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THE PERIODIC COMET *COMAS SOLÁ* (1926 f) AT ITS RETURN IN THE YEAR 1935

WITH SOME REMARKS ABOUT THE METHOD OF DIRECT INTEGRATION IN CO-ORDINATES AND THE METHOD OF PERTURBATIONS

BY

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Comet Comas Solá (1926 f) was discovered on 1926 Nov. 4 by Comas Solá in Barcelona. Using 199 observations from 1926 and 1927 the writer undertook the computation of a definitive orbit[1] for this comet with the following result:

Date of osculation 1926 Nov. 30.0

$$T = 1927 \text{ March } 22.1929 \text{ U. T.}$$

$$\left.\begin{aligned} \omega &= 38^\circ\, 27'\, 50''.8 \\ \Omega &= 65^\circ\, 35'\, 41''.0 \\ i &= 13^\circ\, 45'\, 43''.3 \end{aligned}\right\} 1925.0$$

$$\varphi = 35^\circ\ \ 6'\, 26''.4$$

$$a = 4.17176$$

$$P = 8.52094 \text{ (jul. years).}$$

Referred to the standard equinox of 1950.0 the elements are:

$$\left.\begin{aligned} \omega &= 38^\circ.47714 \\ \Omega &= 65^\circ.93112 \\ i &= 13^\circ.76307 \end{aligned}\right\} 1950.0$$

and the equatorial constants:

$$x = -0.970009 (\cos E - e) - 3.235433 \sin E$$

$$y = +3.433358 (\cos E - e) - 1.060704 \sin E$$

$$z = +2.162110 (\cos E - e) + 0.232816 \sin E$$

[1] Det kgl. Danske Videnskabernes Selskabs Mathematisk-fysiske Meddelelser X, 13. (Publikationer og mindre Medd. fra Kbhvns Observatorium Nr. 74).

From the elements the return of the comet may be expected in 1935, and it was planned to compute the perturbations by Jupiter and Saturn in the years 1926—1935, and to procure new osculating elements for a date near the perihelion in 1935.

In 1931 I was asked by Dr. L. J. Comrie, Superintendent of the Nautical Almanac Office, Greenwich, if I would help him to illustrate the use of the extensive tables[1], that were about to be published by the Nautical Almanac Office. The principal part of these tables is the co-ordinates of the major planets (except Mercury), both the spherical co-ordinates (longitude, latitude and radius vector) and the heliocentric equatorial rectangular co-ordinates, together with the attractions of the planets on the Sun, all referred to the standard equinox of 1950.0. Everybody who has carried through calculations of perturbations over a long period of time knows how inconvenient it was to have to change the equinox during the computation, which in former times happened every 10 years. With the publication of these new tables this difficulty disappears for in the tables are found:

for Jupiter and Saturn (the only perturbing bodies usually taken into consideration): the co-ordinates for the years 1800—1940 (for 1800—1900 only the spherical co-ordinates),

for Uranus and Neptune: the co-ordinates for 1903—1940,

for Venus, the Earth and Mars: the co-ordinates for 1920—1940.

It is planned to continue these tables up to 1960, and later to 1980. The volume also gives a number of very

[1] «Planetary co-ordinates for the years 1800—1940, referred to the equinox of 1950.0».

useful auxiliary tables, e. g. an 8-figure table of $\frac{1}{r^3}$ with r^2 as argument ($2 \leqq r^2 \leqq 20$).

I gladly agreed to this proposal of co-operation and the more so as the calculations I had planned for comet 1926 f would then get an excellent check. It was decided to carry through the calculations of perturbations for comet 1926 f using two methods: the classical ENCKE method and the so-called COWELL method. All calculations were done with calculating machines.

Both methods are based on the differential equations:

$$\left.\begin{aligned} \frac{d^2x}{dt^2} &= -k^2\frac{x}{r^3} + k^2 \sum{}' m_1 \left(\frac{x_1 - x}{\varrho_1^3} - \frac{x_1}{r_1^3}\right) \\ \frac{d^2y}{dt^2} &= -k^2\frac{y}{r^3} + k^2 \sum{}' m_1 \left(\frac{y_1 - y}{\varrho_1^3} - \frac{y_1}{r_1^3}\right) \\ \frac{d^2z}{dt^2} &= -k^2\frac{z}{r^3} + k^2 \sum{}' m_1 \left(\frac{z_1 - z}{\varrho_1^3} - \frac{z_1}{r_1^3}\right) \end{aligned}\right\} \qquad (1)$$

where x, y, z are the heliocentric co-ordinates of the perturbed body, x_1, y_1, z_1 the co-ordinates of the perturbing body and $r^2 = x^2 + y^2 + z^2$, $r_1^2 = x_1^2 + y_1^2 + z_1^2$, $\varrho_1^2 = (x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2$.

