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ON THE LICHTENBERG FIGURES

PART III. THE POSITIVE FIGURES

BY

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WITH 28 PLATES

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CHAPTER I

Introduction.

The determination of the spreading-out-velocity of the LICHTENBERG Figures has been discussed in some previous publications which have also touched upon the problem of the formation of these figures[1].

In the meantime the photographic LICHTENBERG Figures have been successfully applied to the study of surges on high tension lines, especially the kind of surges due to lightning[2]. They have also been used for the measurement

[1] P. O. PEDERSEN: "On the Lichtenberg Figures": Part I. Vidensk. Selsk. Math.-fys. Medd. Vol. I, No. 11. Copenhagen (February 1919); Part II. Vol. IV, No. 7, Copenhagen 1922; referred to as L. F. I and L. F. II respectively. — "Die Ausbreitungsgeschwindigkeit der Lichtenbergschen Figuren und ihre Verwendung zur Messung sehr kurzer Zeiten". Ann. d. Physik (IV) Bd. 69, p. 205—230, 1922.

[2] J. F. PETERS: "The Klydonograph" "El. World" Vol. 83, p. 769—773, 1924. — J. H. COX and J. W. LEGG: "Trans. A. I. E. E." p. 857—870, 1925. — K. B. MCEACHRON: "Trans. A. I. E. E.", p. 712—717, 1926. — J. H. COX, P. H. MCAULEY and L. GALE HUGGINS: l. c. p. 315—329, 1927. — J. H. COX: l. c. p. 330—338, 1927. — R. J. C. WOOD: "Trans. A. I. E. E." p. 961—968, 1925. — EVERETT S. LEE and C. M. FOUST: "Trans. A. I. E. E." p. 339—348, 1927 and "Gen. Elec. Review" Vol. 30, p. 135—145, 1927. — W. W. LEWIS: "Trans. A. I. E. E." p. 1111—1121, 1928. — E. W. DILLARD: l. c., p. 1122—1124, 1928. — J. G. HEMSTREET and J. R. EATON: l. c., p. 1125—1131, 1928. — PHILIP SPORN: l. c., p. 1132—1139, 1928. — N. N. SMELOFF: l. c., p. 1140—1147, 1928. — H. MÜLLER: Mitteil. d. Hermsdorf Schomburg Isolatoren G. m. b. H., Heft 27, p. 813—829, 1926. — P. O. PEDERSEN: "Ingeniøren", p. 201—209, 1928. "Danmarks Naturvidenskabelige Samfunds Skrifter", A. No. 18, Copenhagen 1928. — MÜLLER-HILLEBRAND: "Siemens Zeitschr." 7, p. 547—551, 605—612, 1927. — E. BECK: "The Electric Journal", p. 591—595, 1928; p. 50—53, 1929.

of very short intervals of time, down to 10^{-10} sec. and even less[1]. Such measurements have been used extensively by the writer[2], by M. Iwatake[3] and others for the determination of time lag in electric sparks. The problem of spark lag and spark formation will, however, be treated of elsewhere.

Fig. 1 shows a sketch of the diagram of connections used for obtaining photographic *L. F.*, compare L. F. I, Fig. 10.

In L. F. I the previous theories of the formation of the L. F. have been mentioned and it is hardly necessary to

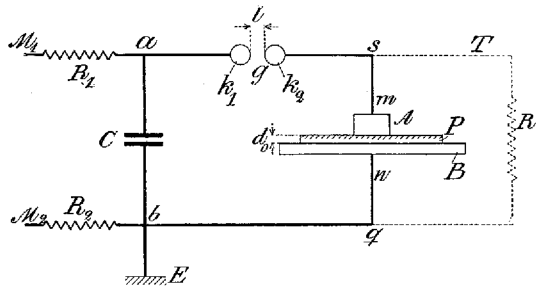


Fig. 1. Diagram of connections for obtaining photographic L. F. M_1 and M_2 are leads from the high tension source. R_1, R_2 and R_3 high resistances (slate pencils or the like). P a photographic plate, B a metal plate connected to earth E.

renew this discussion, especially because no sound theory could be worked out before the velocity of the figures was known.

[1] P. Heymans and N. H. Franck: "Phys. Review" (II). Vol. 25, p. 865—869, 1925.

[2] P. O. Pedersen: (a): Vidensk. Selsk. Math.-fys. Medd. Vol. IV, No. 10, Copenhagen 1922. — (b): l. c. Vol. VI, No. 4, Copenhagen 1924. — (c): "Teknisk Tidskrift (Elektroteknik)", p. 174—184, Stockholm 1923. — (d): Ann. d. Physik (IV). Bd. 71, p. 317—376, 1923. — (e): l. c., Bd. 75, p. 827—847, 1924.

[3] M. Iwataka: "Technology Reports Tôhoku Imp. University" Vol. 7, Nr. 1, p. 57—86, 1927. This paper contains an extensive bibliography on the time lag of electric sparks.

There are, however, one or two exceptions to be mentioned below.

U. YOSHIDA[1] has given a theory of the formation of the negative figures and of the characteristic dark radii in these figures which is in the main satisfactory. His theory is, with one important exception to be mentioned later, identical with that given by the writer in L. F. I[2]. According to YOSHIDA the negative figures are due to negative ions which the electric field drives away from the electrode and which cause ionization by collision along their paths. YOSHIDA does not state the nature of these ions and he does not point out that they must necessarily be electrons, as shown in L. F. I. But this necessity did not exist at the time when YOSHIDA worked out his theory because the great velocity of the spreading out of the L. F. was not known then.

K. PRZIBRAM, who has contributed a long series of important papers[3] on the L. F. has accepted the same view of the formation of the negative figures[4], while M. TOEPLER[5, 6] has been led to a somewhat different interpretation of the negative figures in his important and long continued investigations of gliding discharges.

All circumstances considered, the main points of the

[1] U. YOSHIDA: (a): Mem. Kyoto Imp. University, Vol. II, p. 105—116. 1917. — (b): l. c., p. 315—319. 1917.

[2] The writer did not know of the two mentioned papers of YOSHIDA at the time he wrote L. F. I, which paper was presented to the Royal Danish Soc. of Science on March 8, 1917.

[3] See bibliography in "L. F. II", p. 35.

[4] K. PRZIBRAM: (a): Phys. Zeitschr. Bd. 20, p. 299—303. July 1919. — (b): Die elektrischen Figuren in Handb. d. Physik Bd. XIV, p. 391—404, 1927.

[5] M. TOEPLER: (a): Phys. Zeitschr. Bd. 21, p. 706—711, 1920. — (b): Arch. f. Elektrotechnik Bd. 10, p. 157—185, 1921.

[6] K. PRZIBRAM: l. c., (b): p. 403—404.

above mentioned theory of the formation of the negative figures are so well founded that it is hardly necessary to discuss this problem further. In what follows we therefore only treat of the negative figures in so far as it is necessary in order to throw light on the formation of the positive figures.

The theory of the secondary and tertiary figures given in L. F. I has lately been fully corroborated in some interesting experiments of U. Yoshida[1] who has also given experimental proof of some further consequences of that theory. M. Toepler[2] has also investigated these secondary and tertiary figures and his results agree fairly well with those obtained in L. F. I and by Yoshida. We therefore need not further discuss the theory of these figures.

The only outstanding problem is therefore the formation of the positive figures, but this question is, no doubt, the most important and also the most difficult of all the problems connected with the theory of the L. F. In the course of the last 10 years we have made many experiments aiming at the elucidation of the nature of the positive figures. Most of the experimental material — containing among other things some 2500 photographic L. F. — was collected in the years 1918—20. But it was only about a year ago that the writer succeeded in putting forth a hypothesis which made it possible to establish a coherent theory of the formation of the positive figures explaining all their peculiarities in a satisfactory manner.

Before entering upon the detailed discussion of the experimental results and their theoretical explanation, it

[1] U. Yoshida and G. Tanaka: Mem. Kyoto Imp. University. Vol. V, No. 2, 145—152, 1921.

[2] M. Toepler: Phys. Zeitschr. Bd. 22, p. 78—80, 1921.

will be convenient to dwell a little on some preliminary questions, namely concerning the nature of the photographic impressions of L. F. in various gases.

With regard to the terminology adopted in the following we have to lay stress on the fact, that LICHTENBERG figures and discharges only refer to the well known, regular figures

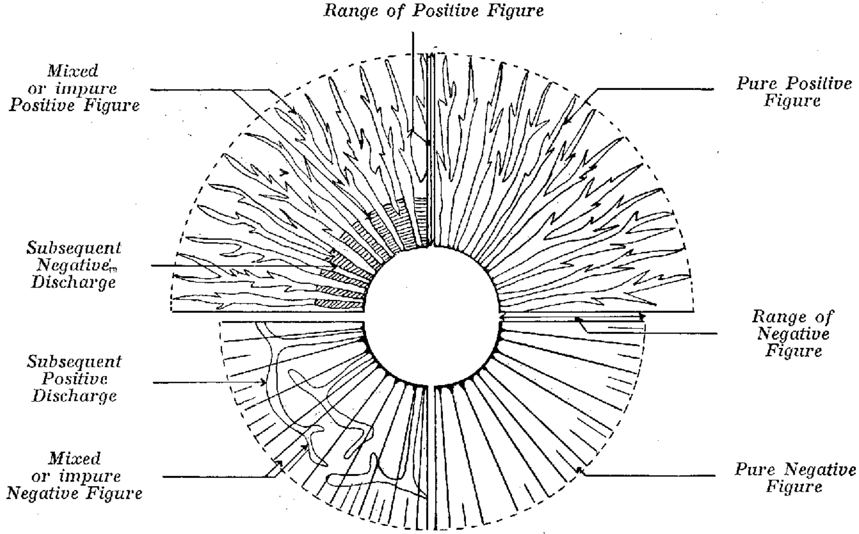


Fig. 2. Regular or simple Lichtenberg Figures. Upper part Positive and lower part Negative Figures. Right hand part pure, left hand part impure or mixed Figures.

showing the characteristic differences between positive and negative discharges and of relatively feeble luminosity but not to the bright sparks or spark tracks which occur if the *p. d.* is sufficiently high and of sufficient duration. Photographs of such spark tracks are to be seen on plate 1, parts VI—VII, and one single track on part III, plates 3 I, 7 II, and 11 I and IV.

The Lichtenberg discharges and figures may start either directly from the electrodes or from the above named bright spark tracks. In the following we will call the first kind regular the latter kind irregular figures and discharges. Samples of the first kind are shown in L. F. I figs. 2, 3, 6, 7, 8, 9, 12, 13, 19, 26, 27, 28 and in this paper on plate 1, parts I and V, plate 2, parts I—III and

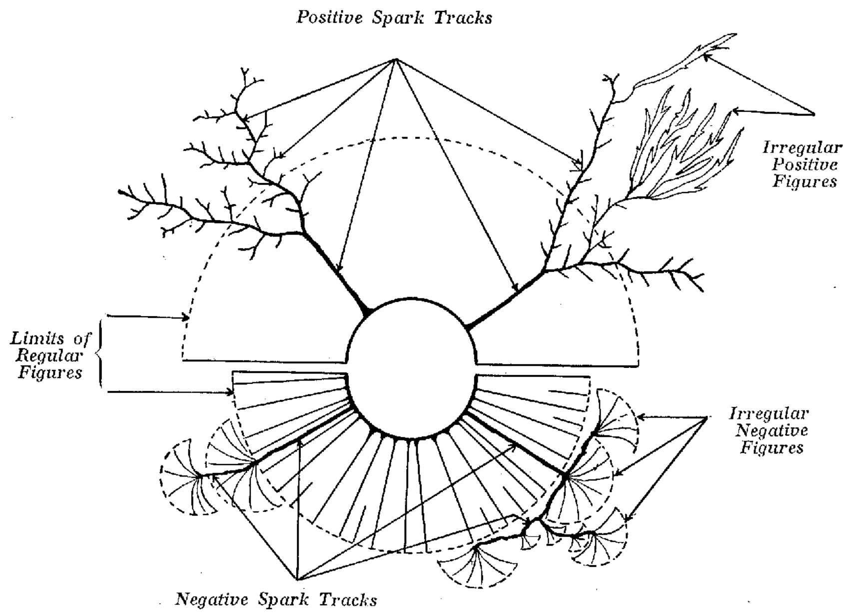


Fig. 3. Composite Lichtenberg Figures.

plate 3, parts I—III. Samples of the latter kind are seen in L. F. I figs. 4 and 5 and in this paper plate 1, parts III, VI and VII and plate 3 part I.

The regular figures will normally always be formed by a potential flash of very short duration. If the *p. d.* is not of very short duration, bright spark tracks will be formed, and from these will again start irregular figures which — especially in case of positive ones — will cover and obscure the regular figure.

A figure consisting of only a regular Lichtenberg figure is often called a simple figure. Such a simple figure is said to be pure if only one single discharge — positive or negative — has taken place; see f. inst. plate 1, parts IV and V, plate 2, part III, plate 3, parts II and III, plate 14, part II and plate 25, part III. If the first discharge has been followed by a second one — generally somewhat weaker — of opposite sign, the figure is said to be a mixed or impure one; see f. inst. plate 1, part II, plate 2, parts I and II, plate 14, part I, plate 15, parts I and II, plate 18, part II and plate 25, part I.

The schematical figures 2 and 3 illustrate this terminology.

CHAPTER II

1. Photographic or "Electrographic" Action?

Are the L. F. due to the photographic effect of the luminosity of the discharge, or to some more direct action of the discharge on the sensitive film — to what may be called an "electrographic" action?

J. Brown[1] considered the luminosity of the discharge to be too feeble to cause the photographic images in the ordinary way and he quoted some experiments in support of this view. It has been proved, however, by U. Yoshida[2] that Brown's experiments are not conclusive.

A. A. Campbell Swinton[3] tried to settle the question by placing discs of non-actinic ruby glass and clear glass on the sensitive film, the small electrode resting on these discs. The ruby glass stopped all action, the clear glass allowed the action to take place all over the plate, though on account of the thickness of the glass, and the consequent intervening distance between the discharge and the film, the details of the figure produced were somewhat blurred and indistinct. Subsequently, using very thin glass, this indistinctness was almost entirely eliminated.

From these results Campbell Swinton draws the con-

[1] J. Brown: Phil. Mag. (5) Vol. 26, p. 503—505, 1888.

[2] U. Yoshida: Mem. Coll. Sc. Kyoto Imp. University Vol. II, No. 2, p. 105—116, 1917.

[3] A. A. Campbell Swinton: "The Electrical Review" Vol. 31, p. 273—275, 1892.

clusion that "the action is due to the ordinary photochemical effect of the light produced by the spark, which though feeble in intensity to the eye, is blue, and must be remarkably actinic".

Even if it is ultimately proved that CAMPBELL SWINTON's opinion is right we cannot consider his experiments conclusive. There is thus no doubt whatever that the light emitted from the spark tracks is strong enough to give the ordinary photographic image, and it is also evident that this light will be cut off by the inactinic ruby glass disc. And with the high potential differences used by BROWN and CAMPBELL SWINTON such spark tracks will occur in every case. The question here discussed only concerns the LICHTENBERG figures proper, and in this case it is much more difficult to attain a decision. With this aim in view we have made a number of experiments of which we shall quote some in the following.

Ebonite discs 0.3 mm thick cut off completely. This is in agreement with CAMPBELL SWINTON's Experiments.

The following experiment will show, however, that the conditions are completely altered with very thin plates. In the figure plate 7, part I the electrode was placed on a mica plate 0.05 mm thick, and the mica covered the photographic film below the line marked *mn*. The mica plate was covered by a dry layer of inactinic red ink. The photographic image shows clearly the Lichtenberg discharges, while the strong light from the spark tracks — as f. inst. *bc*, *de—f—g—h and hij* — is completely cut off. These spark tracks were certainly on the upper side of the mica plate and the Lichtenberg discharges are seen to start from these tracks. (At a few points, f. inst. that marked *g*, the red ink coating has been defective and an image of a short portion of the spark track is to be seen).

As the strong light from the spark tracks in this case has been unable to penetrate the inactinic coating of the mica plate, it is altogether impossible that the feeble luminosity of the Lichtenberg discharges on the upper side of the mica plate can be the cause of the photographic L. F. below this plate.

The photographic L. F. may in this case be due to: (a) the influence of the strong electric field on the photographic film; or (b) the ordinary photographic effect of the light emitted from points where there is a strong ionization by collision, such ionization taking place on the lower side of the mica plate directly below the Lichtenberg Figures which are formed on the upper side of the mica plate; or (c) possibly a combination of (a) and (b), the sensibility of the photographic film being increased by the strong electric field.

The proposition (a) cannot be true because an electric field does not in itself give any photographic image, but without further evidence it is not possible to choose between (b) and (c). This point is illustrated by plate 7, part I. From an inspection of this figure it appears that the Lichtenberg figures cross the boundary *mn* without any discontinuity[1]. It is also evident that the Lichtenberg discharges in the mica-covered part of the figure have started from certain spark tracks along the upper surface of the mica plate. The photographic L. F. cannot be due to an ordinary discharge between the mica plate and the photographic film, since if this were the case, the spark

[1] This fact is quite evident in the original photographs, somewhat less so in the reproduction.

tracks *abc*, *de—f—g—h* and *hij* would have been very bright in the image, and actually they do not show at all. The photographic L. F. below the mica plate must therefore be due to one of the effects (b) or (c).

Plate 7, part II shows the result of another experiment. The parts *VP 1* and *VP 2* of the photographic film were covered respectively with one and two layers of thin (0.02 mm) violet transparent paper, and other parts, *BP 1* and *BP 2*, respectively with one and two layers of 0.04 mm thick opaque, black paper.

The paper strips were soaked in clear vaseline and pressed against the photographic film, no air being left between the strips and the film. All superfluous vaseline was removed before exposing the film to the discharge, and all strips and vaseline removed before developing the photographic plate. The resulting figure is seen in plate 7, part II.

Below the violet paper both the L. F. and the spark tracks are to be seen, clearest of course with only one layer of the paper. Below the black paper strips, on the contrary, there is no image of either L. F. or spark tracks. (In the case of one layer there are some faint spots of light beneath the spark which has passed over the upper surface of the strip, these spots being no doubt due to small holes in the papers).

