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CONSTRUCTION AND CALCULATION OF A VARIABLE ACOUSTIC IMPEDANCE

BY

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Introduction.

In the field of electricity, pure ohmic, inductive and capacitive resistances can rather easily be constructed; similar possibilities are, however, absent within acoustics. Here, the acoustic values corresponding to ohmic resistance, inductance and capacity rarely occur alone, *i. e.* independently of one another, but mostly interdependent, *viz.* the change of one of them involves changes of one or all others. The need of an acoustic standard became a claim after SCHUSTER's construction of an acoustic Wheatstone bridge[1]. Also at Breslau, in WAETZMANN's laboratory, a variable acoustic impedance[2] seems to have been constructed which, however, was troublesome to handle and which, moreover, was only approximately correct; its description never was published. The impedance used by SCHUSTER in his bridge is continuously variable; however, its calculation is rather complicated[3] and it is difficult to make it comprise both the small absorption coefficients from 0 to 10 % and the great ones exceeding 90 %; finally, its reactive part (the felt tube) and its ohmic part (the piston tube) do not work quite independently of one another. A variable, radiation-damped acoustic impedance including the impedance values which probably exist for the human ear was earlier suggested by THORSEN[4].

In the following, the writer wishes to account in detail for the construction of a variable acoustic impedance which is relatively simple both in its mode of action and its manipulation and

1) K. SCHUSTER: Phys. Zt. **35**, 408, 1934.
2) K. SCHUSTER: E. N. T. **13**, 164, 1936.
3) K. SCHUSTER and W. STÖHR: Akust. Ztschr. **4**, 253, 1939.
4) V. THORSEN: D. Kgl. Danske Vidensk. Selskab, Mat.-fys. Medd. XX, 9, 1943, (in the following denoted as Essay I).

which, furthermore, covers a large range of absorption, *viz.* that from practically 100 % nearly down to 0 %. Besides, it is almost purely radiation-damped, whence its damping is easily measurable, for example, with a condenser microphone. This is of importance not only when its calculations are to be controlled, but it will likewise increase its range of applicability.

1. The Tube as Acoustic Impedance.

According to the theory of the acoustic tube-line, every smooth tube without dissipation represents an acoustic impedance of the magnitude

$$\frac{Z_i}{\varrho c} = \frac{\frac{Z_u}{\varrho c}\cos kl + i\sin kl}{\cos kl + i\frac{Z_u}{\varrho c}\sin kl}\text{.}^{1)} \tag{1}$$

Here, $\frac{Z_i}{\varrho c}$ is the inlet impedance, *i. e.* the impedance at the mouth of a tube of length l, which at the other end is terminated by the impedance $\frac{Z_u}{\varrho c}$ (the outlet impedance). When introducing the amplitude reflection coefficient r of the outlet impedance, which is determined by

$$re^{i\vartheta} = \frac{\frac{Z_u}{\varrho c} - 1}{\frac{Z_u}{\varrho c} + 1},$$

and its phase change ϑ, (1) may also be written

$$\frac{Z_i}{\varrho c} = \frac{1 - r^2 - i\,2\,r\sin(2\,kl - \vartheta)}{1 + r^2 - 2\,r\cos(2\,kl - \vartheta)}. \tag{2}$$

1) Cf. V. Thorsen: D. Kgl. Danske Vidensk. Selskab, Mat.-fys. Medd. XX, 10, 1943 (in the following denoted as Essay II).

$\frac{Z_u}{\varrho c}$ then being written in the form $w_0 + iq_0$, (1) passes into

$$\frac{Z_i}{\varrho c} = \frac{w_0 + i\left(q_0 \cos 2\,kl - \frac{1}{2}\,[w_0^2 + q_0^2 - 1]\sin 2\,kl\right)}{(\cos kl - q_0 \sin kl)^2 + w_0^2 \sin^2 kl}. \tag{3}$$

Thus, we have

$$\frac{Z_i}{\varrho c} = w + iq\,,$$

where

$$\left.\begin{aligned} w &= \frac{w_0}{(\cos kl - q_0 \sin kl)^2 + w_0^2 \sin^2 kl},\\[2ex] q &= \frac{q_0 \cos 2\,kl - \frac{1}{2}\,[w_0^2 + q_0^2 - 1]\sin 2\,kl}{(\cos kl - q_0 \sin kl)^2 + w_0^2 \sin^2 kl}. \end{aligned}\right\} \tag{4}$$

Finally, when introducing the energy absorption coefficient a, we find

$$a = \frac{4\,w}{(w+1)^2 + q^2} \quad \text{and} \quad \operatorname{tg}\vartheta = \frac{2\,q}{w^2 + q^2 - 1}.\ ^{1)} \tag{5}$$

The equations (5) in a (w, q) coordinate system with a and $\operatorname{tg}\vartheta$ as parameters represent the known system of circles intersecting one another at right angles, as shown in Fig. 1.

We shall now look at some simple cases. For the absolutely rigid, perfectly reflecting wall, we have

$$a = 0,$$

i. e. the very q axis and, therefore, $w = 0$. Further, $\vartheta = 0$, *i. e.* an infinite large phase circle, whence $q = \infty$. For the absolutely compliant wall, we again have

$$a = 0,$$

1) Cf. Essay I, p. 5, and Essay II, p. 9.

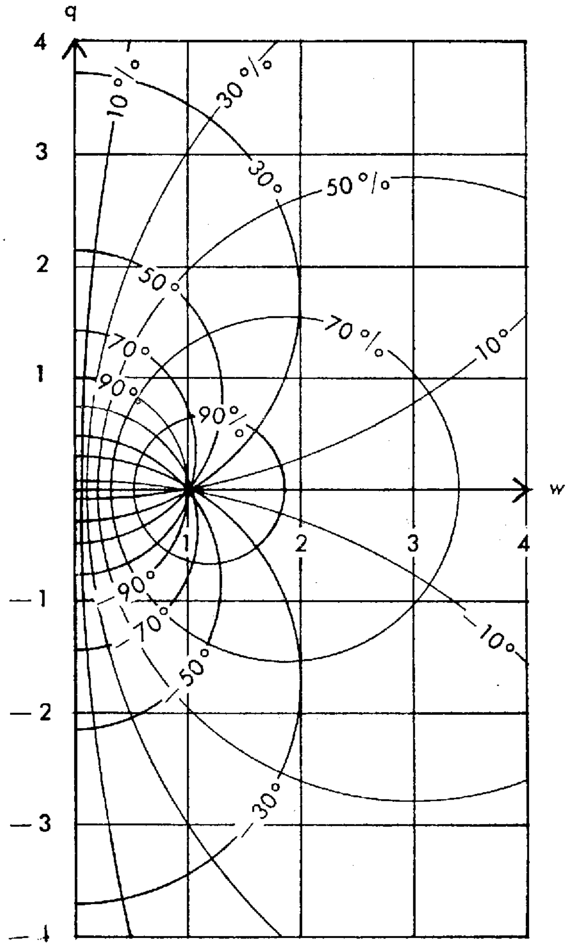

Fig. 1.

hence, also $w = 0$, but in this case $\vartheta = \pm \pi$, *i. e.* $q = 0$. The point (1.0) corresponds to a substance of the same acoustic resistance as the air.

Summarizing, we thus have,

for the rigid wall: $w = 0, \quad q = \infty$,

for the compliant wall: $w = 0, \quad q = 0$.

For the perfectly absorbing substance: $w = 1, \quad q = 0$.

Now, in a tube, we have a combination of acoustic elements, generally both phase change and absorption. If the tube is closed at one end (Fig. 2) with a rigid 100 % reflecting plate,

$$\frac{Z_u}{\varrho c} = w + iq,$$

where

$$w = 0, \quad q = \infty.$$

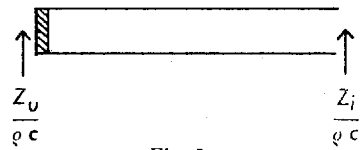

Fig. 2.

The inlet impedance Z_i then, according to (1), becomes

$$\frac{Z_i}{\varrho c} = \frac{\cos kl + i \frac{1}{\frac{Z_u}{\varrho c}} \sin kl}{\frac{1}{\frac{Z_u}{\varrho c}} \cos kl + i \sin kl} = -i \cot kl. \tag{6}$$

If $l = 0$, we find $\frac{Z_i}{\varrho c} = \infty$, which result is obvious. If $l = \frac{\lambda}{4}$, $Z_i = 0$, a result which also should be evident, since the tube now is a closed organ pipe measuring $^1/_4$ wave-length. The tube thus represents a pure reactive acoustic resistance which, if l varies from 0 to $\frac{\lambda}{4}$, itself varies from 0 to ∞.

In the same way, the open tube (Fig. 3) can be treated as an acoustic resistance. If we primarily suppose that the mouth represents an entirely compliant wall, we find for $\frac{Z_u}{\varrho c}$

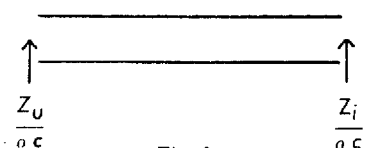

Fig. 3.

$\frac{Z_u}{\varrho c} = w + iq$, where $w = 0$, $q = 0$, which, inserted in (1), gives

$$\frac{Z_i}{\varrho c} = \frac{i \sin kl}{\cos kl} = i \operatorname{tg} kl. \tag{7}$$

For $l = 0$ and $l = \frac{\lambda}{2}$ become $\frac{Z_i}{\varrho c} = 0$, which is also quite obvious. In the latter case, the tube is an open organ pipe measuring $^1/_2$ wave-length. For $l = \frac{\lambda}{4}$, $\frac{Z_i}{\varrho c} = \infty$, whence this acoustic impedance too is purely reactive, varying between 0 and ∞.

In electric analogy, the tube thus in an acoustical conduit acts as a pure reactance (Fig. 4), and the electric generator from the field of electricity may thus be an acoustic generator, for example a telephone or another sound source sending its sound

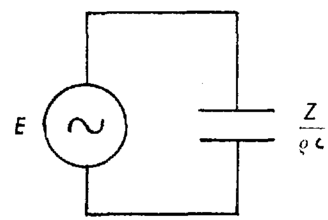

Fig. 4.