In both methods the first term in the symbol of summation has to be computed for each of those planets whose attractions the computer intends to take into account. The last term in the summation can be taken directly from the N. A. O. tables.

In the ENCKE method the three differential equations are not integrated directly by numerical integration, but putting:

$$x = x_0 + \xi$$
$$y = y_0 + \eta$$
$$z = z_0 + \zeta$$

where x_0, y_0, z_0 are the co-ordinates of the unperturbed orbit and ξ, η, ζ the perturbations, and introducing in (1) the differential equations from the unperturbed motion:

$$\frac{d^2x_0}{dt^2} = -k^2\frac{x_0}{r_0^3}$$
$$\frac{d^2y_0}{dt^2} = -k^2\frac{y_0}{r_0^3}$$
$$\frac{d^2z_0}{dt^2} = -k^2\frac{z_0}{r_0^3}$$

three differential equations in $\frac{d^2\xi}{dt^2}$, $\frac{d^2\eta}{dt^2}$, $\frac{d^2\zeta}{dt^2}$ result, which after some transformations and introduction of auxiliary quantities can be integrated by numerical integration[1] eventually using the «trick» given by OPPOLZER[2]. It will be seen that in ENCKE's method the first thing to be done is the computation of the unperturbed rectangular co-ordinates, which is a usual ephemeris computation, and then the perturbations of the unperturbed orbit must be computed step by step by numerical integration.

In COWELL's method, on the other hand, the equations (1) are integrated directly by numerical integration. Such direct integration of the differential equations of motion has been known and has been used for a very long time in the more general problem: «problème restreint»; the pioneer here was TH. N. THIELE[3], who about 1890 first introduced this method, which was instantly used by C. BURRAU[4]. G. H. DARWIN used the method in his

[1] See for instance G. STRACKE: Bahnbestimmung der Planeten und Kometen, page 261.

[2] Loc. cit. page 263.

[3] Oversigt over det Kongelige Danske Videnskabernes Selskabs Forhandlinger, 1889 page 28, 1892 page 20.

[4] See for instance Astronomische Nachrichten 3230 and 3251.

«Periodic Orbits» (1897), and in 1900 ELIS STRÖMGREN[1] applied the method to the still more complicated problem where the masses of all three bodies are of the same order (it is also this direct co-ordinate method that has been constantly used at the Copenhagen observatory under the guidance of E. STRÖMGREN in the extensive investigations of «problème restreint»). When used in comet work the method is generally called COWELL's method, because COWELL, then Chief Assistant at the Royal Observatory at Greenwich, was the first who (together with CROMMELIN) applied it to a real celestial body, namely Halley's comet at its return in 1910[2] (for an application of the method to Jupiter's 8th satellite see «Monthly Notices» LXVIII, 1908).

All computations in the present work have been made independently, with comparisons at suitable intervals, by D. H. SADLER, of the Nautical Almanac Office, with whom it was a pleasure to co-operate, and the writer. The agreement of our results is exact to the last decimal.

The chief points of the experience gained by the calculations shall be mentioned here.

Both methods are admirably suited to machine work, and the new tables from the N. A. O. have proved a most valuable help during the computations. Their merit is not merely that the inconvenience of changing equinox has

[1] «Über Mechanische Integration und deren Verwendung für numerische Rechnungen auf dem Gebiete des Dreikörper-Problemes» (Öfversigt af Kongl. Vetenskaps-Akademiens Förhandlingar 1900 No. 4. Medd. fr. Lunds Astr. Obs. No. 13). Continued in Astr. Nachr. 4356 (Sept. 1909) and Monthly Notices, Nov. 1919.

[2] P. H. COWELL, A. C. D. CROMMELIN: Investigation of the motion of Halley's comet from 1759 to 1910 (Appendix to Greenwich observations, 1909).

now disappeared, but whereas in former times the computer himself had to compute both the rectangular co-ordinates of the perturbing planets from the spherical co-ordinates in the Almanacs and the attractions on the Sun, now all these quantities can be taken directly from the tables.