This experiment proves that a strong electric field does not give any image in the case where there is no air, and therefore no ionization by collisions at the surface of the photographic film. But the experiment does not absolutely prove that the image of the L. F. is due solely to the light emitted by the Lichtenberg discharge, because the photographic film is subjected to a strong electric field simultaneously with the exposure to the light from the discharge. This question can be settled, however, by means of the experiments illustrated in Fig. 4. Upon the film of the ordinary photographic plate P were placed some small pieces of photographic plates with the film downward, either as at P', with a small distance δ between the films, or as at a, b, c, with the two films in direct contact. Even in this case the films only touch each other in a number of points since the films of small broken pieces a, b, c have somewhat projecting edges. In all these cases the Lichtenberg discharges have taken place in the space between the two films. Plate 8, part I shows the result of such an experiment with three pieces: a, b, c, the main plate P and the three pieces of plate being developed in exactly the same manner. It appears that there is very little difference between the photographic intensity of the image on the main plate and on the small plates. Even in cases such as P' in Fig. 4, where there is a considerable distance between the two films, the intensity of the image on the upper film may be almost as great as on the main film, see plate 7, part III. Up to $\delta = 1$ to 1.5 mm the image on P' is quite distinct; for greater distances it becomes blurred.

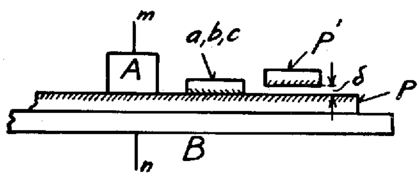


Fig. 4. Ordinary arrangement for obtaining photographic L. F., compare Fig. 1. P' and a, b, c are small pieces of photographic plates with the film downwards.

Since the intensity of the electric field at the film of the pieces P', a, b, c, is very small in comparison with the field at the film of the plate P, and since the photographic intensity is almost the same in the two cases, it is proved that the photographic L. F. are due to the ordinary photographic effect of the light emitted by the discharge. But the light may come from discharges in a very thin layer of air between the photographic film and the covering plate, these discharges being either ordinary Lichtenberg discharges —, as in plate 7, part III and plate 8, parts I—II, — or in cases where the covering plates are very thin, very intense ionizations due to very strong fields at right angles to the film, as in plate 7, part I.

This point of view is in accordance with all the previously known facts and with a number of further experiments and observations of which only a few will be mentioned in the following.

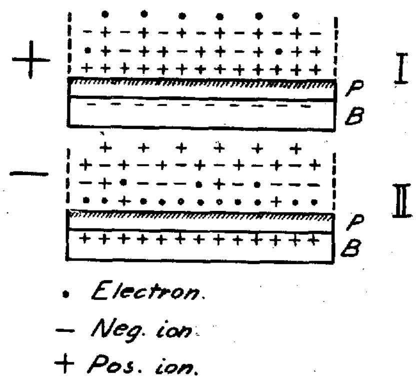


Fig. 5. Schematic representation of the distribution of electrons and positive and negative ions over the cross-section of positive and negative streamers.

With regard to the distribution of the photographic intensity it is to be remembered that the emission of light is mostly caused by the recombination of positive ions with either electrons or negative ions. Fig. 5 gives a schematical sketch of the distribution of electrons and positive and negative ions over the cross-section of positive and negative streamers. But the question of intensity-distribution will be taken up later on in Chap. IV 1 (d).

2. Lichtenberg Figures in Various Gases.

Air, nitrogen and argon give strong photographic L. F. both positive and negative, the last being by far the strongest. In oxygen the luminosity of the Lichtenberg discharges — both the visual and the photographic — is very small. It has not been possible to get positive photographic figures in almost pure oxygen — containing about one per cent of hydrogen —. In oxygen containing small amounts of atmospheric air or nitrogen very feeble positive photographic figures have been obtained, as for inst. plate 1, part II[1]. Part III is the corresponding negative figure.

It may be supposed that the positive discharge in oxygen is very feeble or even that there is no discharge at all in this gas, thus explaining the failure to obtain positive figures. But this supposition would be wrong, for there are strong positive discharges even in the purest oxygen, as may be proved by means of the dust method, using an electrically clean plate of ebonite instead of the photographic plate *P*. A photograph of such a dust figure is shown in plate 1, part I.

The photographic negative figure in oxygen is similar to the negative figure in air, but fainter, see plate 1, part IV. Owing to the faintness there is some difficulty in determining the range of the figures in oxygen. Table 1 contains some approximate values of such ranges in air and in oxygen[2].

[1] This positive figure is impure. The strong light in the neighbourhood of the electrode is due to a negative discharge taking place after the formation of the positive figure, see Chap. III 1 (d) and Chap. IV 1 (d).

[2] K. Przibram. (Wien. Ber. (II a) (a) Bd. 127, p. 395—404, 1918 and (b) Bd. 129, p. 151—160. 1920; (c) Phys. Zeitsch. Bd. 20, p. 299—303. 1919) found that the ratio $\frac{R+}{R-}$ was smaller in oxygen than in air (l. c.) (a) p. 402, (b) p. 151—2) while the writer previously came to the opposite result (L. F. I, p. 35 and 42). The new investigations have fully con-

Table 1. $p = 350$ mm Hg.; $l = 3$ mm; $d_0 = 1.5$ mm.

Air.	$R+ = 34$ mm[1]	$R- = 17$ mm	$\frac{R+}{R-} = 2.0$
Oxygen ($98\ O_2 + 2\ H_2$).	$R+ = 35$ mm[1]	$R- = 13$ mm	$\frac{R+}{R-} = 2.7$

The photographic positive figures are also very faint in hydrogen, see f. inst. plate 2, parts I and II. But large and finely branched dust figures may be obtained in this gas. The photographic negative figures are also fainter in hydrogen than in air or nitrogen, see plate 2, part III.

All these circumstances agree well with our previous result, namely that the photographic L. F. are caused by the light emitted by the discharge, the visual luminosity in hydrogen being considerably less than in air but greater than in oxygen.

For general information, some figures in other gases have been included. Thus plate 3, parts II and III show regular positive and negative figures in CO_2[2] while part I shows some positive spark tracks in the same gas[3].

firmed our previous results. The difference between Przibram's and our results is perhaps due to the circumstance that our measurements refer to simple, regular Lichtenberg figures both in air and in oxygen — compare plate 1, part IV — while the oxygen figures in Przibram's papers (l. c. (a) Figs. 4 and 5; (c) Figs. 5 and 6) are complicated figures with strong spark tracks similar to the figures in plate 1, parts VI and VII. On the other hand Przibram's positive nitrogen figure (l.c. (a) Fig. 3) is a regular simple L. F.

[1] Other experiments also indicate that the range $R+$ is almost the same in air and oxygen.

[2] S. Mikola (Phys. Zeitsch. Bd. 18, p. 161, 1917) says: "In den anderen untersuchten Gasen (O, H, CO_2 und Leuchtgas) entwickeln sich die Strahlungsfiguren kaum sichtbar". This is strictly speaking only true of the positive figures in oxygen.

[3] The positive figure by Przibram (Wien. Ber. (II. a). Bd. 108, 1161—1171, 1899) indicates the presence of some air or nitrogen besides the gas CO_2.

We have not succeeded in obtaining photographic L. F. in pure helium. Plate I, part V shows a figure taken in a mixture of air and helium and plate 4, part II a figure from a mixture of air and argon.

The above remarks concerning the intensity of the luminosity refer only to the regular, pure L. F. The light from the spark tracks is very strong in all gases[1].

Plates 4—6 show parts of a number of positive figures in mixtures of N_2, O_2 and H_2 in various ratios. These figures will, however, be discussed later (Chap. IV. 3 (b)).

[1] Compare also PRZIBRAM l. c. (a) p. 399.

CHAPTER III

The Properties of the Positive Figures.

1. Differences of Form and other Features between Positive and Negative Figures.

With regard to the main points of these differences we may refer to the L. F. I and for ease of reference we repeat here figs. 6 and 7, which show respectively a positive and a negative photographic L. F. obtained by means of the experimental arrangement shown in Fig. 1.

Besides the very obvious differences between the two kinds of figures, which need no further comment, we shall in the following give some particulars about some specific points.

(a) Width of the Positive and Negative Spreaders.

The width of the negative spreaders varies greatly, the broadest ones having often ten times the width of the narrowest ones, compare f. inst. plate 25, part I and L. F. I Table 3, p. 28.

The positive spreaders on the contrary have in all cases almost the same width at the same distance from the tip, and the width is very nearly inversely proportional to the pressure of the gas if measured at distances from the tip which are also inversely proportional to the pressure. This relation is illustrated by plate 6, part I and by the figures in the following table.

The width of the positive spreaders measured at a certain distance from the tip depends very little upon the voltage or upon the thickness of the insulating plate.

Table 2. Effect of Gas Pressure on the Width of the Positive Spreaders.

Pressure p in mm Hg	Width of Spreaders t in mm	Distance from Tip to Test Point in mm	$p \cdot t$
5×760	0.027	0.10	103
3×760	0.043	0.17	99
2×760	0.07	0.25	108
760[1]	0.12	0.5	91
300[1]	0.29	1.3	87
150[1]	0.59	2.6	89
75[1]	1.42	5.2	106
34[1]	3.60	11.0	119
30	3.60	13.0	108
17	7.0	22	119
		Mean value	103

[1] From Table 7 in L. F. I p. 36.

(b) The Ramification of Positive and Negative Figures.

The negative spreaders show no ramification at all[1]. The positive spreaders ramify extensively and the number of branches per unit length of spreader is very nearly proportional to gas pressure, see Table 6, L. F. I p. 35.

This number of branches is greatest in H_2 and decreases in the following order: A, N_2, Air, CO_2 and O_2,

[1] In very broad negative spreaders the end is often divided by dark radii, see f. inst. plates 21 II, 25 II—III and 27 VI. These dark lines are quite similar to those issuing from the electrode, and the explanation of them offers no further difficulty. U. Yoshida (Mem. Coll. Sc. Kyoto Imp. Univ. Vol. II, No. 2, p. 114—15, 1917) has treated this question in a very thorough and convincing manner.

compare plates 1—6. The number of small branches about the middle of the spreaders is comparatively great in mixtures of H_2 and N_2, see plate 5, XI—XVI. An addition of some O_2 to such mixtures reduces the number of branches very considerably, see f. inst. plate 4 I and 5 I—V and XVII—XVIII.

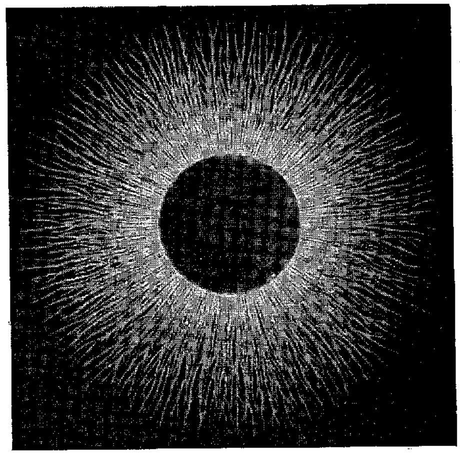


Fig. 6. Impure Positive Figure in Air.

(c) Boundary of the Positive and Negative Spreaders.

The photographic intensity of the negative figures falls off gradually at all points of the boundary but especially so at the outer edge. For the positive figures, on the contrary, the intensity drops down very abruptly to zero all along the boundary line. These properties of the positive and negative figures are illustrated by part II on plate 25.

(d) Distribution of the Photographic Intensity over the Area of the Positive and Negative Spreaders.

The photographic intensity of the negative spreaders has its greatest value at the electrode and on passing outwards it decreases gradually, becoming almost zero at the outer edge, see f. inst. plates I, parts III—IV, 2 III, 3 III, 12 I—IV, 25 I—IV, 26 I—IV, and 27 I—VI.

For pure positive spreaders, however, the photographic intensity has very nearly the same value over the whole area of the spreaders, see f. inst. plates 1 V, 4 I—II, 5 I—XVIII, 6 I, III, 10 I—IV, 12 V—VII, 13 II, 14 I—II, 21 I, 22, 23 and 25 II.

This intensity of the positive spreaders is much less than the maximum intensity of the corresponding negative spreaders.

The innermost parts of the positive spreaders very often, however, show a comparatively very strong intensity which at some distance from the electrode drops down rather abruptly to the normal positive intensity, see plates 2 I—II, 6 II, 12 VIII, 14 I, 15 I—II, 18 I (lowest part), 20 I, III, and 21 I and III. This high intensity is due to a subsequent negative discharge caused by electrical oscillations in the discharge circuit. Such oscillations may also cause a subsequent positive discharge in a previously formed negative figure, see f. inst. plate 18 II (the right hand part) and plate 25 I.

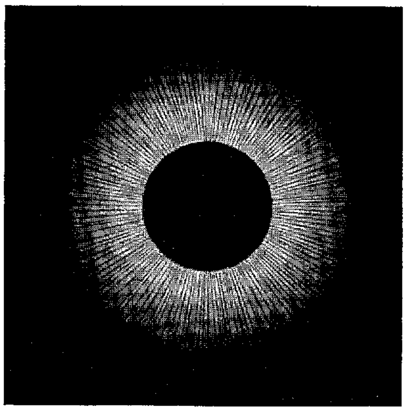


Fig. 7. Pure Negative Figure in Air.

In a positive figure, a subsequent negative discharge will evidently take place mainly out along the positive spreaders, and the resulting high intensity will therefore, be confined mainly to the area of these spreaders. By increasing the damping of the discharge circuit and by making the conditions unfavourable for the formation of negative figures, these subsequent negative discharges may be partly or completely eliminated. In the last case pure positive spreaders and figures are obtained. The damping is increased by reducing the leak resistance R, see Fig. 1, or by increasing the area of the positive figure, f. inst. by reducing the air pressure or by increasing — up to a certain point — the thickness of the insulating plate, compare L. F. I Fig. 34, p. 30. The range of the negative figure is, on the contrary reduced by increasing this thickness, as also appears from Fig. 34 in L. F. I.

It thus appears that pure positive figures are most easily obtained at reduced pressures and with great thickness of the insulating plate. These conclusions are in complete agreement with the experimental results.

With regard to the formation of subsequent positive discharges in previously formed negative figures the conditions are quite otherwise. We shall see later that negative discharges can start even with very low voltages — and from sharp points or edges probably down to almost zero voltage — while positive discharges do not start before the voltage has reached a certain minimum value, which decreases with decreasing pressure. The fact that impure negative figures generally only appear at low pressures is in complete agreement with this idea, for only at low pressures will the succeeding positive voltage be high enough to start a discharge.

U. Yoshida[1] considers the innermost bright part of the positive spreaders as due to an ionization caused by positive ions, while the outermost faint part is due to an ionization caused by negative ions, both these ionizations being essential to the formation of the positive discharge. We have seen, however, that the bright part is not a ne-

[1] U. Yoshida: Mem. Coll. Sc. Kyoto Imp. Univ. Vol. II, No. 2, p. 113, 1917.

cessary feature of the positive spreaders but may be eliminated altogether, and we think there can be no doubt that the bright part of the positive spreader here considered is due to a subsequent negative discharge.

On the other hand, if the voltage across the Lichtenberg gap is kept on long enough, both the positive and the negative ions will cause ionization, but the corresponding discharges and their images differ considerably from the Lichtenberg discharges and figures. We shall return to this question later on.

We thus come to the conclusion that the normal simple positive figure is pure and has almost the same photographic intensity over the whole area of the spreaders.

2. Range and Spreading-out-Velocity of the Positive and Negative Figures.

(a) Velocity of Positive and Negative Discharges.

The spreading-out-velocity is considerably greater for the positive than for the negative figures, f. inst. 2 to 4 times as great, see L. F. I p. 60.

(b) Relation between Air Pressure and Velocity.

The velocity, U, increases with decreasing pressure, p, for both positive and negative figures, but the manner in which U depends upon p is otherwise very different for the two kinds of figures, see fig. 8. At low pressures the velocity of the negative figures increases rapidly with decreasing pressure and this rapid increase continues down to a pressure of about 20 mm, which is about the lowest possible pressure at which the velocity can be measured in this manner[1]. For high pressures the velocity of the negative figures decreases slowly with increasing pressure.

The velocity of the positive figures, on the other hand, approaches a certain maximum value U_m with decreasing

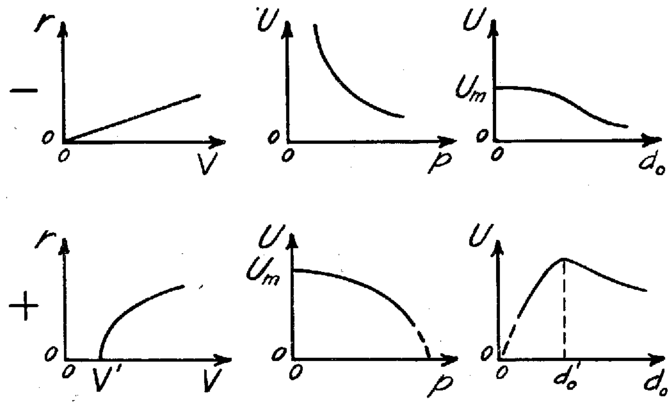


Fig. 8. Schematic representation of (1) the Range r against the Voltage V; (2) the Velocity U against the Pressure p; (3) the Velocity U against the total Thickness d_0 of the insulating Plate.

pressure and drops down to zero if the pressure is increased above a certain critical value, depending mainly on the voltage.

(c) Relation between Thickness of the Insulating Plate and Velocity.

For negative figures the velocity decreases with increasing thickness of the insulating plate; having its greatest value U_m for $d_0 = 0$. For positive figures the velocity seems to be zero for $d_0 = 0$, and for small values of d_0 the velocity increases rapidly with increasing values of d_0. At a certain thickness the velocity attains its highest value and decreases with further increase of d_0.

[1] See figure 24, p. 53.

(d) Relation between Voltage and Range of the Figures.

For negative figures the range, r, seems to go down to zero together with the voltage. In the case of positive figures there is a certain minimum voltage below which there is no discharge, and below which the range is accordingly equal to zero.

3. The Starting of the Positive and of the Negative Figures.

(a) Starting of the Negative Discharges.

From a finely pointed electrode placed directly on the photographic plate, even the smallest negative potential seems to start a figure at least if the potential is above some two hundred volts but the figures are very small at low potentials where the radius of the figure is proportional to the applied potential. This is shown in plate 21, part II to which again corresponds the straight line marked (—) in fig. 9.