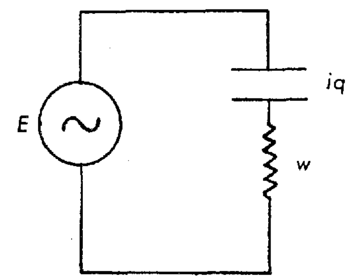

Fig. 5.

energy into the tube. In reality, however, the mouth of an open tube will not be a perfectly compliant wall, so that we have a w of a given, though small, value; q, consequently, is not either quite equal to zero. Thus, neither the real nor the imaginary part of $\frac{Z_i}{\varrho c}$ becomes equal to zero, if $l = \frac{\lambda}{2}$ and the electric comparison picture appears as in Fig. 5. If the tube length varies, w changes along an iso-absorption circle in Fig. 1, and from (4) it is seen that the impedance becomes real, if

$$q_0 \cos 2\,kl - \frac{1}{2}\left[w_0^2 + q_0^2 - 1\right] \sin 2\,kl = 0,$$

or

$$\operatorname{tg} 2\,kl = \frac{2\,q_0}{w_0^2 + q_0^2 - 1} \quad \text{(cf. the 2nd equation (5)).}$$

The corresponding values of w (w_1 and w_2), if $q = 0$, thus become the two values where the iso-absorption circle intersects the w axis. For these values it holds that $w_1 w_2 = 1$. Thus, the tube as an acoustic impedance is likewise arranged so that, with varying lengths, both the real and the imaginary parts change, however, in such a way that the tube has the same absorption coefficient for all values of l. Hence, w and q change simultaneously so that

$$a = \frac{4\,w}{(w+1)^2 + q^2} = \text{const.}$$

Therefore it may be used as absorption standard, although the values of w are different for each tube length.

A presupposition for the correctness of the above statement is that no other ohmic resistances occur than the radiation resistance. If the tube is so narrow or so long that further dissipation resistance is found in the form of friction, the relations become different. This possibility is treated in detail elsewhere[1].

2. **Other Combinations of Tube Impedances.**

If we have a combined system of tubes, as shown in Fig. 6, it must obviously have certain peculiar properties. It consists chiefly of a tube which, for theoretical reasons, we imagine to be divided into two parts, l_1 and l_3, and in whose joining-plane (A) is placed a side-branch which, with the help of a tight-fitting metal piston, may be given different lengths (l_2). A sound-wave which enters from the right will divide into two parts at A. That part which enters the side-branch is entirely reflected from the piston and returns to A with a phase difference relative to that passing on through l_3. If l_2 is exactly equal to $\frac{\lambda}{4}$, the wave reflected from the piston will, when reaching A, be exactly in the phase opposite to that which passes

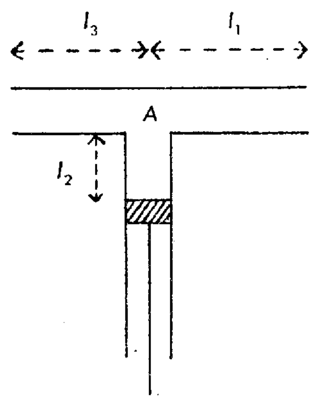

Fig. 6.

1) Essay I, p. 8 *et seq.*, and Essay II, p. 13 *et seq.*

along l_3, and will thus completely extinguish this wave. In this case, the impedance of the side-branch will short-circuit the impedance of l_3. If the piston is pushed up to A, no effect of the side-branch is observed. Its impedance is infinitely great and connected in parallel to the impedance of l_3. Other peculiar relations should be found if either $l_2 + l_3$ or $l_1 + l_3$ are multiples of half wave-lengths. This side-branch principle was set up by QUINCKE[1].

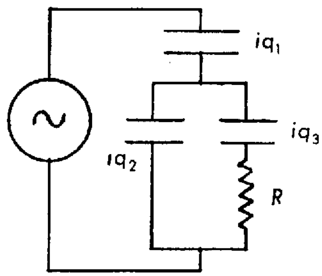

Fig. 7.

The electric equivalent is shown in Fig. 7. R is the radiation at the open (left) end of l_3. If q_2 is equal to ∞, q_1 and q_3 are connected in series; if $q_2 = 0$, R and q_3 are short-circuited and the impedance is equal to iq_1. Roughly spoken, this manner of connecting seems thus to satisfy the above stated demands; yet there are some difficulties which will appear from the following calculation of the impedance.

We find

$$\left.\begin{aligned} Z &= iq_1 + \frac{iq_2\,(R + iq_3)}{iq_2 + R + iq_3} \\ &= \frac{Rq_2^2}{R^2 + (q_2 + q_3)^2} + i\,\frac{R^2\,(q_1 + q_2) + (q_2 + q_3)\,(\Sigma q)}{R^2 + (q_2 + q_3)^2}, \end{aligned}\right\} \quad (8)$$

putting as a simplification

$$q_1 q_2 + q_1 q_3 + q_2 q_3 = (\Sigma q).$$

Further, we find

$$|Z|^2 = \frac{R^2\,(q_1 + q_2)^2 + (\Sigma q)^2}{R^2 + (q_2 + q_3)^2}.$$

From (8) it is evident that Z is not far from being real, if

$$q_2 + q_3 = 0.$$

Therefore it is worth while examining this case somewhat more closely. We find

[1] G. H. QUINCKE: Pogg. Ann. **128**, 177, 1866.

$$Z = \frac{q_2^2}{R} + i(q_1 + q_2). \tag{9}$$

This simplified impedance according to (5) has the absorption coefficient

$$a = \frac{4\frac{q_2^2}{R}}{\left(\frac{q_2^2}{R}+1\right)^2 + (q_1+q_2)^2} = \frac{4\,Rq_2^2}{(R+q_2^2)^2 + R^2(q_1+q_2)^2}.$$

If R is small, and particularly if $q_1 + q_2$ moreover is small, this latter expression becomes

$$a = \frac{4\,Rq_2^2}{(R+q_2^2)^2}. \tag{10}$$

Under these presuppositions, *i. e.* $q_2 + q_3 = 0$ and R small, (9) represents a pure ohmic acoustic resistance with an absorption coefficient given in (10). Forming $\frac{da}{dq_2}$, we find

$$\frac{da}{dq_2} = 4\,R\,\frac{(R+q_2^2)^2 \cdot 2\,q_2 - q_2^2\,(R+q_2^2)\cdot 2\,q_2}{(R+q_2^2)^4}, \tag{11}$$

which, put equal to zero, besides $q_2 = 0$ gives

$$R = q_2^2. \tag{12}$$

If this condition is fulfilled, we find

$$a = 1,$$

i. e. 100 % absorption. Now, if q_2 varies so that $q_2 + q_3$ still is equal to zero (q_3 must thus likewise be changed), a varies in accordance with (10). Then it depends to some degree on the rate at which a varies with q_2 for according to (9), Z is no longer purely ohmic if $q_1 + q_2$ deviates considerably from 0. This question, however, will be easiest to elucidate on the basis of experiments, and experiments show that Z within fairly wide absorption limits may be regarded as purely ohmic. This is supported if Z in its tube lengths is arranged so that, at 100 % absorption where $l_2 + l_3 = \frac{\lambda}{4}$, $l_1 + l_2$ is somewhat greater than $\frac{\lambda}{4}$.

When q_2 increases, *i. e.* l_2 decreases $\left(\text{however, } l_2 + l_3 = \frac{\lambda}{4}\right)$, $q_1 + q_2$ will pass from a small positive value through 0 to a small negative value. In this way the range of absorption of the impedance is increased essentially. How far we may go can be decided with the aid of experiments.

We can also see theoretically that the imaginary part of Z is of minor importance. Since, from (9), we find

$$\operatorname{tg} \vartheta = \frac{(q_1 + q_2) R}{q_2^2}, \tag{13}$$

and, if $\frac{d(\operatorname{tg}\vartheta)}{dq_2}$ is formed, we find

$$\frac{d(\operatorname{tg}\vartheta)}{dq_2} = R \frac{q_2^2 \cdot 1 - (q_1 + q_2) \cdot 2 q_2}{q_2^4} = -R q_2 \frac{2 q_1 + q_2}{q_2^4},$$

which, besides $q_2 = 0$, gives

$$q_2 = -2 q_1. \tag{14}$$

If q_1 is not chosen too small, this value for q_2 brings (12) to a fairly flat minimum of a shape represented in Fig. 8. Experiments also

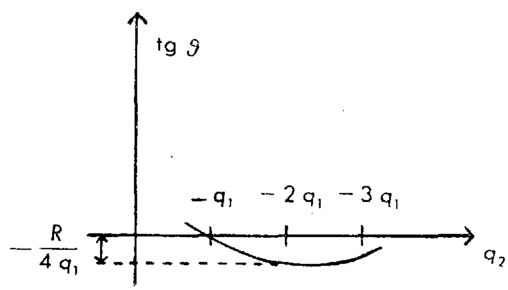

Fig. 8.

prove this to be correct. Evidently, the value (12) gives a maximum for (10)—which should indeed be obvious—since, if q_2 increases from a value smaller than R through R and to a value greater than R, $\frac{da}{dq_2}$ passes from plus through 0 to minus. It appears from (9) that, with decreasing q_2, the ohmic resistance $\frac{q_2^2}{R}$, from being greater than 1, becomes equal to 1 (*viz.*, for $R = q_2^2$) and finally assumes values below 1. Since two values of $\frac{q_2^2}{R}$, the product of which is equal to 1, lead to the same absorption coefficient, the absorption coefficient as a function of l_2 must become a somewhat symmetrical curve with a maximum for $q_2^2 = R$. It must commence in the vicinity of zero, if l_2 is very small, and again approximate zero, if l_2 approximates $\frac{\lambda}{4}$. It is also important to note that it is possible to get all resistance values, both those greater than 1 and those smaller than 1; however, the greater an absorption coefficient we want to obtain at the maximum, the shorter is the range over which q_2 varies, if resistance values smaller than 1 are wanted, since R is a small magnitude, and the resistance is equal to $\frac{q_2^2}{R}$. The falling branch of the absorption coefficient curve as a function of l_2 therefore becomes very steep. If it is unnecessary to obtain as much as 100 % absorption, the falling branch may be made less steep, the adjustment thus becoming less critical. These relations are also substantiated by the measurements.

The impedance determined by (9) we may call the central point impedance of the combined tube-system.

If we look for inlet and outlet impedances in the two tubes l_1 and l_3, respectively, the former is identical with the inlet impedance in an open tube of length l_3. The inlet impedance is thus determined by the tube which is situated to the left of the central point impedance, the outlet impedance by the tube which is situated to the right of the central point impedance (Fig. 9). The impedance at a given value of the absorption coefficient varies as a function of the tube lengths l_1 and l_3 along the absorption circle determined by $\frac{q_2^2}{R}$, and the w and q values of the inlet impedance are found by intersection of this absorption circle with the line $q = \operatorname{tg} kl_3$; in the same way the w and q values of the outlet impedance are found as the intersection of the absorption circle with the line $q = \operatorname{tg} kl_1$. The q values of the inlet and the outlet impedances naturally become q_3 and q_1. The shorter the tube lengths l_1 and l_3, the more the two impedances approximate one another and, at the same time, the central point impedance. If $l_1 = l_3$, the inlet and the outlet impedances are equal. However, not before both are equal to zero does the impedance become reactance-free and thus purely ohmic. This is not quite realizable in practice, but if the pipes l_1 and l_3 are

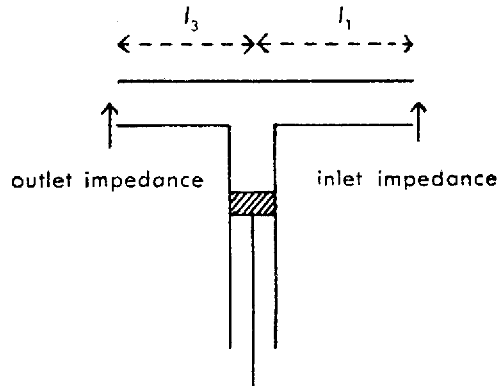

Fig. 9.

short[1], it can approximately be realized, the approximation being best at low frequencies.