The direct co-ordinate method is, from the point of view of formulae, much the simpler; it is very transparent and the scheme of calculation for each interval is quite short. The Encke scheme of calculation is considerably longer and — especially when the Oppolzer trick is used to safeguard against re-calculation of an interval — not very transparent; also the unperturbed co-ordinates have to be computed separately. On the other hand it is generally possible in the integration to use double the interval of that used in the direct co-ordinate method and also to use one or two figures less in the computation itself even if the integration schemes show the same number of decimals or even one more in the Encke scheme. In our illustration the Encke scheme was started with an interval of 20 days and in units of the 8th decimal, and a four-figure computation was sufficient as the so-called indirect terms are very small at the beginning. Later on they grew considerably, so that we ended with a six-figure computation, even after the 8th decimal had been dropped about the middle of 1934. The interval was changed several times; for nearly 4 years it was possible to use an interval of 80 days.

The Cowell scheme was started with an interval of 10 days and only in units of the 7th decimal. A seven-figure computation was used most of the time, and the interval was changed several times, the largest interval used being

40 days. The defect of the COWELL scheme is that near perihelion, when the body approaches the Sun, it is very difficult to handle because, even when small intervals are used, the differences in the integration scheme, which form the check on the calculation, may grow so considerably that the extrapolation is illusory. If the comet gets nearer to the Sun than 1.5 astronomical units I do not think it advisable to use the direct co-ordinate method. It has been proposed[1]) that in such a case the two methods may be combined, so that the integration near perihelion is started by ENCKE's method, and when the comet has reached a suitable distance from the Sun the change over to the direct co-ordinate method is effected. This change is a very simple operation; an illustration of it is given in the N. A. O. volume page xiv.

In the following table the co-ordinates resulting from the two integration schemes are shown shortened to 6 decimals. The unperturbed co-ordinates are not given, as they were computed to 5 decimals only.

Date 0^h U. T.			Direct co-ordinate method (Cowell scheme)			Perturbations in co-ordinates (Encke)		
			x	y	z	ξ	η	ζ
1926	Nov.	1	+1.643 513	+1.437 770	+0.383 128	−0.000 001	−0.000 001	0.000 000
		11	1.519 779	1.478 546	0.437 399			
		21	1.391 617	1.515 010	0.490 393	0.000 000	0.000 000	0.000 000
	Dec.	1	1.259 105	1.546 731	0.541 851			
		11	1.122 378	1.573 266	0.591 491	0.000 000	0.000 000	0.000 000
		21	0.981 633	1.594 163	0.639 008			
		31	0.837 143	1.608 966	0.684 082	−0.000 001	−0.000 001	0.000 000
1927	Jan.	10	0.689 259	1.617 234	0.726 377			
		20	0.538 418	1.618 552	0.765 548	0.000 004	0.000 003	−0.000 001
		30	0.385 146	1.612 547	0.801 253			
	Febr.	9	+0.230 055	+1.598 906	+0.833 163	−0.000 008	−0.000 006	−0.000 002

[1] G. MERTON: The periodic comet Grigg (1902 II) — Skjellerup (1922 I) (1902 to 1927). Mem. R. A. S. Vol. LXIV, part III, 1927.