(b) Starting of the Positive Discharges.

Under similar conditions, positive figures are only started if the potential is above a certain limit, the value of which increases with increasing air pressure. On the other hand, if a positive figure starts at all, it has always a finite and not inconsiderable range. For small potentials there is thus no proportionality between size of figure and potential. This is shown in plate 21, part I which again corresponds to curve (+) fig. 9.

This question is of such importance for the understanding of the formation of the positive figures that a closer consideration was necessary.

The production of a positive discharge — a positive figure — through the influence of a transitory potential depends not only upon potential and the air pressure, but also upon a series of other conditions, although the two first named factors are the most influential. We shall therefore first consider the influence of these two.

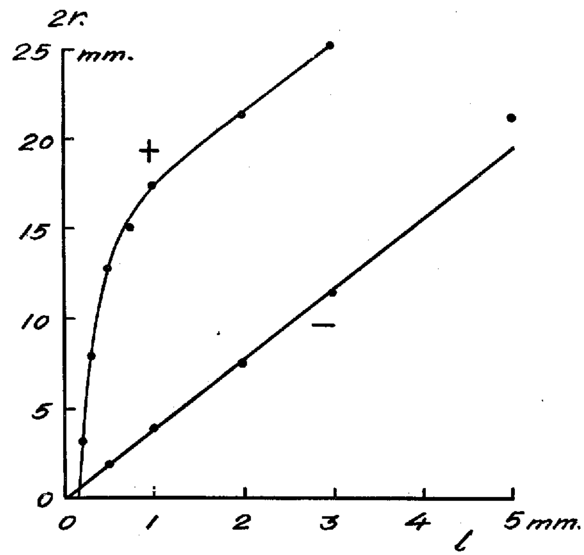


Fig. 9. Range r of positive (+) and negative (—) Figures against the spark length l.

Plate 21, part I shows how the size of the positive figures depends upon the potential at atmospheric pressure, but the conditions near the limit where figures may or may not appear are more favourably elucidated at lower pressures.

For general information plate 22 shows a number of figures produced at constant pressure (p = 100 mm Hg) but applying different potentials.

Where potentials here and in what follows are stated in volts, they are generated by a high-voltage D. C. dynamo, and the spark gap g in fig. 1 is replaced by a discharge key, the design of which we shall come back to later. The potential stated is the voltage across the discharge key before this is closed. Different forms of electrodes have been used; the figures reproduced on plate 22 and 23 were taken with a sharp-edged 3 mm cylindrical brass rod resting directly on the photographic plate.

At 100 mm pressure figures only appeared at potentials above 1160 volts. $V = 1366$ produces a pronounced figure having a radius of about 15 mm, as is also the case with the voltages 1382, 1400, 1410, 1430, 1446, 1478, 1536, 1541, 1581 and 1593 volts. At 1556 volts no discharge appeared at all, while all potentials above 1600 volts produced a figure. An inspection of the pictures on plate 22 shows that all the branches are of nearly equal length at potentials from 1366 to 1593 volts, and that the number of branches increases, although somewhat irregularly, with increasing potential. At 1556 volts the number of branches dropped to zero, no discharge appearing. This indicates a certain irregularity in regard to number of branches, whereas their range is nearly independent of the potential value within the critical interval.

The figures on plate 23 correspond to the constant potential 1190 volts, while the air pressure is varied from 40 to 101 mm Hg. At $p < 80$ mm the discharge appears as a uniform disc near the electrode with a number of teeth or branches stretching outward from the disc edge. At $p > 80$ the branches start directly from the electrode and they are of practically equal length at pressures from 80 to 99 mm, while their number decreases, though somewhat irregularly, with increasing pressure. At $p > 100$ the number of branches is zero, i. e. no discharge occurs at all.

Those series of figures presented on plates 22 and 23 are not especially selected or arranged but include all records taken in these particular series, which series again have been selected arbitrarily from a greater number.

To elucidate these conditions further we have in figs. 10—12 presented graphically the range r and the number of branches n as functions of either potential or air pressure. These curves also confirm the fact that the discharge — i. e. formation of the image — does not fail to appear because the length of the branches decreases toward zero but, on the contrary, because their number becomes zero. They also confirm the above mentioned irregularity in the dependency of n on pressure or potential.

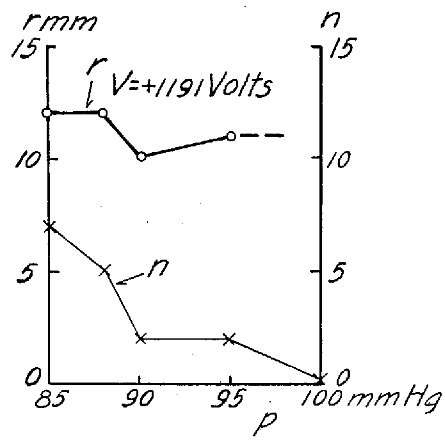


Fig. 10. Range r and number of spreaders n against the pressure p. ($V = +1191$ Volts).

The electrode used in the tests recorded in figs. 10—12 was a 3 mm round brass-rod the end of which was a plane surface, but nothing was especially done in order to keep the electrode sharp-edged. On the other hand a series of investigations have been carried out in order to ascertain how the shape and the state of the electrode may influence the formation of figures within the critical interval. The results of some of these investigations are shown in figs. 13—16. Four different forms of electrodes have mainly been tried: a 6 mm brass ball has been used for some of the tests in all of the figures; a rounded 3 mm brass rod is used in some of the tests in fig. 13; a sharp-edged electrode was used in some of the tests in figs. 13—15[1]; an electrode consisting of a

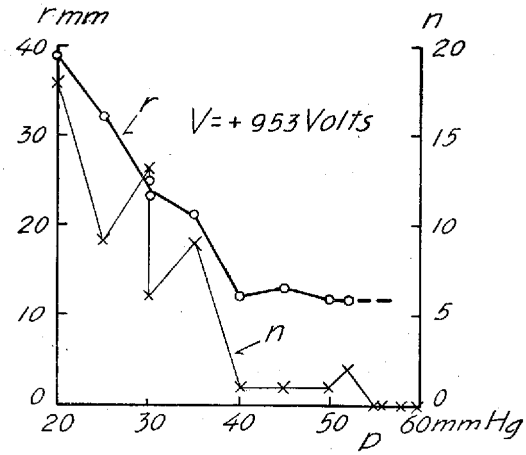


Fig. 11. Range r and number of spreaders n against the pressure p. ($V = + 933$ Volts).

brass tube with very thin walls — 0.15 mm thick — which was kept very sharp edged by grinding, was used in some of the tests in fig. 16; finally we have for some tests in

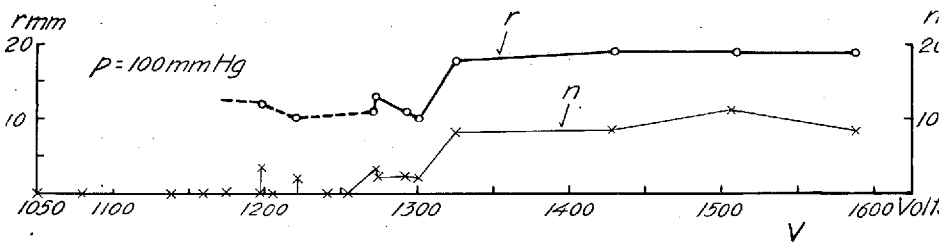


Fig. 12. Range r and number of spreaders n as functions of the p. d. V. ($p = 100$ mm Hg).

fig. 16 used a pointed electrode kept carefully pointed by repeated grinding. If nothing is stated to the contrary all

[1] This shape of electrode will gradually lose its sharp edge by the repeated cleaning process. For this reason we adopted the next electrode shape, a thin-walled tube.

the electrodes were kept "clean" by rubbing with carborundum[1]. The shape of the electrodes is stated in each figure. In some of them is indicated, in the same manner as in fig. 13, whether the figures consist of a uniform disc near the electrode. Finally in fig. 15, for some cases, the number of spreaders is indicated by figures. In fig. 16 we have only stated the number of spreaders n but not their length r, the values of which are given in figs. 13—15.

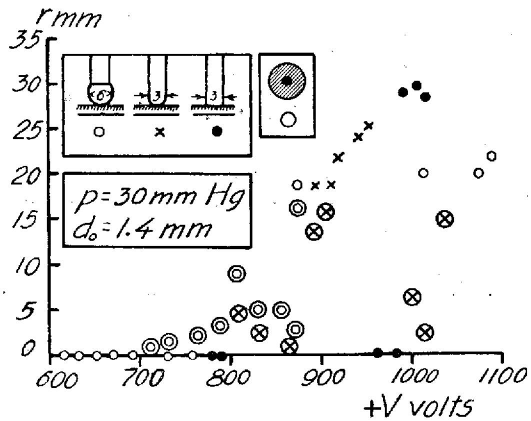


Fig. 13. Range r as function of the $p.d.$ V for three different shapes of the electrode. The large circles indicate that the figure has a continuous disc surrounding the electrode. (p = 30 mm Hg; d_0 = 1.4 mm).

Fig. 13, which refers to the conditions at 30 mm pressure, indicates that a figure is formed somewhat more easily from a spherical electrode than from a rounded or a sharp-edged rod, although the difference between the two first mentioned is not very pronounced. These tests also prove that with sharp-edged electrodes there are formed either spreaders of considerable length or no spreaders at all. With the spherical electrode and with the rounded rod the conditions change more gradually, since with these elec-

[1] See P. O. PEDERSEN: l. c., (a) p. 25, (d) p. 336.

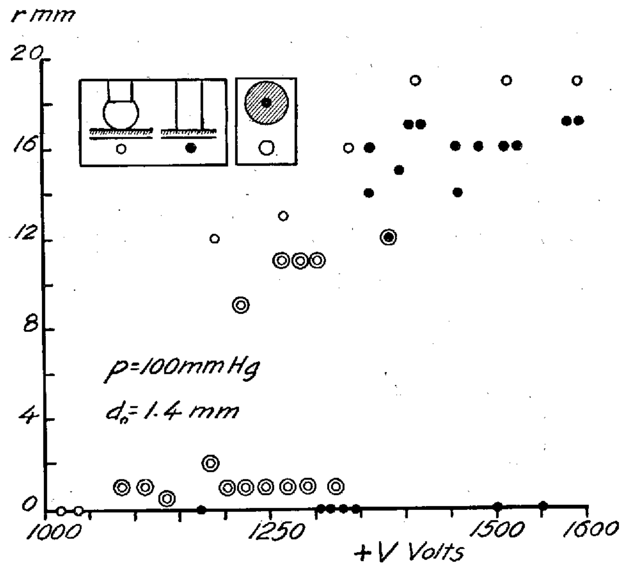


Fig. 14. Relation between Range r and Voltage V.

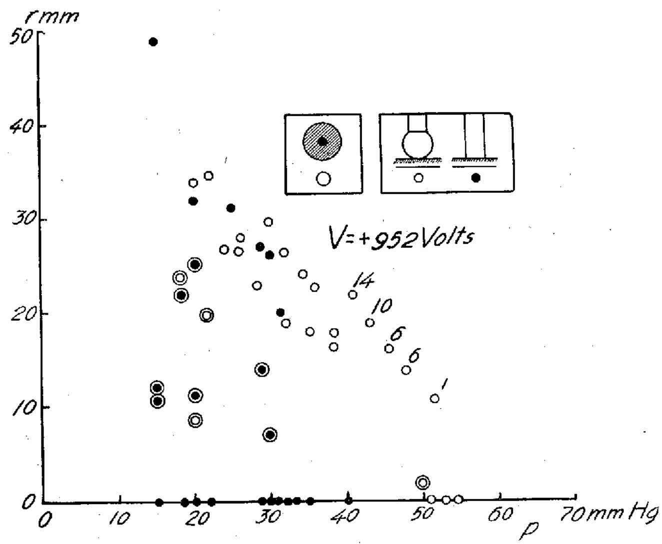


Fig. 15. Range r as function of the pressure p. ($V = +952$ volts). The figures at some of the representative points indicate the number of spreaders.

trodes figures of small range may be obtained within the critical interval. We will come back to this question later on.

Fig. 14, referring to tests at 100 mm pressure applying different potentials, and fig. 15, where the potential is kept constant at 952 volts but the air pressure is varied, show exactly similar relations with regard to the spherical elec-

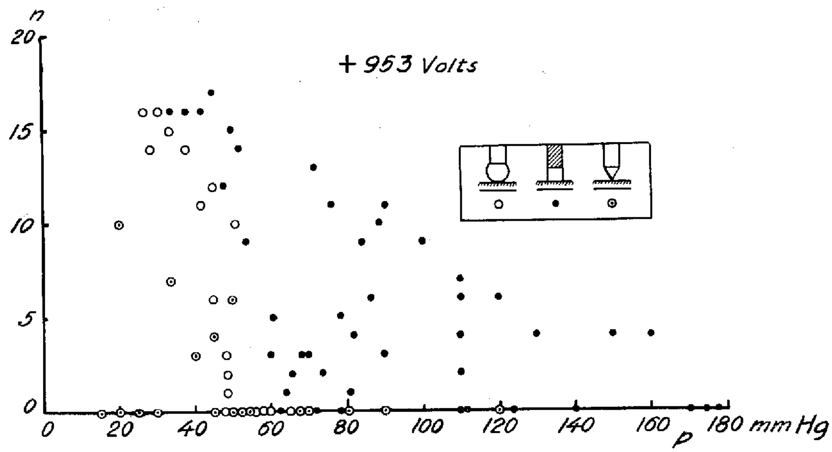


Fig. 16. Number of spreaders n as function of the pressure p. ($V = +953$ volts).

trode and the sharp-edged rod. Finally, in fig. 16 is shown the number of spreaders (n) formed with spherical, tubular and pointed electrodes, for $V = 953$ volts and varying air pressures. It appears from this figure that figures are most easily formed from the tube electrode, less easily from the spherical one and least easily from the pointed electrode. For negative figures the reverse is the case; they are formed by far more easily from a pointed than from a spherical electrode.

The small range sometimes attained by the figures within the critical interval with the spherical electrode and with the rounded rod may possibly be due to the vertical electric field which forces the discharges, started beneath these electrodes, down towards the film of the photographic plate, thus preventing them from spreading out in the normal way. The considerable decrease in range of the positive figures when the plate thickness d_0 tends to zero

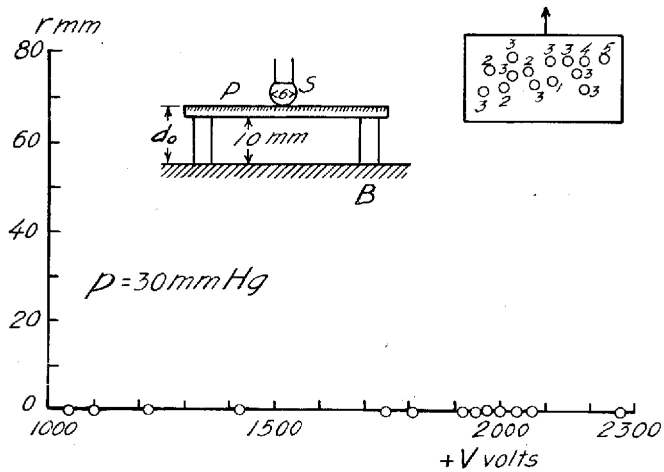


Fig. 17. Range r of positive figures from a spherical electrode. Thickness d_0 of insulating medium 11.4 mm. Experimental points marked by small circles. The figures at the experimental points denote the number of spreaders.

(see L. F. I fig. 34) suggests such an explanation. To elucidate this behaviour we have among others made the test shown in fig. 17.

The spherical electrode rests as usual directly on the film of the photographic plate P which, however, does not rest directly on the earthed plate B, but is raised 10 mm by means of the ebonite blocks shown. In this case the vertical field beneath the electrode will not be very strong, and, as expected, we find that under these conditions the spherical electrode gives either no discharge at all or forms comparatively very long spreaders as with the sharp-edged electrodes[1].

We have also investigated whether the design of the discharge key may influence the starting of the figures. Among others we have tried ordinary discharge keys and also mercury vacuum keys which in other respects show very special behaviour[2]. We have, however, not been able to ascertain any difference in the effect of the various keys.

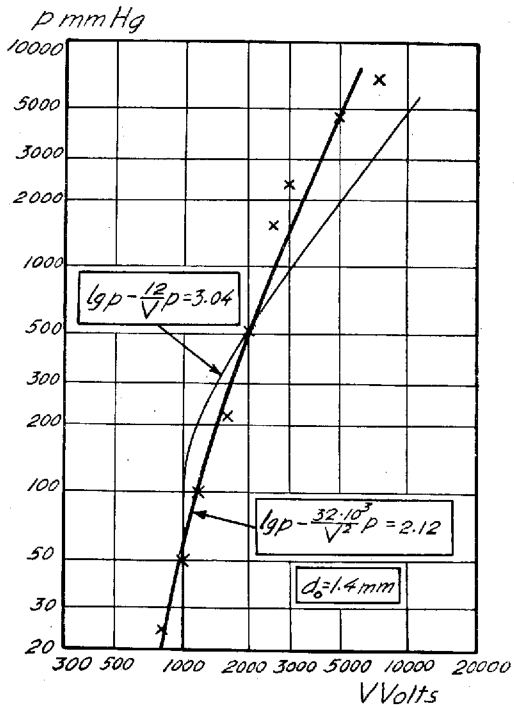


Fig. 18. Corresponding critical values of pressures p and $p.\,d.\,V$.

From the foregoing it appears that it is not possible to state with any great certainty the maximum air pressure at which a given potential may start a discharge, or conversely, at what minimum potential a discharge may be started at a given air pressure. The uncertain or fortuitous nature of the formation of figures

[1] The ranges represented by the points in the square frame are really greater than stated but the full range could not be determined as the figures — when discharges were started at all — passed beyond the edges of the photographic plates.

[2] P. O. PEDERSEN: Videnskabernes Selskabs Math.-fys. Medd. IV, Nr. 5, 1922. — Proc. Inst. Radio Eng. Vol. 13, p. 215—243, 1925.

within the critical interval prevents a true determination of a (p, V)-curve for this critical transitory interval. It is possible, however, on account of the large amount of experimental material available, to fix fairly correct corresponding values of p and V within the critical interval. Such sets of values are marked with crosses in fig. 18 for values of p from 22 to 7000 mm and of V from 700 to 7000 volts. We shall subsequently take up a discussion of this figure.

