly in the case of single particle wave functions for the nuclei. In the low velocity approximation for the Lorenz transformation we get

$$\begin{aligned}\langle f| &\approx \langle f'|\exp\left(-\frac{\vec{v}_R}{2}\sum_{i=1}^{A}\vec{\alpha}^{(i)}\right)\exp\left(-iM\vec{v}_R\sum_{i=1}^{A}\vec{r}^{(i)}\right)\\ &\approx \langle f'|\left(1-\frac{\vec{v}_R}{2}\sum_{i=1}^{A}\vec{\alpha}^{(i)}-iM\vec{v}_R\sum_{i=1}^{A}\vec{r}^{(i)}\right).\end{aligned} \tag{14}$$

The first exponential transforms the spinors while the second transforms the space part of the phase. By the insertion of (14) into (1) and the identification of corresponding terms in (1) and (6) one is again led to (13).

For the neutron the series expansion of the last factor in (14) is not appropriate, and the only contribution will arrive from the recoil term

$$\langle f'|\exp\left(-iM\vec{v}_R\cdot\vec{r}\right)\beta\vec{\sigma}\cdot\vec{\nabla}L|i\rangle$$

since the term

$$\langle f'|\beta\gamma_5\exp\left(-iM\vec{v}_R\cdot\vec{r}\right)L|i\rangle$$

vanishes for free particles.

As specific applications of the above estimates of the pseudoscalar matrix elements we consider the relatively simple cases of allowed transitions with maximum spin change, and first forbidden $0\rightarrow 0$ (yes) transitions. In these cases one may restrict the calculations to a mixture of tensor and pseudoscalar interactions.

The shape of allowed β-spectra can easily be found from (13) and has been evaluated for $\Delta J=\pm 1$ (no) transitions. The second term in (13.2) has been omitted. In the plane wave approximation we get

$$\begin{aligned}P(W)\,dW &= \frac{1}{(2\pi)^3}q^2 p\,W dW g_3^2\left|\int\beta\vec{\sigma}\right|^2\\ &\left\{1+\left[\frac{p^2+q^2}{3}-\frac{2}{9}\frac{p^2 q}{W}\right]\left(\frac{g_5/g_3}{2M}\right)^2-\frac{2}{3}\left(\frac{p^2}{W}-q\right)\frac{g_5/g_3}{2M}\right\}\end{aligned} \tag{15}$$

where g_3 and g_5 are the tensor and pseudoscalar coupling constants respectively. It is seen that the main effect of the pseudoscalar interaction is proportional to $(p^2/W-q)$ and therefore large for large maximum β-energies. Thus disintegrations like the He^6 and B^{12} decays provide the most sensitive tests.

For $(g_5/g_3)/2M=\pm 1/20$ the spectrum (15) has been compared with the experimental He^6 spectrum[8] in Fig. 1. Such a

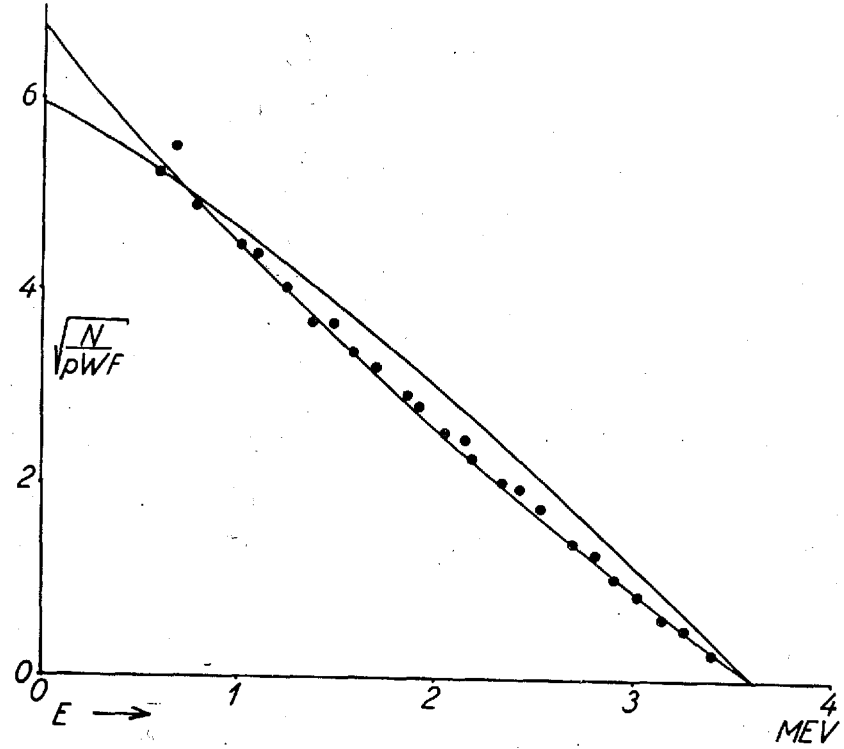

Fig. 1. Comparison of the spectrum (15) for $\frac{g_5/g_3}{2M}=\pm 1/20$ (full drawn curves) with the experimental data for He^6. The data are fitted best possibly to the curve corresponding to $(g_5/g_3)/2M=+1/20$.

value of g_5 would lead to a significant deviation from the observed spectrum and the figure shows that

$$|g_5/g_3|\lesssim 100$$

represents an upper limit to g_5.

[8] C. S. Wu, B. M. Rustad, V. Perez-Mendez and L. Lidofsky, Phys. Rev. 87, 1140 (1952).