Date 0h U. T.	Direct co-ordinate method (Cowell scheme)			Perturbations in co-ordinates (Encke)		
	x	y	z	ξ	η	ζ
1927 Febr. 19	+0.073 835	+1.577 393	+0.860 970			
March 1	−0.082 756	1.547 866	0.884 402	−0.000 014	−0.000 009	−0.000 004
11	0.238 915	1.510 290	0.903 235			
21	0.393 812	1.464 744	0.917 301	0.000 020	0.000 013	0.000 006
31	0.546 620	1.411 426	0.926 499			
April 10	0.696 538	1.350 643	0.930 797	0.000 028	0.000 017	0.000 008
20	0.842 821	1.282 806	0.930 235			
30	0.984 799	1.208 408	0.924 916	0.000 037	0.000 021	0.000 010
May 10	1.121 893	1.128 009	0.915 005			
20	1.253 625	1.042 213	0.900 718	0.000 047	0.000 025	0.000 012
30	1.379 625	0.951 643	0.882 308			
June 9	1.499 626	0.856 929	0.860 057	0.000 059	0.000 027	0.000 014
19	1.613 457	0.758 682	0.834 263			
29	1.721 035	0.657 488	0.805 233	0.000 072	0.000 029	0.000 015
July 9	1.822 354	0.553 897	0.773 271			
19	1.917 469	0.448 416	0.738 673	0.000 089	0.000 029	0.000 015
29	2.006 490	0.341 505	0.701 725			
Aug. 8	2.089 564	0.233 577	0.662 694	0.000 109	0.000 028	0.000 014
18	2.166 868	0.125 000	0.621 832			
28	2.238 603	+0.016 098	0.579 368	0.000 133	0.000 027	0.000 012
Sept. 7	2.304 983	−0.092 845	0.535 517			
17	2.366 227	0.201 584	0.490 471	0.000 162	0.000 025	0.000 009
27	2.422 562	0.309 908	0.444 406			
Oct. 7	2.474 211	0.417 634	0.397 480	0.000 196	0.000 023	0.000 005
17	2.521 395	0.524 608	0.349 835			
27	2.564 329	0.630 700	0.301 600	0.000 235	0.000 021	−0.000 001
Nov. 6	2.603 222	0.735 799	0.252 888			
16	2.638 274	0.839 815	0.203 802	0.000 279	0.000 021	+0.000 004
26	2.669 677	0.942 670	0.154 433			
Dec. 6	2.697 615	1.044 302	0.104 864	0.000 328	0.000 022	0.000 009
16	2.722 261	1.144 661	0.055 165			
26	2.743 781	1.243 707	+0.005 403	0.000 383	0.000 024	0.000 014
1928 Jan. 5	2.762 331	1.341 407	−0.044 366			
15	2.778 060	1.437 737	0.094 091	0.000 443	0.000 029	0.000 019
25	2.791 106	1.532 679	0.143 724			
Febr. 4	2.801 602	1.626 222	0.193 227	0.000 509	0.000 036	0.000 023
24	2.815 433	1.809 080	0.291 700			
March 15	2.820 468	1.986 294	0.389 267	0.000 655	0.000 059	0.000 029
April 4	2.817 519	2.157 892	0.485 736			
24	2.807 311	2.323 936	0.580 958	0.000 821	0.000 096	0.000 032
May 14	2.790 488	2.484 509	0.674 813			
June 3	2.767 629	2.639 710	0.767 212	0.001 004	0.000 147	0.000 029
23	2.739 248	2.789 649	0.858 082			
July 13	2.705 811	2.934 441	0.947 371	0.001 206	0.000 214	0.000 021
Aug. 2	2.667 735	3.074 204	1.035 036			
22	2.625 396	3.209 056	1.121 050	0.001 423	0.000 300	+0.000 005
Sept. 11	2.579 135	3.339 116	1.205 390			
Oct. 1	−2.529 262	−3.464 500	−1.288 042	−0.001 655	−0.000 406	−0.000 018