So far we have only considered "clean" electrodes but we have found that within the critical interval the formation of positive figures is independent of whether the electrodes are clean or slightly greased (unclean). Corresponding tests with negative figures show that these are formed much more easily from "clean" than from unclean (slightly greased) electrodes.

4. The Conductivity and the Spreading-out-Conditions of the Positive and Negative Figures.

There is a very pronounced difference in the manner in which the positive and the negative spreaders conduct electricity and in their ability to promote and initiate spark formation.

To elucidate this behaviour we have made the experiments described in the following.

(a) Conductivity of Positive and Negative Spreaders.

As soon as the edge of a negative figure reaches an outer electrode, a spark will pass between the two electrodes even where the outer electrode comes only very slightly inside the final range of the figure; see for example plate 26, parts I—IV. The spark discharge brings the two electrodes to a nearly equal potential.

In the case of positive figures, spark formation occurs only when the outer electrode comes far inside the range of the figure; see for instance plates 14 II and 15 I—II and also "L. F. I" figs. 45—50, L. F. II fig. 18 and "A. d. Ph." fig. 18. These relations were previously observed by U. YOSHIDA[1] and are also mentioned in "L. F. II" p. 22 and "A. d. Ph." p. 220.

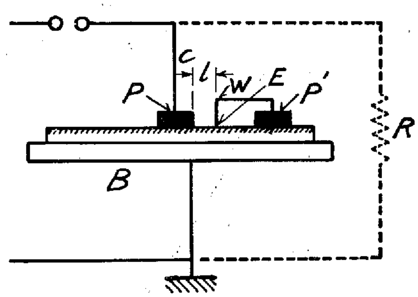


Fig. 19. Diagramatical representation of the electrical connections used in the experiments referred to in fig. 20 and in plate 12, parts I—VII.

For the further investigation of these relations, we have made among others the following experiments: Besides the usual electrode P, see fig. 19, a free electrode P' was placed on the film of the photographic plate. From P' a metal wire W is run to a point E of the film distant l from P. An account of the experimental results is given in fig. 20, parts III—V, and here are also shown the shapes of the electrodes employed, parts I—II. In part III the abscissa shows the ratio $\frac{l}{R}$, where l is the distance from the end E of the wire W to the electrode P, and R is the length of the spreaders from this electrode. The ordinate shows the ratio $\frac{R'}{R}$ of the spreaders from P and P' and for the positive figures also the ratio $\frac{n'}{n}$ of the number of spreaders from P' and P respectively.

[1] U. YOSHIDA: Mem. Coll. Sc. Kyoto Imp. Univ. Vol. II, No. 2, p. 115—116. 1917.

From parts III—V it appears that the conditions are essentially different for positive and negative figures with

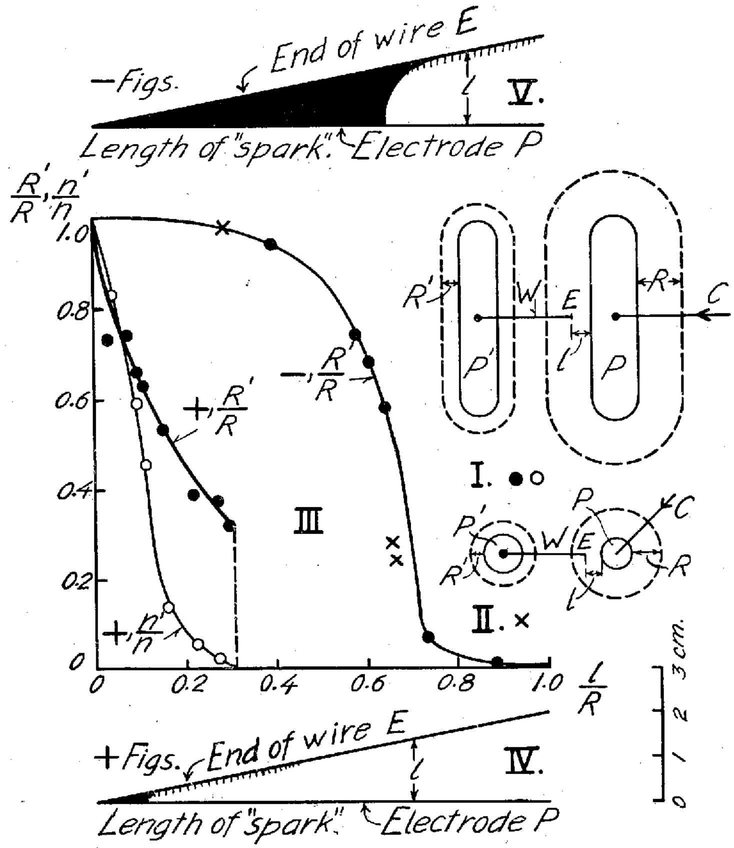


Fig. 20. Conductivity and sparking within positive and negative figures. In parts IV and V the black portions indicate that sparking has taken place. Air; $p = 760$ mm. Hg.

regard to conductivity and spark formation. With negative figures, pronounced spark formation occurs between the end E of the wire W and the electrode P when $\frac{l}{R} < 0.65$, and the ratio $\frac{R'}{R} > 0.9$ as long as $\frac{l}{R} < 0.5$. Not until

$\frac{l}{R} > 0.7$ does R' attain very small values and still at $\frac{l}{R} = 0.8$ to 0.9, R' has not decreased quite to zero. For positive figures conditions are entirely different. Here the formation of figures from P' stops completely when $\frac{l}{R} \cong 0.3$, and sparks are only formed when $\frac{l}{R} < 0.12$. Further the value of $\frac{R'}{R}$ decreases quite rapidly with increasing values of $\frac{l}{R}$ so that $\frac{R'}{R}$ has gone down to about 0.3 at $\frac{l}{R} = 0.3$ while for negative figures $\frac{R'}{R}$ remains nearly constant ($\cong 1$) up to $\frac{l}{R} = 0.4$.

In plate 12, parts I—VII, are reproduced some figures taken with the electrode arrangement shown in fig. 20, part I, though they only show the conditions existing near the end E of the wire W. (In these pictures the electrode P is placed to the left of E). From these pictures it appears that the spark formation is started at E, see for example plate 12, parts I and V, and 13 II. This relation is indicated schematically in fig. 20, parts IV and V.

From fig. 20, part III, it thus appears that "negative" sparks formed in this manner have a comparatively high conductivity, since the subsidiary figure starting from P' has nearly the same range as the one from P, as long as $\frac{l}{R} < 0.5$. Contrary to this the conductivity of the "positive" sparks is comparatively small, since both the length and the number of spreaders decreases very rapidly with increasing values of $\frac{l}{R}$ even at very small values of this quantity. A comparison of parts III and IV, fig. 20, on the other hand, shows that positive spreaders may allow a certain passage of electricity without the formation of a spark, since under these particular circumstances positive sparks are only formed when $\frac{l}{R} < 0.12$ while the subsidiary figure does not fail to appear until $\frac{l}{R} > 0.3$.

Fig. 20 at all events shows that both conductivity as well as tendency to spark formation is much smaller for positive than for negative figures.

(b) Influence of the Duration of the Pulse upon the Positive and the Negative Spreaders.

A corresponding difference of very pronounced character is found in another connection, namely the manner in

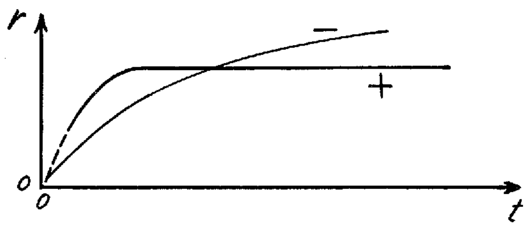


Fig. 21. Schematical representation of the range r of positive and negative figures as functions of the duration t of the *p. d.*

which the positive and the negative figures behave when exposed to potentials of comparatively long duration. This may be varied f. inst. by varying the length L_0 of the lead to the electrode from which the figure starts, L_0 being reckoned from the condenser C fig. 1. The longer L_0 is, the longer will the electrode be subjected to the potential. This point has already been investigated in "L. F. I" p. 32 (figs. 36 and 37). It is found that the radius of the negative figures increases with increasing length of the wire L_0 — at all events for lengths of wire up to about 25 m — as shown in fig. 21. This ability of the negative figures to grow larger is also illustrated in plate 28, part II. The needle-shaped Lichtenberg electrode in the diagram (part III) denoted by K, is first exposed to the potential corresponding to the spark length L of the spark gap shown. A surge travels out along the open line 8 m long and the *p. d.* is nearly doubled by reflection at the end. The Lichtenberg gap K is consequently exposed to a *p. d.* corresponding to the spark length L during $\frac{16}{3 \cdot 10^8} = 5.3 \cdot 10^{-8}$ sec. after which the *p. d.* momentarily increases to about the double value. In the negative figure, the growth of which had

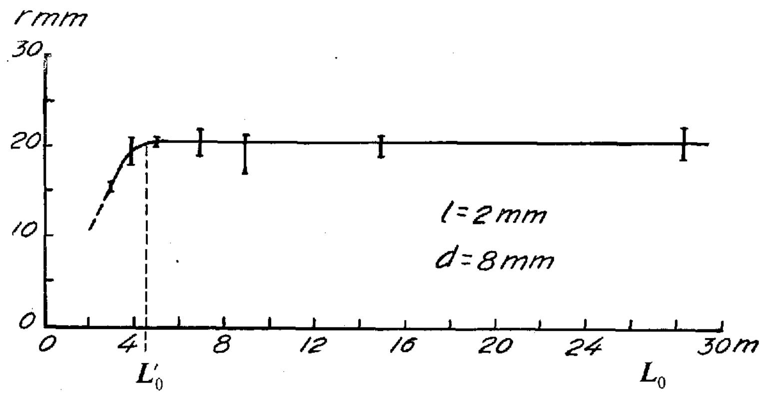


Fig. 22. Range r of positive figures as a function of the length L_0 of the connecting wire. (All experimental determinations of r fall within the heavy vertical lines shown).

practically ceased after the lapse of $5.3 \cdot 10^{-8}$ seconds at the lower *p. d.*, this is evinced by a continued growth under the influence of the higher *p. d.* A darker ring in the figure marks clearly the boundary between the original and the subsequently formed part of the figure. It is clearly seen that the increased growth occurs exclusively as a continuation of the original spreaders.

We shall see later that positive figures formed under similar conditions behave entirely differently; they have already attained their full range at $L'_0 = 4.5$ m, as appears from fig. 22, which shows the results of experiments carried out recently.

If we assume the *p. d.* in the wave front to increase gradually from zero to its maximum value V_0 over the length L' of the wave front, then the time τ during which the Lichtenberg gap is exposed to the maximum *p. d.* $2\,V_0$ will be determined by

$$\tau = \frac{2\,L'_0 - L'}{3 \cdot 10^{10}} \text{ sec.,} \tag{1}$$

L'_0 and L' being measured in cm and assuming the capacity of the Lichtenberg gap to be negligible.

If we assume the range of the positive figures to depend only on the value of the maximum *p. d.* and to be entirely independent of its duration then, according to (1), the length of the wave front must be determined by $L' = 2\,L'_0 = 9$ m.

If, on the contrary, the wave front were perfectly steep i. e. $L' = 0$, then a value of $\tau = 3 \cdot 10^{-8}$ sec. would correspond to $L'_0 = 4.5$ m.

Actually L' must have a finite length and the duration τ_0 of the maximum *p. d.* necessary for the positive figures to attain their full range must thus have its value between

$$0 < \tau_0 < 3 \cdot 10^{-8} \text{ sec.} \tag{2}$$

If the value of L' is known, then τ_0 may be determined by means of (1), but this question we will return to elsewhere.

(c) Irregular Figures caused by Pulses of long Duration.

Beside the regular, simple figures mentioned, there will, however, be formed other peculiar figures if the *p. d.* is maintained for a sufficiently long time. In this respect too positive and negative figures show great differences.

In the negative figures the discharge tracks formed when the *p. d.* is maintained long enough will follow the already formed negative spreaders. In no case have we observed such discharge tracks in the spaces between the original spreaders. Plate 27 shows some enlarged reproductions of negative discharge tracks; they mainly follow the centre line of the spreader concerned, but many show some smaller irregular bends. In some cases they may jump from one spreader to another. Such a case is seen in part VI. Here the spreader in which the discharge track starts is very short and separated from its neighbours by narrow and little ionized spaces. If the discharge track followed this spreader, it would find areas having little pre-ionization, and it is therefore found easier to jump to one of the neighbouring spreaders and then to continue along this. It is such jumps which form the sharp bends in the negative spark tracks, see f. inst. plate 26, part I and L. F. I fig. 5.

These discharge tracks will develop into sparks if the *p. d.* is maintained somewhat longer, but if a spark has been formed, the light from it will generally blur the figure and make it impossible to discern the details of the L. F. which existed before the formation of the spark.

From plate 27 it appears that the negative discharge tracks are very narrow.

With positive figures the conditions are entirely different. If the positive *p. d.* is maintained for some time, a new figure is often formed with its spreaders — trunks as well as branches — fitting themselves in proper order into the spaces between the trunks and branches of the first figure.

The new figure, figure no. 2, may, according to circumstances, pass beyond the boundary of figure 1, but it may also stop earlier.

An example is shown in plate 28, part I. This figure is taken under conditions quite analogous to those obtaining for the formerly mentioned negative figure shown in part II. Under the influence of the *d. p.* corresponding to the spark length L the figure has reached its full range (corresponding to this *p. d.*), within the $5.3 \cdot 10^{-8}$ sec. during which this potential was maintained. The outer boundary of the corresponding regular figure is marked by the drawn circle. The maintaining of the *p. d.* during the time mentioned, and its further increase for a short space of time has caused both some special discharges around and near the electrode, and also the starting of a number of new spreaders, which have had to "squeeze" themselves out between the original spreaders as is clearly recognised in the figure. Many of these subsequently formed spreaders have a greater range than those first formed.

The particular discharge phenomena which take place directly at the electrode will be treated of later.

At low pressures figure number 1 will often cover practically the whole surface, particularly after it has passed a little away from the electrode. In that case figure number 2 is not easily formed. Under especially favourable conditions, figure number 2 may be formed even down to a pressure as low as 150 mm. An example is shown in plate 6, part III. This figure shows clearly 2 sets of spreaders: a set of shorter ones which was formed first and a set of longer ones formed afterwards. That the order of formation is as stated may be inferred with certainty from the form of the spreaders, since the trunks and the branches of figure no. 2 "squeeze" themselves into the spaces between the trunks and branches of figure no. 1.

From the tracks followed by the trunks and branches of figure no. 2, it appears clearly that the trunks and branches of figure no. 1 were present with a sharply localized positive charge at the moment when figure no. 2 was formed. On the other hand, the spreaders of figure no. 1 cannot have possessed ability to conduct to any greater extent at the moment in question, since, if they had, they would simply have continued to grow in length because the field would then be strongest at their tips. The new spreaders start directly from the electrode in the spaces between the former ones, although the electrical conditions here are very unfavourable for a start because the existing charge on the first spreaders reduces the field at the starting points.

At atmospheric pressure this phenomenon is produced very easily and examples of such are given in plate 10, part III, plates 16 and 17. In these figures a, b, c . . . mark spreaders of figure no. 1 while a', b', c' . . . refer to figure no. 2, a'', b'', c'' to figure no. 3 and so on. On plate 17 4 successive discharges are clearly seen.

Beside this number of successive Lichtenberg discharges, another kind of positive discharge, essentially different from the Lichtenberg ones, will occur if the positive potential is maintained for a sufficient length of time. For instance, it has not their regular and sharply defined forms but is of a blurred character, while it has a higher brilliancy and has no well defined range, but spreads out further the longer the potential is maintained. Such discharges are seen near the electrode on plate 9, part I (especially at the electrode A_2), plate 16, parts II and IV, plate 17, plate 18, part II, and plate 20 II.

This positive discharge, (which is of a kind quite different to the Lichtenberg discharges, and will be treated of elsewhere in connexion with the question of spark formation) develops into highly luminous positive spark tracks if the *p. d.* is maintained long enough, and such sparks are shown schematically fig. 3 and examples of actual figures are shown plate 3, part I, plate 7, plate 11 I and IV and L. F. I fig. 4.

This discharge form must not be confounded with the previously mentioned flow of negative electricity from the electrode out along the positive spreaders, which often occurs when the discharge is so slightly damped that oscillations take place in the discharge circuit; see above under 1 (d).

Comparison shows that the character of this subsequent negative discharge is entirely different from the irregular positive discharge just mentioned: The negative flow occurs only out along the already positively charged spreaders, while the spaces between them remain untouched. This is not so in the case of the positive discharge. The boundary of the subsequent negative discharge is fairly sharp; this, also, is not the case with the positive one. The irregular positive discharge is often the starting base for the Lichtenberg discharges no. 2, 3 ....

Very often subsequent negative discharges and irregular positive discharges appear in the same figure, giving the innermost part of the figure a highly luminous and irregular appearance; see plate 9, part I (especially electrode A_2), plate 16 IV and 20 II.

It is of importance in this connexion again to emphasize that neither the subsequent negative nor the irregular positive discharges are necessary parts of the positive Lichtenberg figures; they are only complications which have nothing to do with the regular L. F., and which may easily be avoided by suitable arrangements, see thus plate 1, part V, plates 4, 5, 14 II, 22 and 23.

U. YOSHIDA[1] seems to consider both the subsequent negative discharge and the irregular and comparatively slow positive discharge as a normal and essential part of the positive Lichtenberg figures, in that he assumes the manner of formation to be somewhat different for the inner and for the outer part of the figure. Thus he says in his paper (b) p. 315: "In the anode figures (Fig. 3 and 13 of the former paper) we notice that every branch consists of two parts; namely a more intense portion near the electrode, and a weaker portion more removed. When a celluloid film is used instead of a common photographic plate, these two portions are more clearly distinguishable as seen in fig. 18 of the former paper, and fig. 1 of this paper, the ends of the intense portions of the branches terminating with a continuous outline".

That the subsequent negative discharge should in the latter case be predominant over the positive one is fully in agreement with the explanation we have given of the phenomenon in section 1 (d) above.