We can also get an idea of the highest absorption coefficient to be obtained with given tube lengths. Suppose, for example, that the longest side-branch has the length l, *i. e.* the q value

$$q_A = \operatorname{tg} kl.$$

If this straight line, which runs parallel with the w axis, is brought to intersect the absorption circle system whose equation with the absorption coefficient a as parameter may be written

$$w^2 + q^2 - 2\,w\,\frac{2-a}{a} + 1 = 0,$$

we find

$$w = \frac{2-a}{a} \pm \sqrt{\left(\frac{2-a}{a}\right)^2 - 1 - q_A^2}\,.$$

Real intersection thus is conditioned by

$$\left(\frac{2-a}{a}\right)^2 - 1 + q_A^2 \geqq 0$$

1) Here as well as elsewhere 'short tube lengths' means short in proportion to the wave-lengths or, in other words, kl is a small angle.

or by

$$a \leqq \frac{2}{1+\sqrt{1+q_A^2}}.$$

Accordingly, the sign of equation leads to the highest a-value which the impedance can attain at a given tube length and which thus corresponds to the line $q_A = \operatorname{tg} kl$ just touching the absorption circle.

3. The Influence of Dissipation Damping.

Since it is impossible to keep all the tube lengths short, it is necessary to examine what damping due to friction in the tubes means. This examination claims a picture of comparison

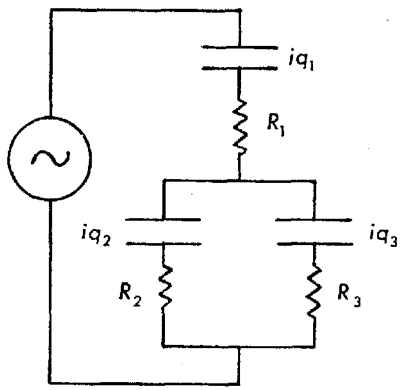

Fig. 10.

as shown in Fig. 10. For the impedance of this system we find

$$Z = R_1 + iq_1 + \frac{(R_2+iq_2)(R_3+iq_3)}{R_2+R_3+i(q_2+q_3)}$$

$$= \frac{R_2R_3 - (\Sigma q) + R_1(R_2+R_3) + i[(R_2+R_3)q_1 + R_3q_2 + R_2q_3 + R_1(q_2+q_3)]}{R_2+R_3+i(q_2+q_3)}$$

and, after multiplication of the numerator and the denominator with

$$R_2 + R_3 - i(q_2+q_3)$$

and some reduction

$$Z = w + iq,$$

where

$$\left.\begin{aligned} w &= \frac{R_2 R_3 (R_2 + R_3) + R_2 q_3^2 + R_3 q_2^2 + R_1 (R_2 + R_3)^2}{(R_2 + R_3)^2 + (q_2 + q_3)^2} \\ q &= \frac{(R_2 + R_3)^2 q_1 + R_2^2 q_3 + R_3^2 q_2 + (q_2 + q_3)(\Sigma q) - R_1 (R_2 + R_3)(q_2 + q_3)}{(R_2 + R_3)^2 + (q_2 + q_3)^2}. \end{aligned}\right\} \quad (15)$$

In case $q_2 + q_3 = 0$, Z assumes the form

$$Z = \frac{R_2 R_3 + q_2^2}{R_2 + R_3} + R_1 + i\left(q_1 + \frac{R_3 - R_2}{R_2 + R_3} q_2\right) \quad (16)$$

and the absorption coefficient in the first approximation becomes

$$a = \frac{4\left[(R_2 R_3 + q_2^2)(R_2 + R_3) + R_1 (R_2 + R_3)^2\right]}{(R_2 R_3 + q_2^2 + R_1 (R_2 + R_3) + R_2 + R_3)^2}. \quad (17)$$

Just as in the simpler case, where R_1 and R_2 were assumed to be equal to zero, we shall find the maximum for a, when a is regarded as a function of q_2. Thereby we find

$$\frac{da}{dq_2} = 4 \cdot \frac{(R_2 R_3 + q_2^2 + R_1 (R_2 + R_3) + R_2 + R_3)\, 2 q_2}{N^2}$$
$$- 4 \cdot \frac{[R_2 R_3 + q_2^2 + R_1 (R_2 + R_3)] \cdot 2 [R_2 R_3 + q_2^2 + R_1 (R_2 + R_3) + R_2 + R_3] \cdot 2 q_2 (R_2 + R}{N^2}$$

which, put equal to zero, besides $q_2 = 0$ gives

$$q_2^2 = R_2 + R_3 - R_1 R_2 - R_2 R_3 - R_1 R_3 = R_2 + R_3 - (\Sigma R); \quad (18)$$

for the sake of simplicity, we put

$$R_1 R_2 + R_2 R_3 + R_1 R_3 = (\Sigma R),$$

i. e. an expression corresponding to (12). If (18) is inserted in (17), we also find $a = 1$, just as before.

R_2 and R_3 being small, (ΣR) is a small magnitude in proportion to R_2 and R_3, whence (17) in good approximation reaches a maximum for

$$q_2^2 = R_2 + R_3.$$

In other words this means that, in (16), R_2R_3 and R_1 may be disregarded so that (16) assumes the following form:

$$Z = \frac{q_2^2}{R_2+R_3} + i\left(q_1 + \frac{R_3-R_2}{R_2+R_3}\,q_2\right). \tag{19}$$

4. Experimental Results.

In order to test the correctness of the formulae developed in the preceding sections, the writer has performed a series of experiments with an impedance of the form shown in Fig. 6. The impedance was made from brass tubes with a lumen measuring 6 mm. in diameter. The tube length l_1 was 2.9 cm., the shortest tube length l_3 being 1.1 cm. The latter could be lengthened with additional tubes of known lengths. The length of l_2 was varied with a piston which, in order to ensure tightness, was supplied with a piston ring consisting of a piano string. The impedance was connected to a calibrated Schuster bridge. Absorption and phase of the inlet impedance were measured for altogether 14 different lengths of l_3, absorption and phase for each value of l_3 were determined as a function of l_2. The frequency applied was 768 Hz, $\lambda = 44.3$ cm. For the determination of every single absorption and phase curve, measurements were performed for about 30 different values of l_2, particularly close around the maximum. Fig. 11 represents an example of the results obtained in a series of measurements of an absorption curve, and Fig. 12 shows a similar measurement for the corresponding phase curve. The accuracy is extraordinarily satisfactory, c. $^1/_2$ $^0/_0$ for the absorption coefficients, and a few degrees for the corresponding phases.

Total results for the maximum absorption coefficients and the corresponding phases are recorded in Table 1; the head lines of the columns are supplied with easily comprehensible symbols referring to those used above. The calculations of a and ϑ are based upon the formulae (5). Finally, the calculations are illustrated by Figs. 13 and 14.

Fig. 13 is very instructive, showing that, for $l_3 = 1.4$ cm., $l_2 = 9.35$ cm., with approximation $R_2 = q_2^2$ so that we here have

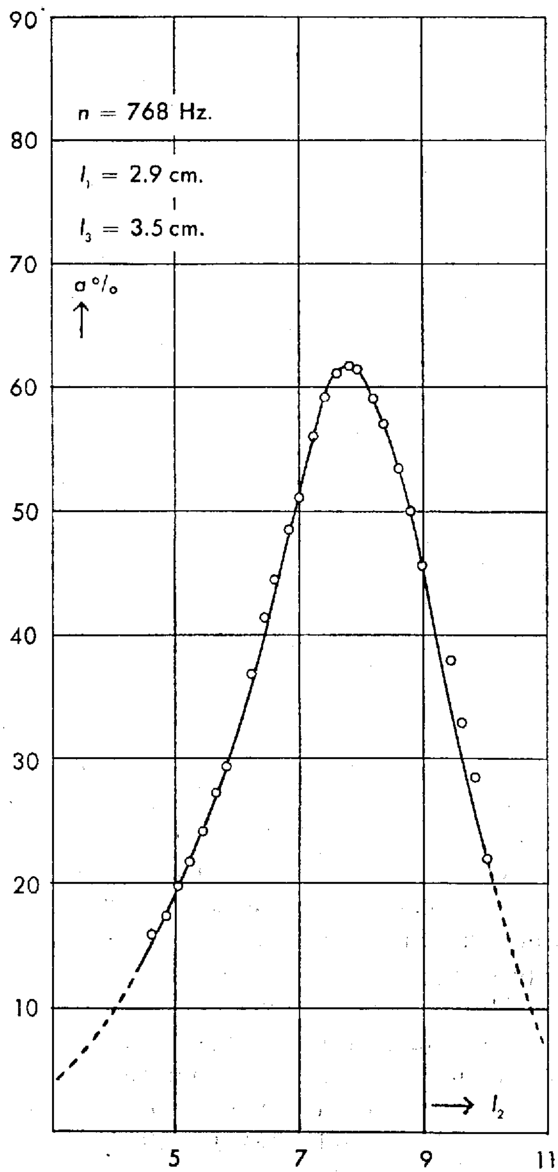

Fig. 11.

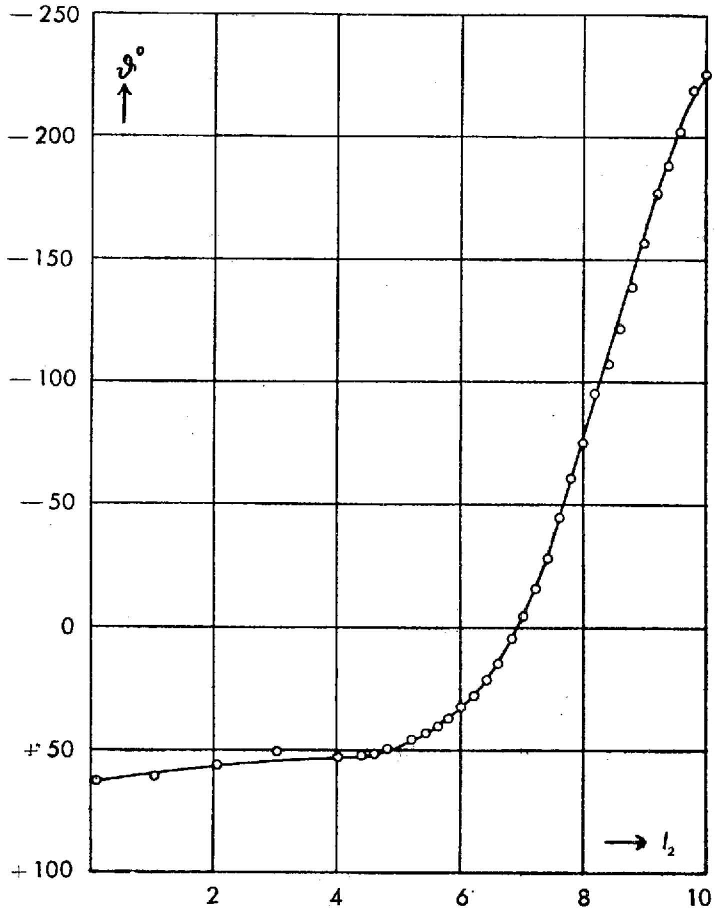

Fig. 12.

nearly 100 $^0/_0$ absorption. At greater l_2 values the maximum absorption curve declines again.