Date 0^h U. T.	Direct co-ordinate method (Cowell scheme)			Perturbations in co-ordinates (Encke)		
	x	y	z	ξ	η	ζ
1928 Oct. 21	−2.476 057	−3.585 321	−1.368 999			
Nov. 10	2.419 776	3.701 689	1.448 256	−0.001 900	−0.000 533	−0.000 050
30	2.360 654	3.813 712	1.525 813			
Dec. 20	2.298 906	3.921 491	1.601 673	0.002 157	0.000 682	0.000 092
1929 Jan. 9	2.234 728	4.025 126	1.675 840			
29	2.168 302	4.124 713	1.748 322	0.002 425	0.000 855	0.000 145
Febr. 18	2.099 796	4.220 342	1.819 127			
March 10	2.029 365	4.312 101	1.888 263	0.002 702	0.001 053	0.000 208
30	1.957 152	4.400 074	1.955 742			
April 19	1.883 290	4.484 340	2.021 573	0.002 987	0.001 277	0.000 283
May 9	1.807 904	4.564 977	2.085 769			
29	1.731 110	4.642 058	2.148 339	0.003 278	0.001 529	0.000 371
June 18	1.653 016	4.715 653	2.209 296			
July 8	1.573 723	4.785 829	2.268 650	0.003 573	0.001 809	0.000 472
28	1.493 327	4.852 650	2.326 414			
Aug. 17	1.411 918	4.916 177	2.382 599	0.003 872	0.002 119	0.000 587
Sept. 26	1.246 390	5.033 580	2.490 274			
Nov. 5	1.077 762	5.138 478	2.591 762	0.004 471	0.002 833	0.000 861
Dec. 15	0.906 590	5.231 269	2.687 141			
1930 Jan. 24	0.733 380	5.312 319	2.776 488	0.005 060	0.003 678	0.001 197
March 5	0.558 592	5.381 964	2.859 872			
April 14	0.382 647	5.440 509	2.937 355	0.005 624	0.004 663	0.001 601
May 24	0.205 937	5.488 235	3.008 994			
July 3	−0.028 827	5.525 397	3.074 837	0.006 148	0.005 794	0.002 076
Aug. 12	+0.148 343	5.552 230	3.134 929			
Sept. 21	0.325 247	5.568 948	3.189 305	0.006 613	0.007 077	0.002 628
Oct. 31	0.501 576	5.575 746	3.237 996			
Dec. 10	0.677 035	5.572 802	3.281 024	0.007 003	0.008 518	0.003 262
1931 Jan. 19	0.851 335	5.560 279	3.318 407			
Febr. 28	1.024 197	5.538 323	3.350 154	0.007 298	0.010 122	0.003 982
April 9	1.195 345	5.507 067	3.376 271			
May 19	1.364 507	5.466 631	3.396 754	0.007 478	0.011 894	0.004 794
June 28	1.531 410	5.417 124	3.411 593			
Aug. 7	1.695 782	5.358 640	3.420 773	0.007 521	0.013 836	0.005 705
Sept. 16	1.857 348	5.291 266	3.424 271			
Oct. 26	2.015 825	5.215 075	3.422 057	0.007 402	0.015 955	0.006 723
Dec. 5	2.170 926	5.130 135	3.414 095			
1932 Jan. 14	2.322 355	5.036 499	3.400 340	0.007 094	0.018 252	0.007 858
Febr. 23	2.469 802	4.934 217	3.380 739			
April 3	2.612 948	4.823 326	3.355 234	0.006 563	0.020 732	0.009 122
May 13	2.751 454	4.703 860	3.323 757			
June 22	2.884 965	4.575 841	3.286 230	0.005 772	0.023 399	0.010 528
Aug. 1	3.013 101	4.439 289	3.242 568			
Sept. 10	3.135 460	4.294 216	3.192 676	0.004 674	0.026 256	0.012 093
Oct. 20	3.251 608	4.140 629	3.136 448			
Nov. 29	3.361 079	3.978 533	3.073 768	0.003 208	0.029 306	0.013 840
1933 Jan. 8	3.463 367	3.807 928	3.004 507			
Febr. 17	+3.557 921	−3.628 815	−2.928 528	−0.001 299	−0.032 550	−0.015 793