It is emphasized above that we cannot agree with U. YOSHIDA on this point.

(d) Resumé.

The results set forth above may be briefly summarized as follows:

[1] U. YOSHIDA: (a) Mem. Coll. Sc. Kyoto Imp. Univ. Vol. II, No. 2, p. 115—116, 1917. — (b) l. c. No. 6, p. 315—319, 1917.

For negative figures the range increases with increasing duration of the *p. d.* and, when the *p. d.* is maintained for a sufficient length of time, spark tracks will be formed along and inside the original spreaders — but never in the spaces between them. These spark tracks begin as fine threads with a somewhat irregular course.

For the positive figures the range of the first formed figure is independent of the duration of the *p. d.* — at all events when its value exceeds $3 \cdot 10^{-8}$ sec. If the *p. d.* is maintained long enough, there may be formed a new positive figure having a range greater or smaller than that of the first one, and in which the spreaders of the new figure fit themselve into the spaces between the spreaders of the first figure.

If the *p. d.* is maintained still longer, irregular positive spreaders, of a kind entirely different to Lichtenberg ones, may be formed. From these irregular positive spreaders there may be started new Lichtenberg spreaders which then fit themselves into the spaces between those already present.

5. Various Questions relating to the Formation of the Lichtenberg Figures, especially the Positive ones.

(a) Influence of Initial Ionization.

In order to solve the question of how positive figures are formed, it is most essential to know what influence an ionization within the area to be covered by the figure may have upon the formation and appearance of the figure. To show the importance of this question we may cite the following extract from U. YOSHIDA[1]: "If the potential of the anode is increased sufficiently, the negative ions, which were present just before the formation of the photographic impression by the ionisation commences, will be pushed toward the portions of the electrode where the electric force is strong, and will ionise the gas molecules with which they collide. The negative ions thus produced will also do the same thing; and many positive and negative ions will be produced. The group of positive ions, which will be left behind as negative ions are pushed toward the anode, will now act as a portion of the anode; and, repeating the same process as before, further branching and elongation of the anode branch will take place. With this explanation, the properties of the anode branches (that they are irregular in their branchings and elongations, and that their branches end in sharp points) will be immediately understood; because the formation of these branches is due to the presence of negative ions which would be distributed irregularly".

We have therefore made this question the subject of a fairly thorough investigation, the result of which was that in no case were we able to ascertain any influence of an existing ionization on the formation of the regular positive (or negative) figures — setting aside such unimportant cases in which the conductivity became so large that the electric fields, determining the course of formation, were liable to considerable distortion.

In the following we shall describe some of these investigations. Plate 16, I shows the result of such a test. Immediately prior to (10^{-6} sec. before) the formation of

[1] U. YOSHIDA: l. c. (a), p. 113.

the positive figure shown, the spot S was illuminated by a strong spark. By this means a strong ionization is caused photo-electrically within the area of the spot, which ionization cannot have vanished entirely before the figure was formed, and consequently the ionization within the spot must have been very much stronger than outside. In spite of this no influence of the ionization on the course or the appearance of the spreaders is to be observed.

Against the validity of this test it may possibly be said — although in our opinion unjustly — that the ionization is located at a place in the middle of the positive spreaders where the spreading out of these is going on with great strength and speed while the influence of such an ionization may be expected to be particularly obvious at the ends of the positive spreaders. Plate 10 III therefore shows the result of another test where the ionized spot A is located at the tips of the spreaders e' and f', but the presence of the ionized spot had also in this case no influence upon the form and course of the spreaders.

We have further investigated the formation of figures over an area where a strong ionization takes place simultaneously with formation of the figure. The results of a couple of these tests are shown in plate 10 I and II. Here the ionization was effected by means of a powerful Radium preparation resting a few millimeters above the photographic plate during the formation of the figure. The action of the radio-active rays was in the main confined to a limited area by means of shielding. The pictures show the ionized portions to be rather intensely luminous but it is also clearly seen that there is no difference in the formation of the positive figure inside and outside the ionized part of the plate.

An ionization by collision of the character mentioned by U. YOSHIDA, but in which the active negative ions are no doubt mainly electrons, may be brought about in various ways of which a few will be mentioned in the following.

With the arrangement shown in fig. 23 a series of pictures has been taken the general character of which appears from plate 6 II. The distance between the electrodes A_1 and A_2 is chosen so that the positive and the negative spreaders only just meet one another. Plate 13 I shows an example of this. As the positive and the negative discharge here take place exactly simultaneously plenty of free electrons will be present at the outer ends of those positive spreaders which reach over — or nearly over — to the outer ends of the negative spreaders. An ionization by collision will then occur at this place, as the positively charged tips of the positive spreaders will attract the free electrons with great strength. But the pictures show clearly that t h e d i s c h a r g e t r a c k s t h e r e b y f o r m e d h a v e a c h a r a c t e r a l t o g e t h e r d i f f e r e n t t o t h a t o f t h e p o s i t i v e s p r e a d e r s[1].

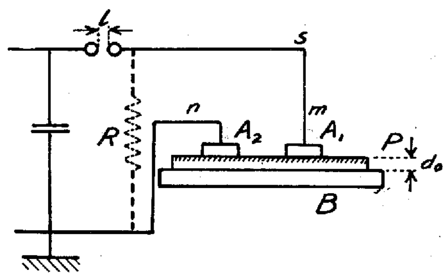


Fig. 23. Arrangement for simultaneous registration of positive and negative figures on one photographic plate. The metal plate B is insulated.

These discharge tracks go partly directly from the ends of the positive to the ends of the negative spreaders, and partly they connect the ends of neighbouring positive spreaders. The outer ends of a few of the positive spreaders have

[1] This feature is already mentioned in L. F. I, figs. 62 and 63.

apparently been bent by the electric field towards the nearest part of the negative figure.

Plate 24 I—IV shows enlarged portions of similar figures taken at atmospheric pressure. We see also here that the presence of a strong ionization, (in which, no doubt, there are a great number of free electrons), may be the cause of discharge tracks starting from the ends of the positive spreaders, but also that these discharges are not similar to regular positive spreaders. We will come back later to the formation and appearance of these discharge tracks; see appendix 3.

Similar conditions are found in the pictures shown in plate 18 I—II. In part I the positive spreaders go far into the area of the negative figure but in spite of this they fully preserve their character. In part II a positive discharge is subsequently started from the negative electrode and the course of this discharge is determined mainly by the electric force from the electric charge in the negative figure. (The same applies to the subsequent positive discharge in the negative figure shown plate 25 I. In these cases also the positive spreaders preserve their typical character.

In very many cases ionization by collision occurs by mutual action between the positive spreaders. This is thus the case in plate 18 I at the points marked 1—9. See also plate 14 I and II which are taken with the apparatus disposed as in fig. 24 — the same as fig. 43 in L. F. I.

In plate 14 I this ionization by collision occurs especially in such places where, *after* the formation of the positive spreaders, a strong electric field arises at right angles to their direction of propagation. In part II these discharges occur especially at the meeting line of the two figures.

The same applies to the ionization by collision in the neighbourhood of the meeting line of the positive figures on plate 15 I and II.

In all cases where such ionization occurs, the discharge has a character essentially different

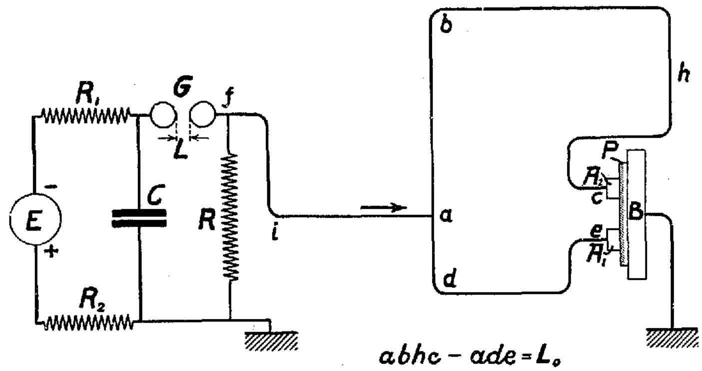


Fig. 24. Arrangement for determining the spreading-out-velocity of the Positive Figures.

to that typical of the positive Lichtenberg spreaders — to mistake the one for the other is impossible.

(b) Influence of Strong Simultaneous Ionization.

If a great number of electrons are set free at the same time and place as that at which spreaders are formed, then a very peculiar phenomenon occurs which we will examine a little more closely. Such a release of electrons may be effected by illumination of the photographic plate by an intense electric spark, preferably of short duration, since the plate will be blackened too much if the spark lasts too long. The number of electrons released will also only increase with the duration of illumination during a very short time interval, as the released electrons are for the greater part captured within 10^{-6} sec. by oxygen or water molecules, or the like, thus being[1] transformed into heavier, less active, ions, whose number will further decrease very rapidly owing to recombination between positive and negative ions.

The most simple experimental method is to apply a spark potential so high that a spark will pass from electrode A to the metal plate B below, see fig. 1. Another equally simple method is to let a spark pass between the electrodes A_1 and A_2 in fig. 23. In plate 11, I and IV show a few examples produced by the first method, II and III by the second. We will first examine the second. In that part of the photographic plate which is exposed to light from the spark, the positive spreaders are surrounded by a "soft", comparatively intensely luminous "veil" or rim, while those parts which are in the shadow have the normal appearance. The results are exactly similar in the first named case where the spark passed between the electrode A and the metal plate below. Part I shows such a picture taken in atmospheric air; part of the positive spreaders have here become strongly distinguished by the very strong rim-formation. Part IV shows a similar discharge in oxygen and is even more significant, since in this case the positive Lichtenberg discharges themselves are so faintly luminous that they cannot be photographed, see p. 16 above. But here they appear very distinctly owing to the strong rim-formation although the latter is weaker than in atmospheric air. This smaller intensity is caused by the fact that negative figures which are formed by ionization by collision are also less luminous in oxygen

[1] P. O. Pedersen: "Propagation of Radio Waves". Copenhagen 1927 (Fig. IV 7, p. 46).

than in atmospheric air. With negative figures — see parts II and III — a luminous ring appears a little outside the photographic boundary of the figure.

The luminous edges around the positive spreaders are caused by ionization by collision, which takes place in the strong fields immediately outside their boundary owing to the presence of numerous free electrons which may instantaneously start such ionization. We shall see later that there is reason to assume that the positive charge is of about equal intensity over the entire cross-section of positive spreaders. The electric field is therefore strongest at the very edge. The fact that the light intensity is greatest here is in good agreement with this.

In case of negative figures the outer boundary is not quite sharp, for, as stated before, the photographic intensity decreases gradually towards zero. We shall show later that there is reason to assume that the negative "electric" figure, a very short time after its formation, is slightly larger than the photographic figure, and that the charge-gradient is steepest at the edge of the "electric" figure. Consequently the strongest field and the most luminous rim is also found here.

It is thus possible fully to explain the source of these luminous rims. The foregoing discussion further shows that the figure formation — positive as well as negative — proceeds in the normal manner in spite of the presence of a great number of electrons and ions, while the luminous rims are caused by ionization by collision analogous to that which U. YOSHIDA and others assume to be the cause of the formation of the positive spreaders. But the discharge caused by this ionization by collision has, as appears from the positive pictures, a character entirely different to that of the positive spreaders.

U. Yoshida[1] has also treated this rim effect but has only tried to find an explanation in accordance with the one given for "dark sparks" ("dunkle Funken") i. e. founded on the Clayden effect[2], but he comes to the conclusion: "that the phenomenae observed are not photographic reversals of any kind ever known" (p. 319), which result is at all events not in contradiction to the one arrived at by us.

Some of the features touched upon in the preceding part are also treated rather more thoroughly in Appendix 3.

(c) Influence of Various Irregularities in the Photographic Film.

We have also investigated the effect of various irregularities prepared in or on the surface of the photographic film over which the positive figure spreads out. For example, plate 10 IV shows a case in which thick, electrically non-conducting ink lines were drawn over the film. They do not appear to affect the spreading-out of the figure. Plate 9 II shows the effect of some pencil lines over the film. It is clearly seen from the appearance of the figure at line 2 that this line is a fairly good con-

[1] U. Yoshida: l. c. (b).

[2] With reference to the formation of these "dark sparks" see P. Metzner (Verh. d. D. phys. Ges. Bd. 13, p. 612—616, 1911). Extraordinary beautiful samples of such dark sparks are found in Lord Armstrong: "Electric Movement in Air and Water", plates nos. 18 and 34 (London 1899). With regard to an explanation of these dark sparks on the basis of the Clayden effect we may refer to R. W. Wood (Phil. Mag. vol. 6, p. 577—590, 1903), K. Schaum (Verh. d. D. phys. Ges. Bd. 13, p. 676—679, 1911) and to M. Volmer und K. Schaum (Zeitschr. f. wiss. Phot. Bd. 14, p. 1—14, 1914).

ductor. On the other hand the positive points ending near line 1 are perfectly normal, proving that a considerable conductivity produced in this way has no effect on the formation of the positive spreaders, as long as these do not come into contact with the conducting lines.

(d) Ionization and Negative Discharges.

In the preceding paragraphs we have mainly treated only the spreading-out-conditions of the positive figures. The formation of negative figures is also, however, within very wide limits, independent of an existing ionization. Our reason for not entering much into this question is that we consider the formation of the negative figures as already fully explained in L. F. I.

As the main result of the investigations described and referred to in this section, we may set forth the conclusion that the formation of regular Lichtenberg figures is independent within very wide limits of the intensity of an existing ionization.

CHAPTER IV

The Formation of the Positive Figures.

A. Preliminary Discussion of various Hypotheses.

At the outset various possibilities can be assumed for the process of formation of positive figures, and several hypotheses to this effect have already been set forth[1]. However, in our opinion at least, none of the hypotheses already advanced can be brought into agreement with the experimental results. In this section we will give a brief account of the reasons which have led us to this view; in section *B* we will then give an account of the hypotheses we have arrived at through our investigations, and we shall here have occasion to give further reasons why we cannot accept the older hypotheses.

1. Formation of Positive Figures as due to Positive Ions moving away from the Electrode.

The process of formation of positive figures may be assumed to proceed analogously to the formation of the

[1] With regard to the various hypotheses for the formation of Lichtenberg figures see L. F. I chapt. I—II and K. Przibram: "Die ionentheoretische Deutung der elektrischen Figuren", Handb. d. Physik. Bd. 14, p. 402 ff. 1927.

negative ones, except that here positive ions, and not electrons, are moving outwards from the electrode, and by ionization by collision produce the necessary number of new positive and negative ions and electrons, the positive charge of the figure being mainly due to the movement of the latter towards the electrode.

The active positive ions may be either ordinary atomic or molecular ions, corresponding to the particular gas in which the discharge takes place[1], or they may be H^+-particles (Protons).

If positive figures were formed in this manner, then (α) their appearance should in the main features agree with that of the negative ones[2], (β) the relation between the spreading-out-velocity and the thickness d_0 of the plate should in the main be the same for positive and for negative figures, and finally (γ) the spreading-out-velocity should be considerably smaller for positive than for negative figures.

In chap. III it is, however, shown that none of these consequences are in agreement with the facts, in fact the spreading-out-velocity is even considerably greater for positive than for negative figures. In section B of the following it will further be shown that the spreading-out-velocity of the positive discharges is so great that the figure formation cannot take place in the manner here considered.

[1] Also S. Mikola (Phys. Zeitschr. Bd. 18, p. 161. a. f. 1917) seems mainly to have this view, though he considers both "Die korpuskulare Strahlung des Kondensators" and further "Die impulsive Strahlung des Kondensators" as active in the formation of figures and the second of these rays he considers to have electro-magnetic character.

[2] We have been unable to feel convinced by the reasons given by K. Przibram (l. c., p. 403) for the great difference in appearance between the positive and the negative figures on the basis of this hypothesis.

2. Formation of Positive Figures due to Negative Ions (Electrons) moving Inwards to the Electrode.

The formation of positive figures could also be assumed to proceed in the following manner: Negative ions from the outer edge of the figure are drawn inward towards the electrode, the necessary number of negative ions being produced by ionization by collision at the outermost ends of the spreaders. The positive charge on the figure is also in this case in the main due to the negative ions moving toward the electrode.

Here again there are several possibilities, in that the ionization by collision may be initiated either (α) by means of the (natural) ionization present in the air directly in front of the outer edge of the figure while this is being formed, or (β) by means of ionization by collision initiated by some positive particles driven out of the tips of the positive spreaders by the electric field.

(α). U. Yoshida is — as mentioned in chap. III 5 (a) — a follower of the first one of the here named theories. But since the experiments discussed in chap. III 5 (a) and (b) show that the circumstances of formation and the appearance of positive figures are independent within very wide limits of an initial ionization either previous to or simultaneous with, the formation of the figure, we cannot suppose the figures to be formed in the manner set forth under (α).

But even setting aside the said experiments — which we indeed consider as conclusive — the hypothesis set forth under (α) would meet with a number of difficulties. It would thus be difficult to give a satisfactory explanation of the very marked difference in appearance of the positive and the negative figures, of the difference in their spreading-out-velocity and initial conditions and of the dependency of the range on the plate thickness. Further, it would be difficult to explain the sharp boundary lines found, and the fact that the width of the positive spreaders is inversely proportional, and their number directly proportional, to the applied air pressure.

Completely decisive for the dismissal of this hypothesis are also the examples given in chap. III 5 (a) and (b), which show that even though such suction or drawing in of negative ions (or electrons) may occur under particular conditions, the discharge-tracks thereby formed have a character entirely different from that of the positive spreaders.

By this means we have, in our opinion, proved the untenability of the hypothesis in question.

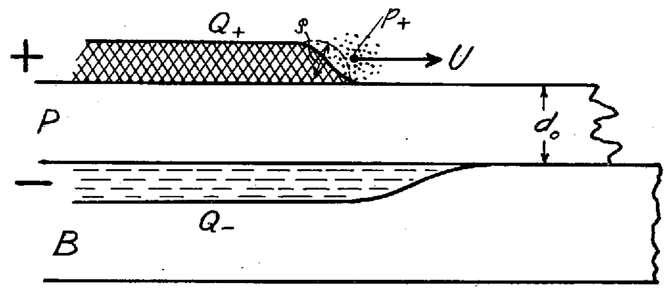
(β). There remains now only the hypothesis mentioned under (β), which we shall subject to a more thorough discussion.