Regarding Fig. 14 the following reflections may be appropriate. ϑ is calculated from the expression

$$\operatorname{tg}\vartheta = \frac{2\,q_3}{\left(\frac{R}{q_2^2}\right)^2 - 1 + q_3^2}.$$

For low values of l_2, $\frac{R}{q_2^2}$ is small, whereas q_3 is fairly great.

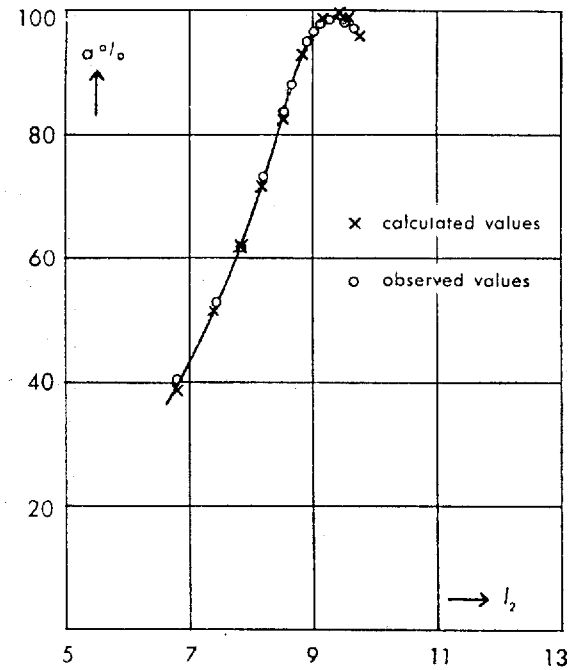
a °/o
calculated values
observed values
l2
100
80
60
40
20
5
7
9
11
13

Fig. 13.

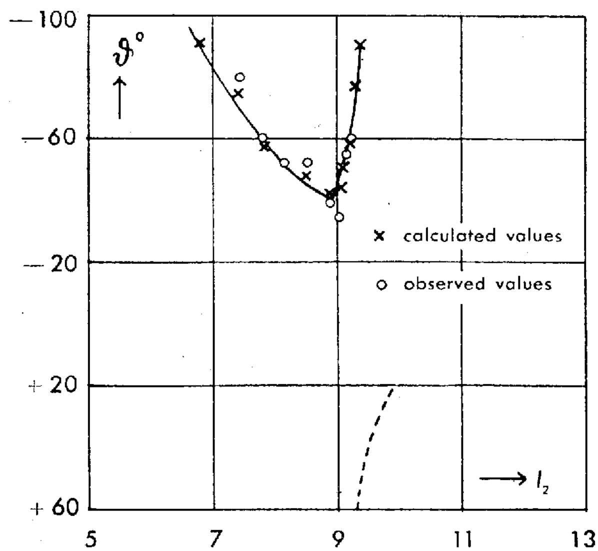
ϑ°
calculated values
observed values
l2
—100
—60
—20
+20
+60
5
7
9
11
13

Fig. 14.

Table 1.

l_3	l_2	q_3	q_2	$\frac{q_2^2}{R}$	$\frac{R}{q_2^2}$	a % Calculated	a % Observed	ϑ^0 Calculated	ϑ^0 Observed
5.5	6.8	1.00	0.695	7.91	0.127	39.5	40.5	ca.–90	–90
4.5	7.4	0.740	0.575	5.41	0.184	52.0	52.7	–74.3	–80
3.5	7.8	0,542	0.505	4.16	0.240	61.7	61.5	–59.1	–60
2.9	8.15	0.440	0.445	3.22	0.310	72.0	73.0	–51.0	–53
2.5	8.5	0.375	0.384	2.41	0.415	82.0	83.5	–47.5	–53
2.0	8.85	0.300	0.327	1.74	0.575	92.5	94.5	–41.2	–40
1.8	9.0	0.265	0.300	1.47	0.680	96.0	96.0	–44.0	–35
1.7	9.1	0.250	0.290	1.36	0.736	97.4	97.5	–50.0	–55
1.6	9.2	0.235	0.275	1.23	0.815	98.5	98.0	–58.0	–60
1.5	9.3	0.220	0.260	1.10	0.91	98.5	98.5	–76.6	—
1.4	9.35	0.200	0.250	1.03	0.97	99.5	99.0	ca.–90	—
1.3	9.45	0.190	0.235	0.90	1.11	98.8	99.0	+	+
1.2	9.5	0.175	0.230	0.85	1.18	98.0	97.5	+	+
1.1	9.65	0.160	0.205	0.69	1.45	95.5	97.0	+	+

Hence, a value must be found for l_2, where $q_3^2 + \left(\frac{R}{q_2^2}\right)^2 = 1$, and the denominator therefore is equal to 0, tg $\vartheta = -\infty$, $\vartheta = -90°$. This point was fortuitously found for $l_3 = 5.5$ cm., $l_2 = 6.8$ cm. If l_2 increases, q_3^2 decreases, while $\frac{R}{q_2^2}$ increases slowly. Here we have

$$q_3^2 + \left(\frac{R}{q_2^2}\right)^2 < 1,$$

whence ϑ becomes negative. For a given value of l_2, we now find on account of the increase in $\frac{R}{q_2^2}$

$$q_3^2 + \left(\frac{R}{q_2^2}\right)^2 = 1,$$

and ϑ again becomes $-90°$. This holds for $l_3 = 1.5$ cm., $l_2 = 9.3$ cm. Hence, in the interval ϑ must have had a (numerical) minimum, *viz.*, for $l_3 = 2.0$ cm., $l_2 = 8.8$ cm. At still higher values for l_2, tg ϑ becomes positive, *viz.*, if

$$q_3^2 + \left(\frac{R}{q_2^2}\right)^2 > 1.$$

This is indicated by the dotted branch of the phase curve in the lower part of Fig. 14. The curve presents, however, a very sharp bend, and the accuracy is but small. Actually the experiments only show that the phase becomes positive. Thus a discontinuity in the phase curve occurs. A closer examination of this relation, which could only be found within the very greatest absorption range between 99 to 100 $^0/_0$, is in progress.

It is clear from the preceding account that it will be possible to produce an impedance which can be brought to assume all possible values of absorption and phase. It is a characteristic feature of this impedance that its damping is a pure radiation damping. This means that we might be able to measure the exact effect, emitted from a telephone, on the human ear. If this impedance is known, and measured for example by means of a Schuster bridge, the variable impedance is adjusted to the value and placed before the telephone. Then the radiation of this telephone through the impedance is equal to the effect produced on the ear. A solution of this problem was suggested before[1], and the program of an investigation was briefly as follows.

(1) The radiation curve of a telephone is measured for a series of frequencies.
(2) The impedance of the ear for the same frequencies is measured with a Schuster bridge.
(3) The variable impedance is adjusted for each frequency as equal to the impedance of the ear.
(4) The impedance is placed before the telephone and, subsequently, the radiation is re-measured.

If the distance between telephone and measuring apparatus (condenser microphone) in case 1 and case 4 is the same, information is obtained as to how great a fraction of the effect, which the telephone is able to emit, is absorbed by the ear. Such an impedance will be much easier to handle than the previously suggested one, and it will moreover be possible to make it cover a far greater range. It must, of course, be adjusted to different standard frequencies, and therefore we possibly may be compelled to make a compromise between the lowest frequency to

[1] Essay I, p. 18.

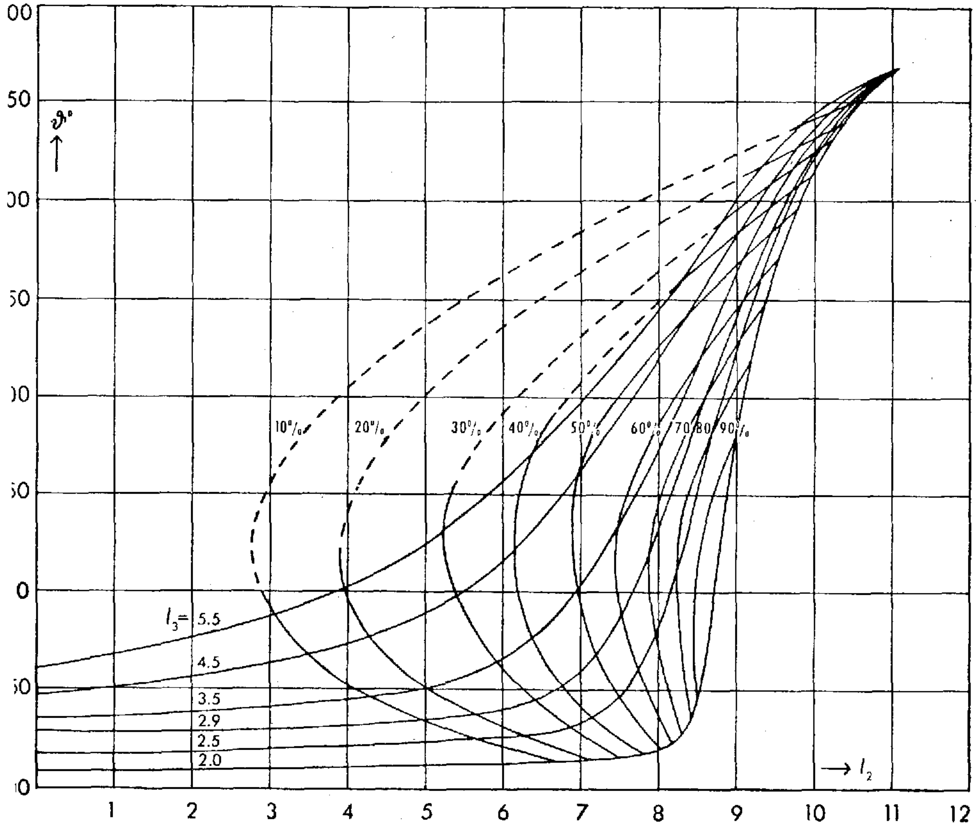

Fig. 15.

be used and the greatest geometric range of the impedance to be allowed.

Fig. 15 was plotted on the basis of all measurements performed. On a series of curves representing the phase as a function of l_2 for different values of l_3 iso-absorption curves for 10, 20, 30, ... 90 $^0/_0$ of absorption are inserted. From these multitudes of curves we may infer, possibly by interpolation, which values l_2 and l_3 must obtain in order to yield a given absorption coefficient and a given phase angle. If we want, for example, $a = 25\ ^0/_0$, $\vartheta = 60^\circ$, we evidently need (about) $l_2 = 5.2$ cm. and $l_3 = 3.3$ cm. In the impedance to be constructed both l_2 and l_3 must be continuously variable. Such an impedance has already been produced, and it has proved to comply with our expectations.