Date 0^h U. T.	Direct co-ordinate method (Cowell scheme)			Perturbations in co-ordinates (Encke)		
	x	y	z	ξ	η	ζ
1933 March 29	+3.644 136	−3.441 193	−2.845 677			
May 8	3.721 349	3.245 068	2.755 787	+0.001 154	−0.035 986	−0.017 984
June 17	3.788 825	3.040 449	2.658 679	0.002 625	0.037 773	0.019 180
July 27	3.845 744	2.827 359	2.554 158	0.004 286	0.039 603	0.020 450
Sept. 5	3.891 191	2.605 837	2.442 014	0.006 163	0.041 473	0.021 799
Oct. 15	3.924 135	2.375 945	2.322 019	0.008 286	0.043 378	0.023 234
Nov. 24	3.943 410	2.137 783	2.193 934	0.010 690	0.045 308	0.024 761
1934 Jan. 3	3.947 686	1.891 496	2.057 503	0.013 418	0.047 253	0.026 387
Febr. 12	3.935 436	1.637 303	1.912 462	0.016 524	0.049 197	0.028 118
March 24	3.904 901	1.375 517	1.758 540	0.020 071	0.051 115	0.029 960
May 3	3.854 034	1.106 589	1.595 468	0.024 139	0.052 972	0.031 916
June 12	3.780 437	0.831 163	1.422 999	0.028 825	0.054 719	0.033 987
July 22	3.681 287	0.550 154	1.240 929	0.034 251	0.056 282	0.036 167
Aug. 31	3.553 237	−0.264 877	1.049 142	0.040 566	0.057 550	0.038 438
Oct. 10	3.392 302	+0.022 782	0.847 682	0.047 955	0.058 361	0.040 762
30	3.298 052	0.166 695	0.743 410	0.052 121	0.058 521	0.041 924
Nov. 19	3.193 733	0.310 092	0.636 871	0.056 642	0.058 466	0.043 066
Dec. 9	3.078 614	0.452 432	0.528 182	0.061 551	0.058 142	0.044 172
29	2.951 900	0.593 058	0.417 509	0.066 883	0.057 487	0.045 214
1935 Jan. 18	2.812 744	0.731 172	0.305 083	0.072 669	0.056 421	0.046 159
Febr. 7	2.660 244	0.865 801	0.191 219	0.078 937	0.054 847	0.046 961
27	2.493 464	0.995 762	−0.076 338	0.085 707	0.052 645	0.047 561
March 19	2.311 457	1.119 617	+0.038 994	0.092 979	0.049 671	0.047 880
April 8	2.113 318	1.235 625	0.154 036	0.100 725	0.045 753	0.047 817
28	1.898 248	1.341 691	0.267 820	0.108 870	0.040 693	0.047 247
May 18	1.665 672	1.435 327	0.379 103	0.117 270	0.034 277	0.046 014
June 7	1.415 398	1.513 638	0.486 317	0.125 681	0.026 291	0.043 939
27	1.147 825	1.573 351	0.587 532	0.133 736	0.016 559	0.040 834
July 17	0.864 207	1.610 940	0.680 461	0.140 926	−0.005 000	0.036 524
Aug. 6	0.566 921	1.622 870	0.762 528	0.146 613	+0.008 295	0.030 898
26	+0.259 660	1.605 994	0.831 036	0.150 107	0.022 993	0.023 962
Sept. 15	−0.052 550	1.558 057	0.883 455	0.150 796	0.038 487	0.015 896
Oct. 5	−0.363 639	+1.478 217	+0.917 771	+0.148 313	+0.053 951	−0.007 060

The new osculating elements were computed for 1935 Aug. 26.0. The quantities necessary for this computation: x, y, z, $\frac{dx}{dt}$, $\frac{dy}{dt}$, $\frac{dz}{dt}$ can be taken directly from the Cowell scheme. In Encke's method ξ, η, ζ, $\frac{d\xi}{dt}$, $\frac{d\eta}{dt}$, $\frac{d\zeta}{dt}$ can be taken from the integration scheme but x_0, y_0, z_0, $\frac{dx_0}{dt}$, $\frac{dy_0}{dt}$, $\frac{dz_0}{dt}$

must be computed separately from the original elements using the formulae:

$$x_0 = A_x(\cos E - e) + B_x \sin E$$
$$\frac{dx_0}{dt} = \frac{k}{r_0\sqrt{a}}(B_x \cos E - A_x \sin E)$$

and the corresponding formulae for the other two co-ordinates.

The result was:

1935 Aug. 26.0.

	Direct co-ordinate method (COWELL scheme)	Perturbations in co-ordinates (ENCKE)
x_0	. . . .	$+0.109\ 3664$
y_0	. . . .	$+1.582\ 9820$
z_0	. . . .	$+0.855\ 0345$
ξ	. . . .	$+0.150\ 1068$
η	. . . .	$+0.022\ 9925$
ζ	. . . .	$-0.023\ 9624$
$\frac{dx_0}{dt}$	. . . .	$-0.909\ 3556$
$\frac{dy_0}{dt}$	. . . .	$-0.137\ 6869$
$\frac{dz_0}{dt}$	. . . .	$+0.154\ 9182$
$\frac{d\xi}{dt}$	. . . .	$+0.006\ 3324$
$\frac{d\eta}{dt}$	. . . .	$+0.044\ 2363$
$\frac{d\zeta}{dt}$	. . . .	$+0.021\ 9346$