B. Theory of the Positive Figure according to the Hypothesis 2 (β).

0. Further Specification of this Hypothesis.

The foremost positive particle p^+, see fig. 25, is assumed to travel with the velocity U in the front of the positive spreader Q_+. The foremost particle has a sufficiently high velocity to act in a strongly ionizing manner, and releases a considerable number of electrons, indicated by dots in fig. 25. These electrons are pulled toward the tip of the spreader by the very strong field existing here, and on their way they release by collision a further large number of electrons, which are then pulled towards the electrode by the field along the spreader.

In this manner the strong electric field will automatically follow the foremost particle. The field strength at the



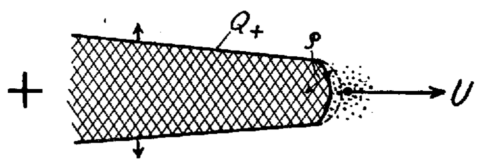


Fig. 25. Schematical Representation of the conditions at the Tip of a Positive Spreader. P is the photographic Plate, B the earthed metal Plate, see fig. 1.

tip of the spreader cannot be directly measured, but its approximate value may be estimated by the following considerations.

Assuming the *p. d.* between the positive electrode and the grounded plate B to be 15 000 volts, this will correspond to a spark length of about 2 mm. A drop in potential will, however, occur out along the positive spreader, and the potential at the tip of the spreader at the moment in question is assumed to be 9 000 volts. It is further assumed that the outermost tip has a spherical shape with a radius ϱ cm. The foremost particle p^+ at a distance $(\varrho + \Delta\varrho)$

from the centre of the spherical spreader tip will then be exposed to an electric field $E_{\varrho + \Delta\varrho}$ which may with approximation be put equal to

$$E_{\varrho + \Delta\varrho} = \frac{\varrho}{(\varrho + \Delta\varrho)^2} \cdot V_0, \tag{1}$$

which for $\Delta\varrho = 0$ reduces to

$$E_\varrho = \frac{1}{\varrho} \cdot V_0. \tag{2}$$

If the front particle is inside the spherical part and the charge density is assumed to be constant, then the field at a distance $(\varrho - \Delta\varrho)$ from the centre will be determined by

$$E_{\varrho - \Delta\varrho} = \frac{\varrho - \Delta\varrho}{\varrho^2} \cdot V_0. \tag{3}$$

If, on the other hand, the total charge is located on the surface of the sphere, we have

$$E_{\varrho - \Delta\varrho} = 0. \tag{4}$$

$E_{\varrho - \Delta\varrho}$ will in general have a value somewhere between those determined by (3) and (4).

$E_\varrho = \frac{V_0}{\varrho}$ is thus the highest value attainable for the field intensity.

If $E_{\varrho + \Delta\varrho}$ at a given moment is greater than the field value E_U, necessary to maintain the velocity U of the front particle, then this velocity will increase. The distance $(\varrho + \Delta\varrho)$ will thereby become greater, but this is again — according to (1) — followed by a decrease in the field intensity.

If, on the contrary, $E_{\varrho + \Delta\varrho} < E_U$ the velocity of the particle will decrease but the distance $\varrho + \Delta\varrho$ will thereby also be decreased and $E_{\varrho + \Delta\varrho}$ will consequently increase. From this it appears that the particle will adjust itself to a distance where the field intensity and the velocity have corresponding values.

This, however, assumes that

$$E_{\varrho} = \frac{V_0}{\varrho} > E_{U_0}, \tag{5}$$

where U_0 is the smallest velocity at which the front particle can be controlled in the manner referred to.

The positive front particle will in fact almost, or possibly completely, lose its ionizing ability if its velocity decreases below a certain value, see App. 1, sections 6 and 7. At this limit the strong ionization at the tip will cease and the intensity of the electric field in which the front particle is located will become much weaker. The particle will then lose its velocity within a very short distance. This process will actually already have set in at a velocity at which the particle still possesses some ionizing ability, and the critical value of U_0 will therefore be so high that considerable ionization still takes place. The actual value of U_0 cannot, however, be calculated beforehand, though such a limiting value no doubt exists.

The front particle may, however, be brought to a standstill by another cause. It will not always contain a positive charge but will sometimes be positive and sometimes neutral, as is well known from investigations of positive rays and other high-speed positive particles, see App. 1. If we call the distance travelled by the particles in a charged state l_1, and the distance travelled in a neutral state l_2, then we know from experience that the ratio $\frac{l_1}{l_2}$ decreases with decreasing velocity. This ratio seems, however, especially in cases where the positive particle is a proton, to approach a limiting value which does not further decrease with decreasing velocity, see for example fig. 2 in App. 1. These relations are, however, not quite certain, compare App. 1, section 6.

The deduced values for the ratio $\frac{l_1}{l_2}$ are at all events only valid for the average values of these distances, and the actual distances may show very considerable differences. If the particle travels 10^{-4} cm without charge, then it will decrease considerably in velocity. If it has been retarded so much that it has come inside the strongly charged spherical front, then the field intensity drops so considerably that the particle, even if it becomes positively charged again, has only very little chance of regaining the necessary velocity; if it does not, it will be drawn back and more into the charged sphere, and completely lose its velocity.

In both of the above described cases the front-ionization ceases and the growth of the positive spreader stops. The strong electric field at the tip decreases simultaneously because the positive charge is spreading out, although comparatively slowly, by which means V_0 decreases and the radius ϱ increases. The drop in V_0 is due to the spreading out of the positive electricity available over a larger area, and as no perceptible ionization now occurs at the tip, no negative electricity is carried away toward the electrode. V_0 must therefore decrease.

We shall now proceed to give a more detailed explanation of the characteristics of the positive figures mentioned in chap. III, but we may remark here that in part 2 of the following we are led to the assumption that the positive front-particles are protons.

1. Differences in Shape of Positive and Negative Figures.

(a) Width of Positive Spreaders.

The width of the positive spreaders is, as was shown in chapter III 1 (a), inversely proportional to the air pressure p[1].

Immediately after the passage of the front-particle the spreader will be very thin and this the more so the smaller is the velocity of the particle. This primary ionization channel is strongly positively charged, because part of the electrons set free through the ionization by collision in the tip of the channel are drawn toward the electrode. The value of the positive charge is mainly determined by the capacity of the channel and by the fact that the potential in the channel is smaller than the potential of the electrode by an amount corresponding to the ohmic drop of potential along the spreader. We will return to this point later. The ionization along the spreader is so intense that the electrons pulled toward the electrode only represent a fraction of the total number set free by the ionization by collision.

Repulsion between the resulting positive charges in the spreader will cause an increase in thickness of the spreader, and this increase will occur with a rapidity inversely proportional to the air pressure, as is more thoroughly shown in App. 2. The thickness — or width — of the spreader a given time after the passage of the front-particle will therefore — ceteris paribus — also be inversely proportional to the air pressure.

As we shall see later the luminous effect must be

[1] Parts of the following accounts apply also to discharges in the free atmosphere — as is easily seen.

assumed to arise a certain time after the original ionization, and the intensity of the photographic picture of the spreader must therefore also as a whole be inversely proportional to the air pressure.

The rate of increase of the thickness of the spreader must also — ceteris paribus — be proportional to the resulting charge per unit length of the spreader, and this charge must have a value increasing with increasing potential across the Lichtenberg spark gap[1]. The *p. d.* between the spreaders and the earthed plate B decreases, as said before, from the electrode towards the edge of the figure, and the drop in potential is the greater the higher the air pressure. But as the width t of the spreaders is measured at a distance from the tip inversely proportional to the air pressure, it may be assumed that the spark potential will have no appreciable influence on the value of t thus measured.

The preceding further shows that the width must increase fairly regularly from the tip towards the electrode. This regular increase in width is only found, however, at such distances from the electrode where the spreaders are so wide apart that they do not interfere with each other's lateral spreading. For Lichtenberg figures, the discharge takes place over the surface of an insulating plate. Here the plate surface will form a geometric boundary to the figure and further the induced charge Q_- on the metal plate B, see fig. 25, will, in proportion to the thickness of the insulating plate, exert a greater or smaller influence upon the electric field immediately in the neighbourhood of the spreader Q_+. In the following we shall consider the

[1] In the neighbourhood of the electrode the field from neighbouring spreaders will hinder a free lateral development of the spreader.

influence which the air pressure and other factors exert upon the formation of the spreader.

We will first, on the basis of the schematic representation in fig. 26, look at the conditions at very low pressures.

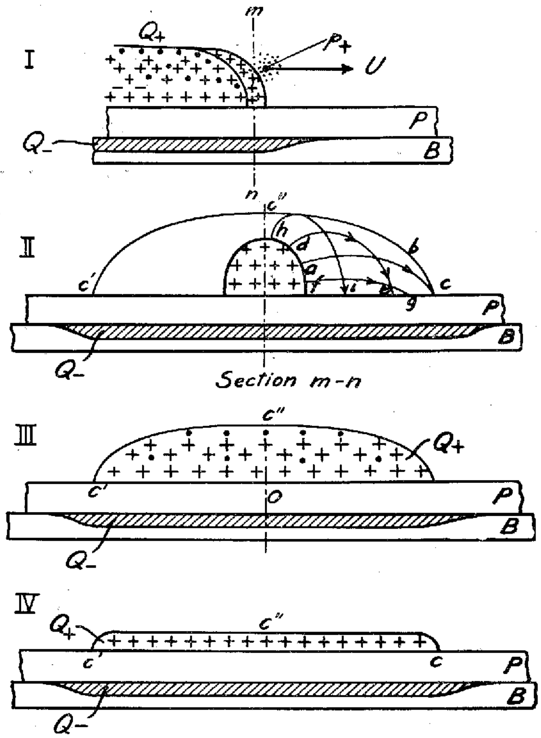


Fig. 26. Schematical Representation of the manner of formation of Positive Spreaders at low Pressures.

Part I represents the tip of a positive spreader. Immediately around the front-particle p_+ will be found some electrons and a corresponding number of positive ions. The number, however, is comparatively small because the air pressure is low. Both electrons and positive ions are indicated by dots in the figure. Immediately behind the front-particle will exist, within a limited layer, a very dense positive space charge, due to the very strong ionization by collision occurring in the very strong field at the tip of the spreader, combined with the great mobility of the electrons which allows their quick removal by the field toward the left — in the direction of the electrode — while the comparatively heavy positive ions may be considered as nearly stationary within the very short intervals of time here in question.

Due to mutual repulsion the positive ions will during the time following move outward — f. inst. a in part II along the path a-c — and the attraction from the induced charge Q_- will finally force the ion down against the surface of the plate P. The smaller the air pressure, and the greater the velocity of the ion, the fewer collisions will it undergo on its way.

Even if the front travels so rapidly that no considerable number of free electrons get time to recombine with positive ions to form neutral molecules, or with neutral molecules to form heavy negative ions, nevertheless, at such low pressures a sufficient number of electrons may easily be carried away toward the electrode, thus enabling the capacitive positive charge on the spreader — and the corresponding induced negative charge Q_- — to increase in accordance with the width of the spreader. The charge density per unit length of the spreader will therefore increase very rapidly. By this means also the forces tending to drive the positive ions outward will become great. Since the ions have at the same time a high mobility, owing to the low pressure, the width of the spreaders must also become great.

In part II it is assumed that the ion a travels the farthest before it impinges upon the plate P at c. Ions which start from higher points such as d and h or from lower points such as f will then impinge upon P at shorter distances from the centre of the spreader.

The positive ions which hit the surface of the plate will be pulled down against it by the electric field and will then be retained by the electric forces between the ion and the molecules in the surface of the solid plate. These positive ions will thus almost completely lose their mobility.

After a short time the state of charge will pass through the state III and approach the state IV, where the density per unit area σ of Q_+ and Q_- (which have the same numerical value) is very nearly constant over the entire width of the spreader in that

$$\sigma = \frac{\varepsilon}{4\pi d_0} \cdot V,$$

where ε is the dielectric constant of the plate and V the $p.\ d.$ between the point of the spreader considered and the plate B.

Those positive ions which are not retained in this way will recombine with a corresponding number of negative ions or electrons and this recombination will in the main occur evenly distributed over the cross section $cc''c'$ — see part II — since the above mentioned scattering of the positive ions occurs with such great velocity[1], and the density of these ions is so great, that the air particles within this cross section are set in such violent motion that effective mixing takes place.

The spreading out occurs in an analogous manner at higher air pressures, but the scattering velocity of the

[1] See also later.

positive ions is smaller and the dispersal of the electrons is slower so that the width of the spreader becomes smaller. Another contributory cause is that the original ionization channel, formed by the front-particle, has a smaller cross-section and accordingly also a smaller charge density. The width of the spreader must therefore decrease with increasing air pressure.

Fig. 27 shows schematically various states of the formation of a positive spreader.

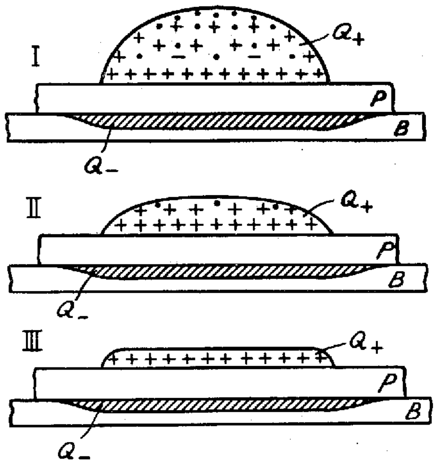


Fig. 27. Schematical Representation of various States of a Cross-Section of a Positive Spreader at Atmospheric Pressure.

With regard to the influence of the spark length on the width of the spreaders, we have already mentioned that the width at a distance from the tip inversely proportional to the air pressure is practically independent of the potential value on the electrode.

Generally speaking, the width will increase with increasing spark length, but to a smaller degree, because the induced charge Q_- increases at the same rate as Q_+, so that the direction of the field will be independent of the value of the two charges. Added to this, we here have near the electrode the above mentioned additional factor that neighbouring spreaders prevent the free spreading out of the single spreader.

The dependency of the width on the thickness d_0 of the insulating plate is evidently partly due to the fact that the induced charge Q_- at thin plates comes up near Q_+, so that the electric field at the edge of Q_+ has a powerful component perpendicular to the surface of the plate, and partly to the fact that Q_+ and Q_- at a given voltage will increase rapidly with decreasing values of d_0. The positive ions will therefore move with great velocity, but owing to the geometrical form of the field they will nevertheless impinge on the plate at a comparatively short distance from the centre of the spreader. It is not easy to say off-hand which of these two causes has the greatest influence. The value of d_0 has actually very little influence upon the width as long as d_0 is not too small. For very small values of d_0, however, the width of the spreaders will decrease with decreasing d_0 but as the original ionization channel has a finite thickness the width will not become quite zero as $d_0 \rightarrow 0$.

In the foregoing we have first discussed the ionization by collision at the tip and thereupon the increase in width of the spreaders. These processes actually occur simultaneously and the total ionization will be almost proportional to the final width of the spreaders.

We shall see later — under section (d) — that the photographic width must be very nearly the same as the width cc' of the charge; see fig. 26 and 27.

The hypothesis set forth thus fully explains the fact that the width of the positive spreaders increases with decreasing pressure, with increasing potential and with increasing plate thickness. An exact quantitative calculation cannot be carried out, but the simple considerations set forth indicate that the width must be inversely proportional to the air pressure. All the results mentioned in chap. III 1 (a) with regard to the dependency of the width on the various parameters are thus explained in a satisfactory manner by the hypothesis set forth.

(b) Number of Branches.

The number of branches per unit of length of a spreader must, ceteris paribus, be proportional to the air pressure. The number of suitable particles at those points where the field is strongest — i. e. at the edges of the spreaders — must in fact be proportional to the number of such particles per unit volume, i. e. to the air pressure.

The number of branches will also depend on the nature of the gas, but this we will return to later.

(c) Boundary-line of the Positive Spreaders.

The limits of the photographic image of the positive spreaders are determined by the limits to which the positive ions have spread at the moment when emission of light occurs. According to the foregoing, the spreading out of the positive ions is coterminous with the resulting positive charge; compare sect. (a) and figs. 26 and 27.

It appears from Appendix 2 that the outer boundary of a charge which spreads out owing to mutual repulsion will have a decided inclination to become sharp. The various conditions mentioned above under sect. (a) regarding the sidewards spreading out of the spreaders also contribute to the same effect.

These circumstances, combined with what will be set forth under the following sect. (d), offers a satisfactory explanation of the comparatively sharp boundary lines of the positive spreaders.

(d) Photographic Intensity.

The difference in luminosity of positive and negative figures is due to the fact that in the case of negative discharges, some of the positive ions set free by ionization by collision travel towards the electrode, while the main part of the negative charge is formed by electrons moving outward from the electrode or its immediate vicinity. This — as we shall see later — results in an exceedingly strong ionization at and near the electrode.

The light is produced in the main by recombination between the positive ions and the electrons and to a much smaller degree by recombination between positive ions and negative ions produced by neutral molecules combining with electrons. Where no positive ions are present no light appears. To this circumstance may be ascribed the fact that the luminosity at the outer boundary of the negative figures decreases gradually towards zero, since here the ionizing intensity, and consequently also the density of positive ions, decreases gradually.

Even when the field at the outer boundary has become so weak that ionization by collision has ceased, the electrons will be driven further out by the field but will in the mean time have combined with neutral molecules to form negative ions. But since no ionization occurs, no positive ions are produced, and therefore no light appears. We shall show later, under sect. 5 (a), that we are able to confirm by other means the fact that the charge has spread beyond the luminous part of the figure.

Corresponding conditions exist with regard to the dark lines of the negative figures. In this case also electrons and negative ions will pass in from the neighbouring parts of the spreader after the ionization by collision has ceased, but no, or at least extremely few, positive ions. The photographic luminosity will therefore also be extremely small.

For positive figures the conditions are entirely different. Here the highly mobile electrons set free by ionization by collision will be driven up from the photographic plate to the surface of the spreader by the field — compare fig. 27 I—II — and the field will then set them moving toward the electrode. The number of electrons absorbed by the electrode will be just sufficient to give the spreader a charge corresponding to the *p. d.* between the spreader and the plate *B*. Part of the rest of the electrons combine with positive ions, while others combine with neutral molecules to give negative ions, which again later combine with positive ones. The light is mainly produced by combination of positive ions with electrons or with negative ions.