As soon as some still unexplained, however less important details are elucidated, an examination of patients is planned.

Summary.

With the aid of a system of acoustic tube impedances a variable acoustic impedance which covers a rather large range of absorption coefficients and phase changes could be constructed and partly calculated. In order to support the theory, numerous measurements of the values of the inlet impedance of the variable impedances were performed. Particularly good agreement with the theoretical expectance was obtained and it also appears from the measurements that the accuracy is significant. The calculations were performed for tubes both with and without dissipation. It is intended to apply the impedance, *inter alia*, to an objective determination of the effect emitted from a telephone on the human ear.

The present work was performed at the Biophysical Laboratory of the University of Copenhagen. To its head, professor Dr. H. M. Hansen, the author wishes to express his gratitude for the good working conditions put at his disposal.

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Færdig fra Trykkeriet den 22. September 1945.

As to the nature of these hard showers it is experimentally found that the showers observed on Wilson photographs consist mainly of mesons together with a few protons. The same is probably true for the hard showers measured by Jánossy and Ingleby, whereas the photographi stars are known to consist of protons and neutrons (together perhaps with a few mesons). The single protons and neutrons also found in cosmic radiation (cf. p. 5) can probably be fully accounted for as resulting from the explosions observed as the stars.[1] We must thus at present conclude that

there are two different types of non-cascade showers, the explosion showers and the evaporation showers.

The explosion showers consist of mesons produced by multiple processes in which presumably a primary, very energetic photon is absorbed. The protons which may accompany these processes arise from a transfer of a certain part of the energy to the nuclei at which the mesons are produced, thus giving rise to a more or less local heating up and a subsequent evaporation of nucleons. The energies of both the incident particle and the mesons produced as a rule being relativistic in these processes, we must expect the angular dispersion to be rather small, but we are unable to judge whether this agrees with experiments or not. In the evaporation showers the processes are presumably the same, except that the primary photons are less energetic than in the explosion showers so that the binding of the nucleons plays a more dominant rôle. Consequently, most of the energy is transferred to the nucleus in the form of greater heating up. As a result, most of the particles emitted are protons and neutrons and only a few particles are mesons. Both the primary particle and the evaporation particles produced having non-relativistic energies, we must in this case expect a more uniform distribution in space of the particles emitted, as is just found experimentally in the Blau-Wambacher stars.

So far as we can judge, only Wilson chamber photographs have been found showing the direct creation of mesons or protons from primary photons, but not from primary mesons, protons, or electrons. Theoretically, the evaporation showers

[1] Cf. the discussion in Heisenberg (1943) p. 124 ff.

may also be produced by primary protons and neutrons by 'nuclear ionization',[1] in which the incident particle gives off part of its energy by inelastic collisions, thereby heating up the nucleus, which then evaporates. This process, however, does not seem to have been directly observed. Also such showers may theoretically be produced by the absorption of slow negative mesons[2] (the slow positive mesons being repulsed by the Coulomb forces); for this process will take place long before the radioactive decay. Although this effect seems indirectly verified by the fact that at sea level more positive mesons are found than negative mesons, and next by the experiment of Rasetti[3] who finds that roughly only half of all mesons decay radioactively, the rest being absorbed without decay, no direct evidence seems to have been found, i. e. a Wilson photograph showing a slow meson being absorbed under the emission of several protons.

Finally, the extended showers found by Auger and collaborators[4] should be mentioned. They measured coincidences between G-M-counters placed up to several 100 meters apart. It has been discussed whether these Auger showers consist of electrons or of mesons. From absorption measurements Auger and Kolhörster assumed them to consist mainly of electrons together with a few mesons, because the number of coincidences was only reduced to about 25 % behind 15 cm Pb. At any rate, from Wilson chamber photographs it follows that most of the particles are very energetic electrons.[5] Assuming the Auger showers to be cascade showers formed at the top of the atmosphere and reaching their maximum at about sea level, it follows that such cascades are not absorbed even in 15 cm Pb. As shown by Molière,[6] the cascade theory can actually account for *all* the particles being electrons. Such showers representing at sea level energies up to 10^{15} e. v., they must have been produced by primary particles of energies even up to 10^{18} e. v.

[1] Heisenberg (1937).
[2] Tomonaga and Araki (1940).
[3] Rasetti (1941).
[4] Auger and col. (1938), Kolhörster and col. (1938).
[5] Jánossy and Lovell (1938), Auger and col. (1939).
[6] Heisenberg (1943) p. 35 ff.

Part 2. The various possible hypotheses regarding the primary component.

Having reviewed the main experimental facts of importance for our fundamental problems, (b) and (c) p. 3, we now turn to the next question: by what hypotheses regarding the primary component can we correlate and explain this vast experimental material? We have the following possibilities regarding the primary constituents:

photons, electrons, neutrons, protons, mesons, and neutrinos,

together with combinations of all these particles.

First of all we can exclude the neutrons and the charged mesons, as these particles are unstable with mean lifetimes of the order of magnitude one hour and 10^{-6} sec, respectively. The latter result is deduced experimentally, the former theoretically, but without being verified experimentally. This verification is presumably also impossible because the neutrons are slowed down and absorbed by the various nuclei even in the atmosphere long before they would have time to decay.[1] Furthermore, we shall at once exclude the hypothetical neutrinos from our considerations, as their existence has not yet been directly demonstrated (cf. however the remark on p. 4).

From the geomagnetic effects it follows that at any rate a certain fraction of the primary particles are charged particles. (We note that these effects obviously operate *outside* the atmosphere, the thickness of which is only of the order of magnitude $\frac{1}{10}$-$\frac{1}{100}$ of the radius of the earth). From the very high values, viz. 70-80 %, for the latitude effect of the total radiation at great altitudes, i.e. practically the soft component, it follows that most of the primary particles of the soft component must be charged particles. Millikan and col.[2] estimate that

the energy brought into the atmosphere by non-charged particles can at most amount to 20 % of that brought by charged particles.

To obtain as simple a description as possible we shall, therefore, also exclude photons and neutral mesons as primary particles of the soft component. Of course, Nature need not at

[1] Heisenberg (1943) p. 141.
[2] Bowen, Millikan and Neher (1938).

all be simple, and in fact cosmic rays have proved to be far more complicated than anybody has at first imagined. Nevertheless it is generally agreed that to begin with we should try the simplest hypotheses before having recourse to the more complicated ones.

For the hard component the latitude effect as a function of altitude, as mentioned above p. 8, has not yet been fully investigated. Thus, we cannot at present exclude the possibility that a more considerable fraction of the hard component is due to non-ionizing primary particles than the soft one. We shall, however, also here, for the sake of simplicity, assume that *the whole* of the hard component is due to primary charged particles. Consequently in both cases only electrons alone, protons alone, or a combination of these particles remain.

(I). *The electron hypothesis.*

According to this hypothesis[1] the soft component mainly consists of cascade showers from the primary electrons, the integral energy spectrum of which must be assumed to be of the form

$$F(E) = \text{const } E^{-\gamma} \qquad \text{for } E > 1\text{-}2 \times 10^{9} \text{ e.v.}, \qquad (4)$$

in which we must insert $\gamma \sim 1{\cdot}8$ in order to fit the experiments.[1] Next, the hard component is assumed to consist of secondary mesons produced by the photons of the soft component. Hence the intensity of the hard component must pass through a maximum and approach zero at the top of the atmosphere. As regards the proportion between positons and negatons, JOHNSON[2] has concluded from the very small east-west asymmetry of the soft component at sea level and at 4300 m altitude that there must be practically the same number of positons and negatons (cf. p. 10). This conclusion does not, however, follow unambiguously from the experiments mentioned. From the cascade theory it follows, firstly, that at sea level a soft component produced as cascade showers from either photons, positons, or negatons can show a latitude effect of at most a few per cent and the

[1] This hypothesis forms the basis of the surveys of EULER and HEISENBERG (1938), HEITLER (1938), and ARLEY (1940).
[2] JOHNSON (1935 a).

same, therefore, applies to the east-west asymmetry.[1] At 4300 m altitude, HEITLER[1] estimates the latitude effect at 17 %. The corresponding east-west asymmetry has not been worked out, but as a zenith angle z at this altitude increases the layer of air traversed by the shower from $l = 17$ to $l = \frac{17}{\cos z} \sim 20$ and 24 (l measured in shower units) for $z = 30^\circ$ and $z = 45^\circ$, respectively, the primary energies necessary to penetrate this distance will certainly be such as to reduce the east-west asymmetry to at most a few per cent (cf. the discussion on p. 12). *Consequently, we can draw no conclusions as to the sign of the primaries of the soft component from its east-west asymmetry in the lower part of the atmosphere.*

From the east-west asymmetry of the hard component it follows, on the other hand, due to its small absorption (the mesons only losing about 2×10^9 e.v. during their passage through the whole atmosphere) that its primaries must consist of more positive than negative particles. If SCHEIN's experiment mentioned above ([3] p. 10) turns out to be reliable, we must even conclude that

all the primary particles of the hard component are positively charged,

as first concluded by JOHNSON.[2]

Thus we must assume either that the primary radiation mainly consists of positons, or that the mesons can only be produced by the primary positons, but not by the primary negatons. The latter possibility must be rejected at once, because the showers produced by primary electrons of either sign are after some distance practically identical in the number of photons, positons, and negatons, respectively, and it is impossible to imagine processes by which the mesons of the hard component should be produced only by the primary, but not by the secondary electrons. Furthermore, we must at present assume that the mesons are produced only by the photons and not directly by the electrons of the soft component.

We are thus forced to assume the primary electrons to be

[1] HEITLER (1937), ARLEY and ERIKSEN (1940).
[2] JOHNSON (1938), (1939 a), (1939 c). Cf. also ALFVÉN (1939 b).

mainly positive. But this conclusion involves some difficulties. Firstly, both the soft, the hard and thus the total radiation should in this case show a very large positive east-west asymmetry at great altitudes in contrast to the above discussed experiment of JOHNSON and BARRY ([1] p. 10) finding only $a \sim +7\,\%$ for the total radiation against an expected value $\sim +60\,\%$ on the hypothesis that the primary radiation consists only of positive particles. *We think this experiment is already a crucial one, which alone is enough to reject the electron hypothesis.* It has, however, been objected to this conclusion that the negative result of the experiment may also be explained by assuming that the direction of the primary particles is not conserved, but is quite blurred by the processes producing the secondary particles.[1] Against this argument it must first of all be pointed out that it seems difficult to understand why this effect should be more pronounced in the upper than in the lower atmosphere or at sea level, where the total radiation shows a considerable east-west asymmetry. If the particles are cascade electrons, most of them will have energies about or rather above the critical energy of air, viz. $1{\cdot}5\times10^8$ e.v., which is much higher than the rest energy of the electrons, and both from the cascade theory of showers and directly from Wilson chamber photographs it then follows that the angular dispersion is very small. Next, by whatever processes particles are created from primary particles of relativistic energies, it follows simply from the Lorentz transformation from the center of gravity coordinate system to that in which the process is observed, that all the particles emitted have very nearly the same direction as the primary particle.[2]

We cannot either agree with the conclusion drawn by JOHNSON[2] from the experiment of JOHNSON and BARRY just discussed, that the primary particles of the soft component are equally positively and negatively charged. We must remember that at the altitude at which this experiment is carried out, viz. 3 cm Hg, the total radiation consists of about $57\,\%$ mesons and only $43\,\%$ electrons (as judged from the curves of PFOTZER and of SCHEIN,

[1] HEISENBERG (1943) p. 45.