	Direct co-ordinate method (COWELL scheme)	Perturbations in co-ordinates (ENCKE).
x	$+0.259\ 660$	$+0.259\ 473$
y	$+1.605\ 994$	$+1.605\ 975$
z	$+0.831\ 036$	$+0.831\ 072$
$\frac{dx}{dt}$	$-0.903\ 015$	$-0.903\ 023$
$\frac{dy}{dt}$	$-0.093\ 398$	$-0.093\ 451$
$\frac{dz}{dt}$	$+0.176\ 882$	$+0.176\ 853$
A_x	$-0.977\ 63$	$-0.977\ 62$
A_y	$+3.435\ 76$	$+3.435\ 74$
A_z	$+2.166\ 73$	$+2.166\ 71$
B_x	$-3.241\ 14$	$-3.241\ 12$
B_y	$-1.065\ 45$	$-1.065\ 44$
B_z	$+0.227\ 07$	$+0.227\ 07$
M	354°.992	354°.993
ω	38. 787 } 1950.0	38. 786 } 1950.0
Ω	65. 708 } 1950.0	65. 708 } 1950.0
i	13. 722 } 1950.0	13. 722 } 1950.0
φ	35°.073	35°.072
a	4.1779	4.1779
μ	0°.115 416	0°.115 417
T	1935 Oct. 8.395 U. T.	1935 Oct. 8.383 U. T.
P	8.5398 jul. years	8.5397 jul. years

The question of the accuracy of the two methods might be raised. The ENCKE method has been in use for many years and is known to yield excellent results. As will be seen from the above, the discrepancies between the rectangular co-ordinates derived from the two methods are not inconsiderable. They can not be due to any error of computation, as the calculations were made independently by two computers, so the cause must be sought elsewhere. The original elements are not accurate to more than 6 figures, so the uncertainty in the rectangular co-ordinates and velocities at the date of osculation will appear in the 6th figure. If a velocity computed for the beginning of the COWELL scheme should be one unit wrong in the 6th decimal, this error would accumulate during the computation of the more than 150 intervals and make itself felt in the 4th decimal of the rectangular co-ordinates. This source of error is not found in ENCKE's integration scheme because at the beginning the perturbations in co-ordinates and velocities are all exactly 0, but the uncertainty arising from the original elements will here appear in the unperturbed co-ordinates, the accuracy of which is diminishing with the time elapsed since osculation corresponding to what happens to the co-ordinates from the direct co-ordinate method. If for instance a in the original elements should be one unit wrong in its 6th significant figure (0.00001) this would make itself felt in the 4th decimal of the unperturbed co-ordinates at the osculation date in 1935. However, the two systems of elements agree within 5 figures which should be ample accuracy for the recovery of the comet in 1935. A decisive comparison between methods is possible only when they are applied to a celestial object that has been followed by observation and calculation for a longer period of time.

With the elements gained from ENCKE's method the following ephemeris for the return in 1935 has been computed:

0^h U. T.

1935	α_{vera}		δ_{vera}		r	Δ
Aug. 2	6^h	$0^m.2$	$+24°$	$20'$	1.893	2.575
6	6	11.3	24	38		
10	6	22.5	24	54		
14	6	33.8	25	7		
18	6	45.1	25	18	1.846	2.436
22	6	56.6	25	26		
26	7	8.1	25	32		
30	7	19.6	25	35		
Sept. 3	7	31.2	25	35	1.810	2.301
7	7	42.8	25	33		
11	7	54.4	25	29		
15	8	6.0	25	22		
19	8	17.5	25	13	1.787	2.174
23	8	29.0	25	1		
27	8	40.4	24	48		
Oct. 1	8	51.7	24	33		
5	9	2.9	24	15	1.778	2.053
9	9	14.0	23	57		
13	9	24.9	23	37		
17	9	35.7	23	16		
21	9	46.3	22	54	1.782	1.939
25	9	56.8	22	31		
29	10	7.1	22	7		
Nov. 2	10	17.1	21	44		
6	10	27.0	21	21	1.799	1.831
10	10	36.6	20	58		
14	10	45.9	20	35		
18	10	55.0	20	13		
22	11	3.8	19	53	1.830	1.727
26	11	12.4	19	33		
30	11	20.6	19	15		
Dec. 4	11	28.5	18	59		
8	11	36.0	18	45	1.872	1.625
12	11	43.2	18	33		
16	11	50.1	18	24		
20	11	56.5	18	17		
24	12	2.5	18	13	1.925	1.527
28	12	8.1	18	11		
32	12	13.3	$+18$	13		

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