The ionizing intensity is much smaller near the electrode than in the case of negative figures. The photographic intensity is therefore also comparatively small but will, on the other hand, be nearly uniform over the entire length of the spreader. The ionization will in fact, as said before, be very nearly proportional to the width of the spreader and will not be influenced by the electrons carrying negative charge from the spreader tip toward the electrode. The voltage drop along the positive spreaders is in fact so small that these electrons will cause no appreciable ionization by collision.

In single cases, the outermost tips or branches may be less luminous than the rest of the spreaders, see plates 4 I, 5 and 6 I which are taken at low pressures. At atmospheric pressure the corresponding branches are comparatively shorter. Possibly here the velocity of the front particles has become so small that the initial ionization which they cause is insignificant.

In this way we have accounted for the very peculiar difference between positive and negative figures with regard to light distribution. But it may still be asked: Can the spreaders really increase noticeably in thickness in the very short time elapsing before the positive and negative ions have recombined and before the light resulting therefrom is emitted?

The greater part of the recombination no doubt occurs within 1 to $2 \cdot 10^{-7}$ seconds. The emission of light probably occurs within a similar time interval after the recombination[1].

If, for example, the *p. d.* between the point in question of the spreader and plate B is 9 000 volts, and the thickness of the photographic plate about 1 mm, then the intensity of the electric field at the edge of the spreader will be of the order of 100 000 volt cm^{-1}. If we put the mobility of the ions at atmospheric pressure equal to 1.8 cm sec^{-1} per volt cm^{-1}, then the velocity of the edge will be $1.8 \cdot 10^{-5}$ cm sec^{-1}. The edge of the spreaders will then in the course of $1 \cdot 10^{-7}$ seconds move a distance of 0.18 mm. The increase in width of the spreader may thus attain the necessary velocity, and this velocity, moreover, is so high that the ionization at the tip may be assumed to be proportional to the width t.

This great velocity, combined with the great density of ions will, as said above, cause very vigorous movements inside the air volume occupied by the spreaders. By this means a very effective mixing occurs through the

[1] J. Franck u. P. Jordan: Anregung von Quantensprünge durch Stösse, p. 201, 1926.

entire cross section of the spreader whereby the density of the positive ions as such and negative ions as such will be practically the same over a cross section. The luminosity will therefore also very nearly be the same over the whole width of the spreader.

The occurrence of this vigorous motion further offers an explanation of various phenomena of a mechanical nature in connection with the formation of the figures. We may just mention the formation of dust figures or the depressions produced in the surface by the spreaders when the figure is formed over water or other liquids. Of a different nature are the permanent indentations made by Lichtenberg figures in the surface of melting pitch, which were noticed by E. W. BLAKE[1], and which are no doubt due to the electric forces between the charges of the spreaders and of the plate B.

2. Range and Velocity of the Positive Figures.

(a) Relation between Range and Velocity and the *p. d.*

According to L. F. II formulae (I)—(II) we have that:

$$x = R(1 - e^{-\alpha t}), \tag{I}$$

and

$$U = \alpha R e^{-\alpha t} = U_0 \cdot e^{-\alpha t}, \quad U_0 = \alpha R \tag{II}$$

where R is the range of the figures and x the length of the spreaders at time t.

From (I) and (II) we have that

$$U = U_0 \cdot \frac{R - x}{R}. \tag{III}$$

[1] E. W. BLAKE: Am. Journ. Sc. and Arts (2) Vol. 49, p. 289—94, 1870.

According to App. 1, formula (7) the maintenance of a velocity U requires an electric force X determined by:

$$X = k\sqrt{U} \tag{IV}$$

where k is a constant dependent upon the nature and the pressure of the gas and also on the nature of the particle.

From (III) and (IV) we have that

$$X = k\sqrt{U_0} \cdot \sqrt{\frac{R-x}{R}}. \tag{1}$$

From this we find the corresponding *p. d.* distribution by putting, according to formula (2) p. 63.

$$X = \frac{1}{\varrho} V, \tag{2}$$

where ϱ is the radius of the strongly charged tip of the spreader. The length of this radius cannot be determined beforehand but it is reasonable to assume ϱ to be proportional to the range of the front particle at the velocity in question. This range, according to App. 1 formula (2), is proportional to $U^{\frac{3}{2}}$ so that the above formula may be written

$$X = \frac{aV}{U^{\frac{3}{2}}}, \tag{2'}$$

where a is the constant from formula (2) in App. 1.

From the above formulae we have that

$$V = \frac{k}{a} U_0^{\frac{1}{2}} U^{\frac{3}{2}} \sqrt{\frac{R-x}{R}} = \frac{k}{a} \cdot U_0^2 \left(\frac{R-x}{R}\right)^{\frac{3}{2}}. \tag{3}$$

When $x \rightarrow 0$ then $V \rightarrow V_0$ and $U \rightarrow U_0$ and we therefore get

$$U_0 = \sqrt{\frac{a}{k} \cdot V_0}. \tag{4}$$

This expression agrees in the main with the experimental results; compare L. F. I p. 50, fig. 52.

According to (II) we have

$$R = \frac{U_0}{\alpha},$$

and since according to Ann. d. Ph. Bd. 69, p. 214 α is very nearly independent of the *p. d.* V_0, it then follows from the above formulae that

$$R \cong c\sqrt{V_0}, \tag{5}$$

which also on the whole agrees with experiment; compare L. F. I p. 39.

Formula (3) determines the dependency of the *p. d.* on the distance from the electrode at the instant when the front particle is passing the point in question. We will then proceed to investigate how the resistance of the spreaders must depend upon the time elapsed since the start of the ionization, in order that by application of Ohm's law we may come to the said expression for the *p. d.* We may here emphasize, however, that these calculations are only rough estimates. The conditions in the discharge tracks are so complicated and so little known, that a theory satisfactory in all details cannot be developed at present.

For the sake of simplicity we will therefore assume that the initial ionization following the tip of the spreader has the same value all along the discharge track. For a length element dx of the spreader the resistance may be $r \cdot dx$. We further assume that after a lapse of time t' reckoned from the commencement of the ionization the resistance has increased to $e^{\beta t'} \cdot r \cdot dx$.

The resistance M_y of the length y of the spreader at the moment when this has the range y is consequently determined by

$$M_y = r\int_0^y e^{\beta t'} \cdot dx = r\int_0^y e^{\beta(t_1 - t_2)} \cdot dx,$$

where t_1 is the time from the start of the discharge until when it has reached the distance y, while t_2 is the time at which the spreader has attained the length x.

From (I) we have that

$$e^t = \left(\frac{R}{R-x}\right)^{\frac{1}{\alpha}}, \tag{6}$$

where t and x are corresponding values of time and spreader length.

Consequently we have

$$M_y = r\int_0^y \left(\frac{R-x}{R-y}\right)^{\frac{\beta}{\alpha}} \cdot dx = \frac{r}{\frac{\beta}{\alpha}+1}\left(\frac{R^{\frac{\beta+1}{\alpha}+1}}{(R-y)^{\frac{\beta}{\alpha}}} - (R-y)\right). \tag{7}$$

For the current i we have the following expressions

$$i = \frac{V_0 - V_y}{M_y} \quad \text{and} \quad i = h\, V_y\, U, \tag{8}$$

where h is proportional to the capacity of the spreader per unit of length and therefore also approximately proportional to the width t of the spreader.

If we put the two quantities in (8) equal we get

$$V_y = V_0 \cdot \frac{\left(\frac{R-y}{R}\right)^n}{\left(\frac{R-y}{R}\right)^n + K_0 R\left(1 - \left(\frac{R-y}{R}\right)^{n+2}\right)}, \tag{9}$$

where

$$K_0 = \frac{hr}{\frac{\beta}{\alpha}+1} \quad \text{and} \quad n = \frac{\beta}{\alpha} - 1. \tag{10}$$

Consequently, for $y=0$, $V_y = V_0$, and for $y=R$, $V_y=0$.

As h is very nearly proportional, and r inversely proportional, to the width of the spreader, then K_0 is very nearly independent of this width.

From (9) we have

$$\frac{dV_y}{dy} = -V_0\cdot\left(\frac{R-y}{R}\right)^{n-1}\cdot\frac{K_0\left(n+2\left(\frac{R-y}{R}\right)^{n+2}\right)}{\left[\left(\frac{R-y}{R}\right)^n + K_0 R\left(1-\left(\frac{R-y}{R}\right)^{n+2}\right)\right]^2}. \quad (11)$$

For

$$K_0 R\,(n+2) = n \quad (12)$$

we have for $y\to 0$ and $y\to R$

$$\left(\frac{dV_y}{dy}\right)_{y\to 0} = \left(\frac{dV_y}{dy}\right)_{y\to R} = -V_0\cdot\frac{n}{R}\cdot\left(\frac{R-y}{R}\right)^{n-1}. \quad (13)$$

If we further put $n=\frac{3}{2}$ then we see not only that V_y at $y=0$ and $y=R$ has values equal to those determined for V by (3) but also that $\left(\frac{dV}{dy}\right)$ at the points considered has the same value in the two cases since, according to (4), we have

$$V_0\cdot\frac{n}{R} = \frac{3}{2}\cdot\frac{R}{a}\cdot U_0^2.$$

With regard to the various assumptions on which the above results are deduced, we must still add a few remarks.

The assumption that the original resistance per unit of length is constant along the entire spreader is not correct. There is reason to assume the resistance to be inversely proportional to the width, but the quantity of electricity consumed by charging of the spreader is at the same time nearly directly proportional to the width. It will, however, in the main be the product of resistance and capacity of the spreader which is the deciding factor, and this product will, as mentioned, be very little dependent on the width. It is, however, to be assumed that the capacity decreases somewhat more slowly than the conductivity with decreasing width, and the product $(r \cdot h)$ will therefore increase a little under these conditions. The error caused by the slightly tapered form of the spreaders will, all things considered, hardly have any considerable value, but for the thinner spreaders there will be a tendency to a smaller velocity and a smaller range than for the wide ones, a result which is confirmed by plate 4 I.

We will now discuss the assumption that the conductivity decreases at the rate $e^{-\beta t}$. The conductivity is mainly due to the free electrons the number of which per cm^3 we will call N. We therefore simply write the conductivity of the spreader as

$$\frac{1}{r} = \lambda = \frac{c}{A} \cdot N, \tag{14}$$

where A is the cross-sectional area of the spreader and c is a constant.

The free electrons may be lost by recombination with positive ions or by forming negative ions with neutral molecules.

In the cases where the ionization is not extremely strong and where no strong outer field is found, we shall have in the first case

$$\frac{dN}{dt} = -\alpha N^2 \quad \text{or} \quad N = \frac{N_0}{1 + \alpha t N_0}, \tag{15}$$

where, at atmospheric pressure, $\alpha \cong 1 \cdot 6 \cdot 10^{-6}$.

In the second case we have

$$\frac{dN}{dt} = -bN \quad \text{or} \quad N = N_0 \cdot e^{-bt}, \tag{16}$$

where b is a constant which is proportional to the air pressure but also to a large extent depends upon the nature of the gas. For hydrogen and nitrogen b will thus be very nearly equal to zero, while for oxygen b will be of the the order of 10^6 at a pressure of 760 mm Hg.

A rough calculation of the maximum resistance shown by the spreaders and the number of ions per cm^2 of a cross section of the spreader — in both cases on the assumption that the thickness of the spreaders (perpendicular to the plate) is about $^1/_4$ mm — leads, however, to the result that the density must be 10^{12} to 10^{13} electrons per cm^3. We have, however, further assumed that we can apply a value of the conductivity calculated on the basis of the kinetic gas-theory. But at such intense ionization and such strong fields as are here considered, neither formula (15) nor (16) can be applied; nor will the general kinetic theory lead to results of even approximately the right order. The conditions in highly conductive air spaces are as a whole very little known; compare f. inst. the efforts to establish a plausible theory for the conductivity of the electric arc[1]. We shall therefore not enter further into this question here but shall merely remark that, considering the agreement found with experiment, when assuming the conductivity to decrease at the rate $e^{-\beta t}$, there is reason to believe that the conductivity of strongly ionized spaces in the main decreases in the said manner, the value of β being about $5 \cdot 10^{-7}$. Presumably this result cannot be dismissed at the outset as unreasonable.

A few more remarks may be made with regard to the decrease of the conductivity as a function of time. First, the conductivity will maintain a considerable value as long as

[1] H. Hagenbach: Handb. d. Phys. Bd. 14, p. 346 1927.

a great number of electrons are set free at the tip of the spreader. We will come back later to this question under 4 (a). Secondly, even when all of the free electrons have disappeared, the discharge track will retain some conductivity owing to the heavy ions still present. But this conductivity is so inconsiderable, at least at higher pressures, that it is of no significance for the passage of the comparatively strong currents existing in the spreaders during their formation.

At comparatively low pressures the conductivity due to the ions may be of such value that it may have some influence upon the formation of the spreaders. The conductivity due to the ions will decrease at a slower rate than the conductivity due to the electrons, and the following conditions may then occur: the growth of the spreader stops completely, or very nearly completely, because the ohmic voltage-drop becomes too high, after which the part of the spreader already formed will be charged comparatively slowly to a higher potential owing to conductivity due to the ions. Especially at the tip the potential may increase considerably. The result of this is that a new spreader starts from the tip of the old one, or that the tip of the latter gains renewed speed. Examples of such cases are quite frequent; see thus plate 21 IV and 23 ($p = 50$ mm Hg.). In some few cases this process may be repeated several times; see plate 12 IX.

The conductivity due to ions at all events plays an important rôle for the continued growth of the spreaders if the potential on the electrode is maintained for a comparatively long time, but this question will be treated later. The leading away of the charge from the figure through the shunt R — see fig. 1 — also mainly depends on the conductivity due to ions.

Although it has not been possible to work out an explicit theory for the conditions in the strongly conductive spreaders, the considerations set forth above show that the formation of the spreaders, as known from experience, may be explained in a reasonable manner by means of the hypothesis set forth about the manner of formation of the positive spreaders.

As for the negative figures there exists in the main no doubt with regard to their mode of formation, and a detailed analysis of the above treated questions is therefore of less interest for these figures. We shall therefore make only a few remarks concerning their mode of formation.

Since their conductivity is mainly due to free electrons, the amount of electricity neccessary to charge the negative figure must for the most part originate in electrons travelling outward from the electrode. A strong ionization must therefore occur at, and in the neighbourhood of, the electrode, with intensity decreasing outwards. A minor part of the charge is due to positive ions moving toward the electrode. On their way outward the electrons will cause ionization and this will be the stronger the stronger is the field. Since the latter increases towards the electrode, this will also be the case for the ionization by collision. The outer edge of the figure is determined by the condition that the field becomes too weak here to maintain ionization by collision.

(b) Relation between Range and Velocity and the Thickness d_0 of the Insulating Plate.

With decreasing thickness d_0 of the insulating plate, the electric field at the tip of the positive spreaders will increase, but its direction will at the same time be bent down towards the plate; the track of the front particle will therefore also have a downward direction as indicated by the arrow U in fig. 28.

When the front particle hits the plate it will be stopped. The tendency to impinge on the plate will increase with decreasing thickness of the plate and consequently the range of the figure will go toward zero simultaneously

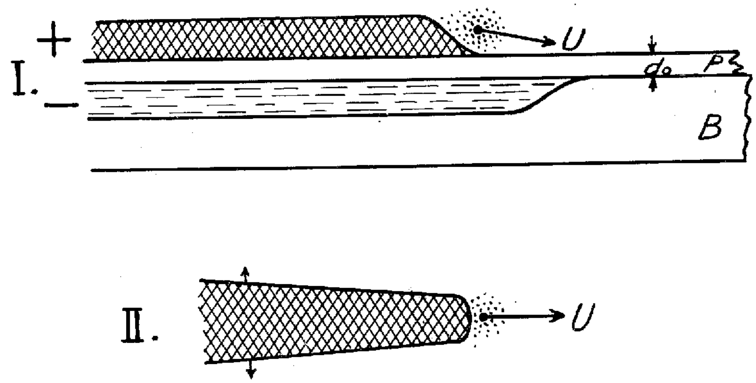


Fig. 28. Schematical Representation of the Conditions at the Tip of a Positive Spreader for small Thicknesses of the Insulating Plate.

with the plate-thickness. For very great values of d_0 the electric field, and therefore also the velocity, will become comparatively small. The velocity consequently attains a maximum value at an intermediate value of d_0. This is in complete agreement with the experimentally determined relation between velocity and plate-thickness; see fig. 8 in this paper and L. F. I, p. 56, fig. 59.

The range of the positive figures as a function of d_0 — if, as is to be expected, α remains constant — must be governed by a similar relation, and this is also in agreement with the experimental results set forth in L. F. I, p. 30, fig. 34.

In the case of negative figures the electrons at the end of the spreader will also be drawn down towards the plate, and the thinner the plate, the stronger will be the field. The front-ionization will therefore also be the stronger, the thinner the plate. This strong ionization will very rapidly set free electrons in sufficient number to supply the proper negative charge to the negative spreader. When this is effected, the surplus of electrons coming from the electrode and its vicinity will not be pulled down toward the plate, but can move freely in the air under the influence of the field. Since the field at the edge increases with decreasing thickness of the plate, the velocity, and simultaneously the range, will approach a maximum when the plate thickness decreases toward zero. This result, also, is in agreement with experience; compare L. F. I, p. 30, fig. 34 and fig. 8 in this paper.

(c) Relation between Range and Velocity and the Pressure. Nature of the Positive Front Particle.

It appears from equation (IV) in section (a) above and equations (2), (6), and (7) in App. 1 that

$$k = k_0 \cdot p, \tag{1}$$

where p is the air pressure and k_0 a constant.

From equation (2) in App. 1 we find that

$$a = \frac{a_0}{p}. \tag{2}$$

Equation (4) in section (a) will consequently give

$$U_0 = \sqrt{\frac{a}{k} \cdot V_0} = \frac{1}{p}\sqrt{\frac{a_0}{k_0} \cdot V_0}, \tag{3}$$

so that according to the theory the velocity of the positive spreaders should be inversely proportional to the air pressure.