[2] Cf. also JOHNSON (1939 a), (1939 b), who reaches the same conclusion from other arguments.

JESSE and WOLLAN[1]); together with SCHEIN's result mentioned above ([3] p. 10) that the east-west asymmetry of the hard component is very considerable, we can thus only conclude from JOHNSON and BARRY's experiment showing a very small east-west asymmetry of the total radiation at great altitude, that *the soft component shows a negative east-west asymmetry at great altitude. This result should, of course, be verified directly.*

A second difficulty in assuming the primary radiation to consist of considerably more positive than negative particles is that it becomes difficult to understand the propagation of the radiation in interstellar space. As pointed out by SWANN,[2] any difference in the space charge of positive and negative particles of any kind would give rise to potential differences quite irreconcilable with the further passage of charged particles through space. (Furthermore ALFVÉN[3] has pointed out that such a difference would also give rise to large magnetic fields. The effect of these fields seems, however, only to be that they make the radiation *isotropic*). Consequently, it is necessary that *in distances far away from the sources of the radiation it must consist of the same number of positive and negative particles.*

Thirdly, the SCHEIN-JESSE-WOLLAN experiment ([2] p. 12) is probably the most crucial experiment which makes the electron hypothesis irreconcilable with experimental facts, quite apart from what detailed picture we may accept of the genesis of the various components. If this experiment is reliable (in spite of the minor objections which, as we have pointed out, may be raised against it (p. 13)), it means partly that the hard component does not pass through any maximum but increases steadily, partly that the primary radiation can at most contain a few per cent electrons, both facts strongly disagreeing with the electron hypothesis.

(II) *The proton hypothesis.*

According to this hypothesis both the soft and the hard component are secondary radiations produced by protons having the same integral energy spectrum (4) as the electrons had previously.

[1] Fig. 1 in SCHEIN, JESSE and WOLLAN (1941), reproduced as fig. 2 in HEISENBERG (1943) p. 41.
[2] SWANN (1933). Cf. also JOHNSON (1939 a).
[3] ALFVÉN (1938), (1939 a).

The intensity of the soft component must, therefore, approach zero at the top of the atmosphere, whereas the hard component must increase steadily since very energetic protons also behave like penetrating particles. As just discussed it is, however, on this hypothesis quite impossible to understand the small east-west asymmetry of the total radiation at great altitude. Next, as also just discussed, it makes the propagation of the radiation in interstellar space impossible. Finally, it makes it quite impossible to understand the latitude effect of the soft component amounting to 70-80 $^0/_0$ at great altitude; for the electrons could only be produced by processes in which they obtain only a fraction of the primary energy. This primary energy must, therefore, be much higher than if the primary particles were electrons. But when the main contribution to the intensity of the soft component comes from the higher part of the energy spectrum (4), the variation of the minimum energy with geomagnetic latitude will be of little importance. Consequently, the latitude effect becomes much smaller, at most a few per cent, as also emphasized by HEISENBERG.[1] That the secondary electrons can in fact obtain only a fraction of the primary energy is clearly seen by considering those processes by which protons could produce soft showers: by knock-on electrons, by bremsstrahlung and through intermediate mesons. In the latter case, it might be suggested that the soft component in the upper atmosphere is mainly due to the radioactive decay of the very short-living vector-mesons with spin 1, the hard component consisting of the longer living pseudoscalar mesons with spin 0 (cf. p. 15). Now it follows both theoretically[2] and experimentally[3] that the mesons are mainly produced in multiple processes, each meson thus obtaining on the average only a fraction of the primary energy. Furthermore, on an average half the energy of each meson is carried away by the neutrinos. As a result most of the shower intensity would be produced by primary protons with energies beyond the field sensitive region, viz. about $2\text{-}15 \times 10^9$ e.v., and the soft component could show practically no latitude effect even at very high altitudes.

[1] HEISENBERG (1943) p. 5.
[2] Cf. e. g. HEISENBERG (1943), SWANN (1941), and others.
[3] Cf. e.g. SCHEIN, JESSE and WOLLAN (1941 b).

The proton hypothesis must, consequently, also be regarded as irreconcilable with the experimental facts.

(III) *The combined electron-proton hypothesis.*

According to this hypothesis[1] the soft component in the upper atmosphere is produced as cascade showers from primary electrons, whereas the hard component is mainly produced from primary protons. From the above discussion of the very small east-west asymmetry of the total radiation together with SWANN's neutrality argument regarding the number of positive and negative particles in the radiation in interstellar space, it follows that the primary *electron* component must consist practically of only *negatons* in a number equivalent to that of the protons. The only crucial experiment which forces us to reject this in all other respects excellent hypothesis is thus the experiment of SCHEIN, JESSE and WOLLAN ([2] p. 12), which shows that there can only be at most a few per cent electrons present at the top of the atmosphere.

Summarizing our discussion, we must thus conclude that *the total present experimental evidence is irreconcilable with any of the hypotheses theoretically possible using the particles known at present.* For this negative result the crucial experiments are those of JOHNSON and BARRY ([1] p. 10), SCHEIN, JESSE and WOLLAN ([2] p. 12), and the latitude effect of the soft component at great altitude. Also the neutrality argument of SWANN ([2] p. 23), necessary for the propagation of a charged radiation in interstellar space, leads to the same conclusion. We may thus say that

there is at present indirect experimental evidence for the existence of a new and hitherto unknown particle in the primary cosmic radiation,

and we think that the most plausible hypothesis which may be set up as to the nature of this new particle is to assume it to be a *negative proton.*

[1] This hypothesis has been favoured by JOHNSON (1938), (1939 a).

Part 3. The hypothesis of the existence of negative protons in the primary cosmic radiation.

As mentioned in the introduction, this hypothesis has been put forward by the author from the arguments discussed above, and by KLEIN from arguments regarding the origin of cosmic rays.[1] In this part, the consequences of this hypothesis, and in the last part KLEIN's theory will be discussed.

We assume on this hypothesis that

the primary cosmic radiation consists of positive and negative protons with the integral energy spectrum given in (4), *p.* 20,

previously assumed to belong to electrons. From SWANN's neutrality argument we assume that

the numbers of positive and negative protons are practically equal.

Next, we assume that most negative protons will be absorbed by the positive protons at the top of the atmosphere or in the very upper part of it, their total kinetic plus rest energy thereby being transformed into 2 annihilation photons which, due to the conservation of energy and momentum, obtain the same energy and equal, but opposite momenta, uniformly distributed in space in the center of gravity coordinate system. (A *one*-quantum annihilation process is impossible for free protons, and less probable for bound protons than the *two*-quantum process). Due to the Lorentz transformation they will then, as discussed above on p. 22, in the coordinate system in which we observe the process, have practically the same direction as the incident negative proton and energies practically uniformly distributed up to $2Mc^2 +$ kinetic energy of the negative proton. These photons then immediately give birth to cascade showers which at higher altitudes constitute most of the soft component. The most energetic of these showers constitute the large AUGER showers, which extend even down to sea level, together with some of the large bursts. Some of the photons may also be absorbed under the emission of mesons, especially more slow mesons.[2]

[1] ARLEY (1944), KLEIN (1945).

[2] This last process seems, however, to occur very seldom as compared with the absorption of photons leading to pair production. This is shown by the fact that the cascade theoretical ROSSI curves fit the experimental curves of

These slow mesons may then be absorbed, if they are negatively charged (cf. p. 18), giving rise to nuclear evaporation processes in the form of BLAU-WAMBACHER stars, most of which, however, are probably produced directly by the absorption of photons.

As the kinetic energy of the incident negative proton is large as compared with the binding energy of the positive protons and the neutrons in the nuclei they meet in the atmosphere, we may neglect this binding and regard the nucleons as being free. By these annihilation processes we therefore assume that no or little heating up of the rest of the nuclei takes place, and therefore presumably few evaporation nucleons will be emitted. The single protons and neutrons found experimentally we assume to be the result of the stars (cf. the discussion on p. 17).

It may also be possible that some of the negative protons are annihilated in other processes by which mesons are created. In such cases, it is most probable from current theoretical ideas (cf. p. 24) that these processes are multiple, whereby several mesons are created in one elementary act. In order not to complicate the theory more than necessarily, and also because of the above discussion of the latitude effect of electrons produced from the mesons of these processes (p. 24), *we shall, however, tentatively assume that only the photon annihilation is of importance.*

Although most negative protons should on our hypothesis be annihilated in the upper part of the atmosphere, some of them might of course happen to penetrate to the lower parts of the atmosphere. *It is, therefore, possible to obtain direct experimental evidence on our hypothesis by looking for negative protons on Wilson chamber photographs from high altitudes such as mountains or airplanes.*

As for the *positive protons* of the primary radiation we set up the same hypothesis as e. g. JOHNSON in the previous proton or electron-proton hypothesis, viz. that in the upper atmosphere they are momentarily or gradually transformed into mesons (which are presumably only pseudoscalar mesons, as discussed

ROSSI and JÁNOSSY (1939), TRÜMPY (1943), NERESON (1942), and others, even up to the highest thicknesses of absorbers employed in these experiments (cf. the theoretical calculation and the comparison with these experiments in ARLEY (1943) chap. 6). On the other hand, the experiments of SCHEIN and col. ([1] and [3] p. 14) seem to show that such processes do occur in the atmosphere.

below p. 29). It may also, of course, be possible that very energetic protons emit bremsstrahlung and knock-on electrons, thereby producing cascade showers which form part of the soft component, but these effects may presumably be entirely neglected. On the other hand, the hard component produces a considerable soft secondary radiation by the radioactive decay electrons of the mesons, and by the knock-on electrons and bremsstrahlung also produced by the mesons, giving at once rise to cascade showers denoted as decay and interaction showers, respectively. These showers presumably form most of the soft component found at sea level.

We shall now discuss the consequences of our hypothesis and the above mentioned assumptions and compare them with the experimental evidence given in part 1.

First, by its very construction our hypothesis is seen to agree with SWANN's neutrality argument. Secondly, the soft component is seen to pass through a maximum, approaching zero at the top of the atmosphere as was found experimentally by PFOTZER and by SCHEIN, JESSE and WOLLAN ([1] and [2] p. 12). Thirdly, the *total* energy of the negative protons is transferred to the soft component produced, and next nearly the same fraction of the negative protons as of the electrons, previously assumed to be the particles having the energy spectrum (4) p. 20, have now energies in the field-sensitive region, viz. about $2\text{-}15 \times 10^9$ e.v. for electrons; for this energy region is practically the same also for high speed protons (although somewhat lower).[1] Consequently, our hypothesis also leads to the same high values of the latitude effect of the soft component at great altitude as did the electron hypothesis, and as is found experimentally. That part of this soft component which reaches sea-level would, however, just as was the case in the electron hypothesis, now be produced mostly by protons in the non-field-sensitive region and would, consequently, show a latitude effect and an east-west asymmetry (although negative) of at most a few per cent at sea level. Both these effects would, on the other hand, increase very much with increasing altitude. On our hypothesis the soft component should thus at high altitude, where the

[1] Cf. e. g. JOHNSON (1938) table II p. 219, also quoted in HEISENBERG (1943) table 1 p. 152.

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ledge of the latitude and the east-west effects of the hard and the soft components *separately*, and the dependence of these effects on altitude, latitude and zenith angle, together with the transition from SCHEIN, JESSE and WOLLAN'S curve to that of PFOTZER. Only such new experiments can decide whether the purely tentative hypotheses, on the existence of negative protons as well as on the cosmic radiation being produced by annihilation processes, contain part of the truth or perhaps even the whole truth of the genesis of cosmic rays.

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promising, since it is a conspicuous fact that the energies of the primary cosmic radiation lie essentially within the region of the annihilation energies of the lighter nuclei known to exist in interstellar space ([1] p. 36). Whether *all* the primary cosmic radiation can be explained in this way or we have to explain some part of it by other processes it is premature to decide at the moment.

Summary.

In this paper we discuss the three main problems of present cosmic ray physics, the origin of the radiation, the composition of the primary component, and the genesis of the various components observed in the atmosphere, at sea level and at great depths. In part 1 we review all the experimental data bearing upon these problems. In part 2 we discuss the three possible hypotheses regarding the primary radiation which involve only particles known at present: (I) the electron hypothesis, (II) the proton hypothesis, and (III) the combined electron-proton hypothesis. It proves that the present total experimental evidence cannot be reconciled with any of these hypotheses. For this negative result the crucial arguments are the experiments of Johnson and Barry, of Schein, Jesse and Wollan, the latitude effect of the soft component at great altitude and, finally, the neutrality argument of Swann, which is necessary for the propagation of a charged radiation in interstellar space. There is thus indirect evidence of the existence of a new hitherto unknown particle in the primary cosmic radiation. In part 3 we discuss the hypothesis, put forward by the author and by Klein, that these new particles are negative protons. It is shown that the results of this hypothesis, together with plausible assumptions regarding the genesis of the soft and the hard components, seem to fit extremely well with all the experimental data. Finally, we discuss in part 4 a related hypothesis of Klein, that cosmic rays are produced by the annihilation of ordinary and reversed matter consisting of negative protons, antineutrons and positons.

In the discussion it is emphasized that the present experimental material is still rather incomplete. Especially we need more know-

according to KLEIN's hypothesis must be electrons (perhaps photons). Roughly one would expect the fraction originating from the collisions between like nuclei to be of the order of magnitude 10 %, because the most frequent collision leading to electrons is the He-He process,[1] the relative frequency of which is of the order of magnitude $10\times10 = 100$, which is just 10 % of the relative frequency of the most frequent collision leading to nucleons, viz. the H-He process, the relative frequency of which is of the order of magnitude $100\times10 = 1000$. Hereto must, certainly, be added those electrons originating from the radioactive decay of the neutrons and the antineutrons, but as the energy hereby liberated is only of the order 10^6 e.v., these electrons will in our coordinate system practically move with the same velocity as the neutrons, i.e. their energy will only amount to the fraction $\frac{m}{M}$ of that of the nucleons. Consequently these electrons may be entirely neglected.

Another crucial point for KLEIN's hypothesis, if it is to explain *all* the primary radiation, is, as stated by himself, whether it is reconcilable with the existence of the large AUGER showers, representing a total energy of the order of magnitude of 10^{15} e.v. at sea level, which energy is by several powers of 10 beyond the upper limit represented by the rest energy of Si^{28}, viz. 26×10^9 e.v. We have already above (p. 31) discussed KLEIN's suggestion for solving this problem, and his rough quantitative analysis does not seem to be unreasonable. This point cannot, however, be decided at present; it must be left for future investigations.

Summarizing, we may say that only further experimental investigations can at all decide on the truth of KLEIN's hypothesis. We can only say at present that *it seems at any rate very*

[1] We note that also the most frequent collision between like nuclei, the H-H process, may in fact lead to electrons with energies above the lower limit $1{\cdot}4\times10^9$ e.v. caused by the blocking effect of the sun. Although a two-photon annihilation can only lead at most to the energy $0{\cdot}9\times10^9$ e.v., and the same applies to the electrons resulting from a two-meson annihilation, an annihilation process of 3 or more mesons may lead to electrons of energies of the order of magnitude of 2 atomic units $= 1{\cdot}8\times10^9$ e.v. As this is, however, only the case if one of the mesons gets practically the whole energy and the same applies to its decay electron, we suppose such a process to be of negligible frequency in spite of the fact that the relative frequency of the H-H collision is of the order of magnitude $100\times100 = 10\,000$.

Just as was the case for the collisions between like nuclei, KLEIN thus assumes the collisions between unlike nuclei to lead to discrete energies. *This last assumption is, however, certainly just as erroneous as the first one,* because the energy must necessarily be distributed more or less at random over the $y-x$ nucleons, thus again giving a *continuous* spectrum extending up to $2x$ atomic units ([3] p. 33). After some time the neutrons and antineutrons produced by these unlike collisions will, furthermore, decay, being transformed into protons + negatons and negative protons + positons, respectively. Due to the Lorentz transformation mentioned above these particles will again have *continuously* distributed energies extending up to some 10^9 e.v.

We may thus conclude that KLEIN's *hypothesis does not lead to a band structure of the primary radiation,* which on his hypothesis consists of electrons (perhaps photons) together with both positive and negative protons having continuously distributed energies of the order of magnitude of some 10^9 e.v. (the maximum energy at any rate not exceeding the rest energy of Si^{28}, i. e. 26×10^9 e.v.). Furthermore, this primary radiation will obviously consist of practically the same number of positons and negatons as well as of positive and negative protons. *Apart from the electrons (perhaps photons),* which particles must necessarily, as far as we can see, constitute a non-negligible part of the primary radiation, KLEIN's *hypothesis just leads to the same result regarding the primary component of cosmic radiation as our analysis of all the experimental data on the behaviour of the radiation in the atmosphere of the earth.* The crucial point for KLEIN's hypothesis is thus, whether the experiment of SCHEIN, JESSE and WOLLAN[1] is compatible with the existence of a certain electron component in the primary radiation or not. We note, however, that primary photons will *not* be measured in this experimental arrangement and perhaps, therefore, we have to assume the photon rather than the meson annihilation (case (a) above, p. 36). On the other hand, it is impossible at the present state of quantum theory to evaluate quantitatively the cross sections for the various processes in question and thus to estimate the fraction of the primary radiation, which

[1] SCHEIN, JESSE and WOLLAN (1941 a).

to the cosmic rays. Now, as mentioned above, the most frequent nuclei are H, He and then C, N, O and Si, which occur in the approximate ratio 100:10:1:1:1:1.[1] Collisions between *like* nuclei will lead to total annihilation, the energy being given off either (a) as two photons, or (b) as two or more mesons. KLEIN assumes that (b) is the dominating process and that just *two* mesons are formed. These mesons will next decay, emitting an electron and a neutrino. KLEIN now argues that, as the nuclei are assumed to move with thermal velocities, each meson will get exactly the rest energy of one nucleus and the electrons therefore practically half that energy, thus just leading to the same discrete energies as postulated by MILLIKAN and collaborators. *This argument is, however, erroneous.* First, it is unlikely that just *two* mesons will be created, because, as discussed by HEISENBERG,[2] the processes with higher multiplicity must be expected to be practically just as probable as the two-meson process. Secondly, whether this is true or not, the mesons will at any rate obtain relativistic velocities and in that case the electrons emitted by the radioactive decay in our coordinate system, due to the Lorentz transformation, will have energies nearly uniformly distributed between 0 and the *whole* meson energy, as previously stated (p. 22),[3] but overlooked by KLEIN. Any such process will thus lead to *continuously* distributed electron energies and not to the band structure postulated by MILLIKAN and col.

Next, as regards collisions between *unlike* nuclei with x ordinary and y reversed nucleons respectively ($x < y$), or vice versa, KLEIN assumes that $2x$ of the nucleons are completely annihilated and that, due to the thermal energies of the colliding particles being small compared with the binding energies of the nuclei, this annihilation energy will, by a sort of internal conversion of either the photons or the mesons produced, be transferred to the remaining $y-x$ nucleons rather than be given off. KLEIN next assumes this heating up to be so violent that all the $y-x$ nucleons are emitted with *equal* energy.

[1] We only wonder whether these figures may be extrapolated to be valid for the inter*galactic* space, as they have, so far as we know, only been deduced experimentally in the inter*stellar* space.

[2] HEISENBERG 1943 p. 115.

[3] Cf. the detailed calculation in EULER and HEISENBERG (1938) § 14.

However, from a theoretical point of view a process in which nuclei are annihilated, just two electrons thereby being emitted, is quite an unknown process. Furthermore it is irreconcilable with the conservation of charge, and in general also of spin and statistics, which conservation laws are just as fundamental as those for energy and momentum. In order to overcome these theoretical difficulties and yet to be in agreement with the banded structure postulated by MILLIKAN and col., KLEIN has put forward the following hypothesis.

From general theoretical considerations one would expect a perfect symmetry between the positive and negative electricity in the world, a symmetry which was much emphasized by DIRAC's electron theory and the subsequent experimental discovery of the positon. Thus, there ought also to exist what KLEIN calls reversed matter, in which all electric signs are reversed, i.e. which consists of negative protons, 'antiprotons, positons and antineutrons, the magnetic moment of which has a direction with respect to the spin momentum opposite to that of ordinary neutrons. Applying the DIRAC equation also to the nucleons, a positive and a negative proton, as well as a neutron and an antineutron, should be able to annihilate each other just as a positon and a negaton can annihilate each other under the emission of two photons (which process is more probable than the one-quantum annihilation process being possible for bound particles), whereby the photons become equal energies and equal but opposite momenta. The annihilation can perhaps also take place under the emission of two or more mesons.

Since the spectra emitted by ordinary and by reversed matter would be identical, it would be impossible to ascertain whether a given star consists of one or the other form of matter. Assuming the stars of each galactic system to have a common origin, KLEIN now also assumes that all the stars of one galactic system consist of the same kind of matter, but of matter different from one galactic system to another. In the intergalactic space nuclei of both kinds may exist together, due to the extremely small density of matter present there. KLEIN next assumes that these nuclei move about with thermal velocities and by their collisions are at once annihilated as soon as different kinds of matter come into contact with each other, thus giving birth

	H^1	min. en.	He^4	C^{12}	N^{14}	O^{16}	Si^{28}	max. en.
Energy in atomic units	0·5	..	2	6	7	8	14	..
Energy in 10^9 e.v.	0·47	1·4	1·9	5·6	6·6	7·5	13·2	16·5
Corresponding geomagnetic latitude		60°N U.S.A.	56°N U.S.A.	42°N U.S.A.	40°N U.S.A.	33°N U.S.A.	20°N India	0° India

In this table we have also given the geomagnetic latitudes at which these energies represent the minimum energy (for electrons) for the direction of easiest access, which is smaller than the minimum energy for the vertical direction.[1] Since the magnetic dipole of the earth is situated excentrically, these minimum energies vary slightly with longitude.[2] For protons the minimum energies are somewhat smaller for the same latitude.[3] The column denoted by min.en. in the table gives the minimum energy found in the primary spectrum for easiest access, which is generally ascribed to the blocking effect of the sun. The column denoted as max. en. gives the largest minimum energy for vertical incidence at the equator.

On MILLIKAN's hypothesis we should thus expect the intensity of cosmic radiation in the stratosphere to have a banded structure, being constant between the geomagnetic latitudes corresponding to these energies and increasing each time such a latitude is passed from north to south. This effect MILLIKAN and collaborators[4] in fact claim to have observed. Their observations are, however, carried out with ionization chambers and the measurements, therefore, give the total effect of both the soft and the hard component and from all directions. *So it would be more adequate to use G-M-counters and thus try to ascertain whether the effect, if real, exists for the soft, the hard, or both components. Furthermore, the east-west asymmetry should then also show a banded structure,* an effect which does not yet seem to have been observed.

[1] Cf. e.g. JOHNSON (1938) fig. 14 p. 219.
[2] Cf. e.g. JOHNSON (1938) fig. 16 p. 222.
[3] Cf. e.g. JOHNSON (1938) table II p. 219.
[4] MILLIKAN, NEHER and PICKERING (1942), (1943).

present quantum theory, as estimated by HEISENBERG ([1] p. 32), this discrepancy may not be so serious, especially when we also keep in mind that the negative protons may certainly participate in quite different processes, the calculation of which is beyond the capacity of the present quantum theory.

Part 4. On the origin of cosmic radiation.

As mentioned in the introduction, the idea of assuming the existence of negative protons in the primary cosmic radiation has also been put forward in a paper by KLEIN.[1] The purpose of this paper, however, is not that of explaining the present experimental data on the behaviour of the radiation in the atmosphere of the earth, but to answer our question (a) p. 3, i. e. to explain the origin of the enormous energies of the cosmic rays. As already pointed out by MILLIKAN and his collaborators,[2] the average energy of the primary energy spectrum (4) p. 20, viz. about 4×10^9 e. v., is just of the same order of magnitude as the rest energy of those nuclei which, from astronomical observations, are known to occur most frequently in interstellar space, namely H, He, C, N, O, and Si. MILLIKAN and his coworkers therefore suggested that the source of the cosmic radiation is simply to be sought in nuclear processes in which these nuclei are annihilated, the rest energy being given off in the form of two electrons. (At least *two* electrons in order to obey the conservation laws for energy and momentum). Due to these conservation laws, the electrons carry each half the energy and have equal, but opposite momenta which are uniformly distributed in space. From this hypothesis we should expect the primary energy spectrum to be not continuous, as assumed in formula (4), but *discrete*, having only the energies corresponding to half the rest energies of the nuclei mentioned, viz.[3]

[1] KLEIN (1945).
[2] BOWEN, MILLIKAN and NEHER (1938).
[3] 1 atomic unit is the rest energy Mc^2 of $\frac{1}{16}$ of O^{16}, i.e. $931 \cdot 05 \times 10^6$ e. v (cf. e.g. BETHE (1936) p. 86). The atomic weight of the proton being 1·00813, the rest energy of H^1 is $0 \cdot 9386 \times 10^9$ e. v., etc.

of reversed matter hit the atmosphere, all their constituents are annihilated successively during a very short time by a chain of annihilation-processes so that a very large number of very energetic particles are produced within a very narrow space. If it is possible that the particles resulting from these annihilations are mostly photons or electrons, or are immediately transformed into such by cascade multiplication of photons or of the electrons from the radioactive decay of intermediate mesons, we think this to be a most promising explanation of the extremely high total energies revealed in the AUGER showers, these energies now resulting from many primary particles which are transformed practically simultaneously, instead of from one single parent particle as in the previous explanation. The result must, however, on whatever explanation given be electrons, as the AUGER showers are experimentally known to consist mostly, if not exclusively, of electrons (cf. the discussion on p. 18).

Summarizing, we think it may be said that *our hypothesis is able to explain, at any rate qualitatively, all the present experimental evidence.* In fact we have not found any experiment directly contradicting it, but we stress that, of course, *only further experiments can show whether our purely tentative hypothesis contains part of the truth or perhaps even the whole truth of the genesis of cosmic rays.*

Regarding the more quantitative side of the hypothesis it is, due to the very incomplete state of the present quantum theory within these high energy regions, premature to try to deduce any numerical results e.g. for the various intensities and the geomagnetic effects. As discussed by the author,[1] our hypothesis demands a cross section of the order of magnitude 10^{-25} cm^2, i.e. nuclear dimensions, for the fundamental process of the two-quantum annihilation of a negative and a positive proton. Against this, the present DIRAC equation, *which, applied to protons, just demands the existence of negative protons,* gives only a cross section of the order of magnitude 10^{-32} cm^2, i.e. smaller by a factor 10^7. Since we are in these processes far beyond the limits of validity of the

[1] ARLEY (1944).

as for the latitude effect amount to at most a few per cent, but in the *opposite* direction of the east-west asymmetry of the part of the soft component produced by the negative protons. Consequently, the east-west asymmetry of the total soft component at sea level must be practically zero, as is just found experimentally. With increasing altitude the cascade part of the soft component becomes more and more dominating, and the east-west asymmetry of the soft component should thus on our hypothesis decrease with increasing altitudes, becoming more and more negative, as is indirectly verified by the experiment of JOHNSON and BARRY (cf. the discussion on p. 22).

As for the meson showers and the nuclear stars, i. e. the explosion and the evaporation showers, respectively (cf. p. 16 ff.), it follows from our hypothesis (p. 27) that their frequency should increase roughly proportionally to the intensity of the soft component, as just found experimentally. KLEIN,[1] however, has also suggested the possibility that the stars may be due to the absorption of slowed down negative protons. As here the binding of the nucleons must come into play, such an absorption would lead to a strong heating up of the nucleus and a subsequent evaporation in contrast to the case of very fast protons (cf. p. 27). Also this process would explain that the frequency of the stars increases very strongly with increasing altitude. Although, as we have seen, it is unnecessary to have recourse to this explanation of the stars, because they are equally well explained as the result of the direct absorption of photons (or perhaps of slow negative mesons), we would not exclude the possibility of the existence of such processes.

KLEIN[1] has also suggested another explanation of the very large AUGER showers in order to account for the occurrence of the enormous energies, viz. 10^{15}-10^{18} e.v., necessary if they are to be explained as cascade showers produced at the top of the atmosphere and penetrating down through the whole of the atmosphere to sea level. KLEIN suggests as another explanation that there may also in the primary radiation exist whole grains or dust particles consisting of reversed matter, i. e. matter the atoms of which consist of negative protons, 'antineutrons' and positons (cf. the last part of the present paper). When these grains

[1] KLEIN (1945).

Combining the latter result with the result of the negative east-west asymmetry of the soft component, we thus see that our hypothesis leads to an east-west asymmetry for the total radiation which decreases with increasing altitude, as is just found experimentally by JOHNSON and BARRY ([1] p. 10; cf. also the discussion on p. 22). *We think that this crucial experiment is a strong argument in favour of our hypothesis.*

We note that it might be thought possible to test experimentally, if our hypothesis is at all accepted, whether the soft component in the upper atmosphere is produced through intermediate vector mesons (cf. above p. 24). For the lifetime of these particles we must presumably assume values of the order of magnitude 10^{-8} sec. In that time, they would on an average move a distance of $10^{-8}.3.10^{10}.\frac{E}{\mu c^2}$ cm $\sim$ 100 m (the velocity being relativistic, and the factor $\frac{E}{\mu c^2} \sim 40$ being the relativistic time factor). Thus, those vector mesons produced in the neighbourhood of the measuring apparatus would pass through it as a hard radiation, but as one showing a *negative* east-west effect. At that altitude at which the hypothetical transformation, negative protons to vector mesons, should take place, we thus might observe a temporary decrease in the east-west asymmetry of the hard component. We think, however, that in view of the fact that such vector mesons must be created in multiple processes, if at all created, this eventual decrease could only amount to a few per cent and thus presumably only be within the measuring errors.

At sea level most of the soft component is presumably due to the decay and interaction showers mentioned above (p. 28),[1] and it could therefore only show a latitude effect of at most a few per cent, these showers representing only a fraction of the energy of the primary particles from which they have been produced. As the same applied to the cascade showers produced from the negative protons, the total soft component at sea level should show a latitude effect of at most a few per cent, as just found experimentally.

As regards the east-west asymmetry of the soft component produced from the hard one, it could also for the same reasons

[1] In HEISENBERG (1943) p. 90, it is estimated that at sea level the soft component is composed of about 62 % decay showers (Z), 17 % interaction showers (W) and 21 % cascade showers (R) (the last originating according to our hypothesis from the negative protons).

contribution to the soft component from the hard component is only small, show a considerable east-west asymmetry in the *opposite* direction of the hard component, i. e. a preponderance of negative primaries, or greater intensity from the east. It should, however, here be noted that this conclusion is based on the assumption that the mesons produced by the positive protons mostly are long-living pseudoscalar mesons. If also a considerable number of short-living vector mesons were produced in these processes, they would already at high altitudes decay into electrons at once giving birth to cascade showers. As a result, the east-west asymmetry of the soft component at great altitude would in this case be less negative or even practically zero. *The direct experimental determination of the east-west asymmetry of the soft component at great altitude is thus of fundamental importance,* although JOHNSON and BARRY'S experiment already gives strong evidence of a considerable *negative* east-west asymmetry of the soft component at great altitude, as discussed above (p. 23). Furthermore, this east-west asymmetry of the soft component should be practically non-increasing with increasing zenith angle (cf. p. 12).

As for the hard component, it is firstly seen that on our hypothesis it does *not* pass through any maximum, but increases steadily up to the very greatest heights. As already stated, the primary protons, having relativistic energies, will behave as a hard component whether they are transformed immediately or gradually into mesons. Next, the hard component now shows the same geomagnetic effects as in the previous proton hypothesis, viz. a latitude effect at sea level of the order of magnitude 10-20 $^0/_0$, which increases with increasing altitude, but less strongly than that of the soft component, because the mesons only lose about 2×10^9 e. v. by their passage through the whole atmosphere. We think that also this statement is in agreement with the experiments although the data are here rather scanty, as discussed on p. 8. Finally, for the same reasons our hypothesis leads to a positive east-west asymmetry already at sea level. Furthermore, this positive east-west asymmetry must increase with increasing altitudes and with increasing zenith angle (cf. p. 12), which statements are both in agreement with the experimental findings.