Their range is accordingly determined by

$$R = \frac{U_0}{\alpha} = \frac{1}{\alpha p} \cdot \sqrt{\frac{a_0}{k_0} \cdot V_0}. \tag{4}$$

Since α decreases somewhat with decreasing pressure, the range will, according to equation (4), increase somewhat more rapidly with decreasing pressure than it would if R were proportional to $\frac{1}{p}$. This is in good agreement with the formulae given by v. BEZOLD and S. MIKOLA[1] and our experimental determination of the dependency of the range on the air pressure, at not too low pressures is also fully in agreement with this.[2]

But from fig. 42 L. F. I it further appears that at very low pressures R increases more slowly with decreasing pressure than it should according to equation (4). Fig. 58 in L. F. I further shows that the velocity approaches a finite limiting value — U_m in fig. 8 above — when the pressure approaches zero.

It is also easily seen that at extremely low pressures the manner of formation must be different from that assumed here. If we f. inst. assume that in the limit the pressure becomes zero, then the front particle will nevertheless by unable to attain a velocity greater than that corresponding to a free fall through the entire $p. d.$ V_0. We consequently have

$$U_{p \to 0} < 1.38 \cdot 10^6 \sqrt{\frac{z}{M} \cdot V_0}, \tag{5}$$

where M is the mass of the particle with the hydrogen atom as unity, and $(z \cdot e)$ its charge.

The value given by the right hand term in (5) is not attained before the particle has travelled through the total

[1] L. F. I, p. 41. [2] l. c., Fig. 42.

$p.d.$ V_0. Right at the electrode the velocity will be zero. On the other hand the method used in these investigations for measuring the velocity, (see L. F. I), determines particularly the velocity near the electrode. Applying this method to the case of freely moving particles will consequently lead to values of U considerably lower than those given by the right hand term of equation (5). Finally, the spreading-out-velocity of the figure at small but finite pressures will certainly be smaller than the velocities found for freely moving particles.

The experimentally obtained values of the velocity must therefore, at low pressures, be supposed to have a limiting value considerably lower than the one determined by equation (5); in fact, hardly more than half that value. We therefore put this limiting value to be

$$U_{m,\, p \to 0} = 7 \cdot 10^5 \sqrt{\frac{z}{M} \cdot V_0}. \qquad (6)$$

Fig. 42 in L. F. I shows the results of a series of such measurements. The limiting value found is about $5.2 \cdot 10^7$ cm sec.$^{-1}$ and the $p.\ d.$ used was $V \cong 7000$ volts. By inserting these values in (6) we get

$$\frac{z}{M} = 0.79.$$

For H^+-particles or protons $\frac{z}{M} = 1$, for α-particles this ratio is $\frac{1}{2}$, and for all other known positive particles considerably smaller.

We are therefore led to the supposition that the front-particles are protons.

In the negative figures, where the active outward-travelling particles are electrons, there exists no such limiting value for the velocity at decreasing pressures, or more correctly, the limiting value is so high that it falls outside the range here considered. This is in complete agreement with experiment, which shows that at decreasing pressures the spreading out velocity of the negative figures may attain very high values, see f. inst. fig. 8 above and L. F. I, fig. 38.

3. The Start of Positive and of Negative Discharges.

(a) The Start of Negative Discharges.

The start of a negative figure presents no remarkable features. With a needle-point as electrode placed directly on the photographic plate the range of the figure will, at low potentials, be proportional to the *p. d.*, and this holds right down to a few hundred volts, when the figure becomes so small that it cannot be seen with the naked eye. The explanation is obvious. In the immediate vicinity of the electrode-point the field has, even at small potentials, such a value that the ionization by collision sets in, i. e. figure formation starts. But at these low potentials the electrode must neccessarily be point-shaped or have sharp edges, and must further be clean in the sense defined elsewhere by the author[1]. If the electrode is unclean, greasy, for example, the air does not come in direct contact with the sharp edges or points, and ionization by collision will thus not occur at low potentials.

There is thus no difficulty in explaining the experimental results, that negative figures start most easily from a clean point, that at low potentials their range is proportional to the *p. d.*, and that no lower limit exists for the effective potential.

[1] P. O. Pedersen: Vidensk. Selsk. Math.-fys. Medd. Vol. IV, No. 10. Copenhagen 1922; Ann. d. Physik (VI) Bd. 71, p. 317—376, 1923.

(b) The Start of the Positive Discharges.

The start of a positive figure, however, presents very peculiar features, as appears from chap. III 3 (b). We shall now proceed to discuss these features in the light of the hypothesis already set forth, and further defined in the preceding section, where we stated that the front particles must be protons.

We will for the present assume that these protons originate from hydrogen molecules, and we will put their number N per cm^3 equal to

$$N = N_0 \cdot p, \tag{1}$$

where p is as usual the air pressure.

The volume of that space near the electrode within which the electric field has sufficient strength to split off protons from hydrogen molecules we will call A. The total number of hydrogen molecules available for the possible giving up of protons is consequently

$$n = A N_0 p. \tag{2}$$

The probability s that a hydrogen molecule will give up a proton, depends upon the field, and therefore also upon the potential V. We put

$$s = s(V), \tag{3}$$

and the number n_p of protons available is consequently

$$n_p = s A N_0 p. \tag{4}$$

We shall next discuss under what conditions an available proton becomes active, that is, causes the start of a positive spreader. These conditions are undoubtedly rather complicated. However, it is excluded from the chance of becoming active if it is first rendered "hors de combat" by any means: for example it may adhere to the first neutral molecule it strikes, thus giving rise to a positive ion, or it may be neutralized by an electron from the molecule with which it collides. The conditions that are neccessary to prevent such occurrences cannot be stated with any certainty, but the probability that the proton remains free after a collision is the greater the greater its velocity. We assume for simplicity that the proton remains free if — and only if — its velocity at the collision is greater than that corresponding to V' volts. Denoting the distance travelled by the proton in the direction of the field since its liberation by x, and the electric field strength by E, the proton will remain free after the first collision if

$$Ex > V'. \tag{5}$$

In order to simplify the calculations we take a mean value of E in the active space A by putting

$$E = \frac{V}{L}, \tag{6}$$

where L is proportional to the thickness of the active space.

Then equation (5) will read

$$x > \frac{L}{V} \cdot V'. \tag{5'}$$

The mean value of the free path l of the proton is inversely proportional to the pressure, that is

$$l = \frac{l_0}{p} \tag{7}$$

and the probability S that the first free path of one of the available protons is greater than x is

$$S = e^{-\frac{x}{l}} = e^{-\frac{LV'}{l_0 V} \cdot p} = e^{-c \frac{p}{V}}, \quad c = \frac{L}{l_0} V_0. \tag{8}$$

The total number n^0 of active protons is consequently

$$n^0 = np \cdot S = sAN_0pe^{-c\frac{p}{V}}. \tag{9}$$

Until anything is stated to the contrary, we shall assume that we have the same electrode and the same kind of gas; in that case A and N_0 are constants and (9) may be written

$$n^0 = sKpe^{-c\frac{p}{V}}. \tag{9'}$$

We will further assume quite provisionally that $s = s(V)$ is a constant. This assumption is, no doubt, not justified, but it will simplify the following considerations, and we shall investigate later the effect of the dependency of s on V.

We therefore write provisionally

$$n^0 = K_0 \cdot p \cdot e^{-c\frac{p}{V}}, \tag{9''}$$

where K_0 is considered a constant.

A positive discharge starts only if $n^0 \geqq 1$. This is the case when

$$\frac{1}{p} \cdot e^{c\frac{p}{V}} \geqq K_0. \tag{10}$$

Here we do not know — and we are unable to calculate beforehand with sufficient accuracy — the value of K_0, but we may try to find out under what conditions the left hand term of (10) is independent of the air pressure. We will write this condition in the form

$$lg\, p - c\frac{p}{V} = \text{constant}. \tag{11}$$

Now the question is: are we able to select such a value of $c = \frac{L}{l_0} V'$ that a (p, V)-curve determined by means of

(11) shows with sufficient accuracy a relation between p and V corresponding to the relation found by experiment? Further, will the value of c applied in such a case be a reasonable one?

In fig. 18 are shown by crosses the corresponding values of potential and air pressure obtained in atmospheric air and with a plate thickness of about 1.4 mm.

The thin curve in fig. 18 represents the equation

$$lg\,p - \frac{12}{V} \cdot p = 3.04\,. \tag{12}$$

We see that equation (12) does not give any very good representation of the experimental results since — especially at higher pressures — the experimentally determined potentials have somewhat lower values than the calculated potentials. Equation (12), on the other hand, represents values of the right order of magnitude for the relation between p and V over a wide range, and it may, therefore, be of some interest to see whether the value given for c is also of the right order of magnitude. If in (7) the free path is given in cm and the pressure in mm, then $l_0 \cong 2.6 \cdot 10^{-2}$ cm. For electrodes resting directly on the photographic plate the value of L in (6) will presumably be about 0.025 à 0.05 cm. V' must presumably have a value of about 10 volts. The values of c corresponding to this are between 9.6 to 19.2, and these limiting values agree very well with the value 12 in equation (12).

It is obvious why equation (12) can give only a comparatively rough approximation. The probability $s(V)$ that a hydrogen molecule gives up a proton is not constant, as we have assumed in the equations (9″) to (12), but must, on the contrary, increase quite rapidly with increasing values of the potential. However, we do not know this relation in any detail, and for the sake of simplicity we therefore replace (9′) by the following equation:

$$n_0 = sKpe^{-c\frac{p}{V}} = K^0 \cdot p \cdot e^{-c^0\frac{p}{V^2}}. \tag{13}$$

This expression is, no doubt, not theoretically correct but — as compared with (9″) — has the advantage that the value of the probability s increases with increasing value of the potential.

In fig. (18) the heavy curve corresponds to the equation

$$lg\, p - 32 \cdot 10^3 \cdot \frac{p}{V^2} = 2.12\,, \tag{14}$$

and we see that this curve furnishes a representation of the experimental results as satisfactory as may be expected when we take into account the considerable uncertainty in the experimental determination.

Fully in agreement with the considerations here set forth is the fact that the positive figure, at potentials near the critical minimum value, often consists of only one single or a few spreaders. But from this it again follows that it is a matter of chance whether or not a figure is formed with potentials near the critical value.

According to (9) the number of active protons is proportional to the volume of the active space A. It is difficult to state a definite value for this, but it is obvious that of the four different forms of electrodes shown in figs. 13 and 16 the spherical one will have the greatest value of A, the pointed one the smallest value and the rounded rod a value between the two. This explains the fact that within the critical interval the spherical electrode shows a greater tendency to produce figures than the point-electrode. The tube-formed sharp-edged electrode shown in fig. 16, however, shows an ability to produce figures, at least equal to the spherical one. For this electrode, not only is the field very strong at the sharp edge, and consequently the probability *s* great, but also the active space has a considerable volume owing to the considerable length of the edge.

Uncleanness (grease or the like), will cause a slight diminution in the active space immediately around edges and points, but if the uncleanness is only small, its influence in this respect will be insignificant. For positive discharges the electrode will therefore be very little sensitive to contaminations even within the critical interval.

The difference between positive and negative electrodes in sensitiveness to contaminations is quite analogous to the peculiar relations which have been stated with regard to retardation of spark-formation as dependent upon the state of the electrodes[1]. This question will, however, be treated in more detail elsewhere.

In chapt. III 3 (b) we have mentioned that the spherical electrode, within the critical interval, if placed on a not too thick plate, shows a tendency to produce abnormally short spreaders, while sharp-edged electrodes do not show this tendency. This is easily understood from the considerations set forth above, section 2 (c), according to which for spherical electrodes the electric field will have a powerful vertical component at the starting points of the spreaders, which drives the protons down into the plate.

A final consequence of the hypothesis here set forth is that the front-particle must have a certain, rather high velocity if a discharge of the kind here considered shall

[1] P. O. PEDERSEN: "Vid. Selsk. Math.-fys. Medd." IV, 10. Copenhagen, 1922; "Ann. d. Phys." (IV). Bd. 71, p. 317—376, 1923.

be started. The positive spreaders must consequently attain at all events a certain, finite length, if they start at all.

There then remains the question of the origin of the protons. It is well known that protons are set free in discharge tubes containing even the smallest trace of hydrogen[1], and in many cases also *H*-atoms in considerable numbers[2]. In an apparatus such as that by means of which Lichtenberg figures are produced there will be found — even in cases where the use of pure gases as f. inst. N_2 or O_2 is aimed at — a not insignificant number of hydrogen molecules, and these are also present in considerable numbers in the ordinary air of the laboratory. Even if we reckon with an admixture of only 0.0001 pCt. hydrogen there will be present — at atmospheric pressure — $3 \cdot 10^{13}$ hydrogen molecules per cm^3. Hydrogen molecules in sufficient numbers will consequently always be at hand.

Another feature may possibly seem peculiar at first sight. The formation of positive figures ceases — as we have seen — because the number of spreaders approaches zero. It is comparatively seldom, however, that figures are obtained having only 1 or 2 spreaders, generally there will be f. inst. 5—10. On the other hand, the number will not exceed f. example 30—40, according to the existing conditions, even if we raise the potential or diminish the pressure very greatly. The above calculated probability increases, however, to a very great extent, and accordingly the number of spreaders should increase highly.

There are, however, other causes which automatically set a limit to the number n of the spreaders. Out of a probable number of spreaders, the individual members will

[1] J. J. Thomson: "Rays of Positive Electricity", see f. inst. p. 15—20.
[2] K. F. Bonhoeffer: Erg. d. ex. Naturwiss. Bd. 6, p. 204, 207—8, 1927.

not start quite simultaneously and those first started will consequently come ahead of the others. When so many have started that at some distance from the electrode they cover practically the entire circumference, then — on account of their charge — they will counteract the formation of further new spreaders, or at least stop any such in their growth.

If we are far outside the critical interval, the number of spreaders will therefore not be determined by the available number of protons, but simply by the available space.

At not too high pressures we therefore often observe also a number of small spreaders which are stopped in their growth by the electrical charge of those first formed, see for example plates 12 V—VII and IX, 13 I—II and 14 I—II.

According to the hypothesis set forth, it may be expected that the number of available protons will be the greatest in hydrogen or in compounds of hydrogen and nitrogen, smaller in nitrogen, still smaller in compounds of nitrogen and oxygen and the smallest in gases having great electron-affinity as for example oxygen.

From the foregoing it will be understood that this difference will not make itself apparent by the number of spreaders of the normal figures, but it manifests itself quite clearly by the number of side-branches and smaller side-spreaders. The greater the number of available protons the greater number of these will be observed. A look over the figures shown in plates 4, 5 and 6 I, which are taken in various gaseous compounds will verify this fact.

The small side-spreaders and branches are presumably formed in this manner: The original spreaders, which are in the process of formation, act as electrodes with a potential somewhat lower than that of the main electrode, but still of a value sufficient to start available protons with the necessary velocity.

Inside the spreaders protons will generally be set free to a great extent and these will be driven by the electric field to the surface of the spreader, so that here there will always be available protons. Spreaders will, however, be formed only if the electric field in the surrounding air is of sufficient value to give the protons the necessary velocity. The field outside in the air will be strongest at the edge of the spreader immediately on the surface of the photographic film. The protons driven out here will therefore also have the greatest probability of becoming active. If the spreader is formed in a narrow space between two planes — for example if there is a second photographic plate as shown in plate 8 I—II — then the lateral spreading out will be small but both the field as well as the proton-density will be great at the edges and therefore the tendency to form side-spreaders will be great.

If the side-spreaders start in directions which are nearly the same as that of the original spreaders, they will be squeezed in between these, and if they start perpendicular to the original ones, they will become very short as they are stopped by the neighbouring spreaders. Such short side-spreaders are observed in great numbers in hydrogen-nitrogen compounds. (In pure hydrogen this phenomenon is very difficult to study because the luminosity is so small). In some cases these side-spreaders may point inward — in the direction of the electrode — if there is more free space in that direction, see for example plate 5 XVI.

In cases where the spreaders are stopped abruptly by an opposing field as f. inst. at the meeting line in velocity-

7*

measurements, the potential at the tip of the spreader will become comparatively high and the tendency to form side-spreaders will therefore also be great. This conclusion is also verified by experiments, see for example plate 14 I, especially just above the dividing line some little distance outside both ends of the lower electrode.

The hypothesis set forth thus explains in a simple manner all the peculiar features of the start of the positive figures.

4. Conductivity of Positive and of Negative Spreaders.

(a) Conductivity of Negative Spreaders.

In fig. 29, A represents a negative electrode resting on a photographic plate P, while $a\,b$ represents, schematically, a negative spreader which at the point b runs into a free — or earthed — electrode B, the potential of which is zero to begin with.

There will be present in the spreader positive ions as well as negative ions and electrons. The sum of the two last named will be greater than the number of positive ions because the spreader has a negative charge. Since the electrons have a much greater mobility than the ions only electrons are shown in I and II of fig. 29.

We assume the conductivity of the spreaders to be so high that at a point just a little to the left of b there is a potential which is a considerable fraction of the value of the potential of A. There will consequently be a strong field immediately near b and this field will pull the electrons toward and into B. This results in a decrease in the negative charge near b as indicated at c in II. A strong field will then be developed near c, which will pull a further number of electrons in the direction of B. The location of the strong field will therefore — owing to the great mobility of the electrons — move to the left with great velocity. This is indicated by the double arrow at c.

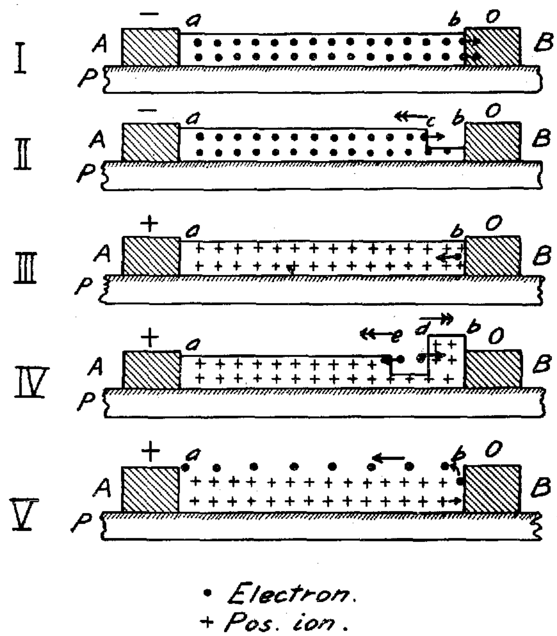


Fig. 29. Schematical Representation of the Conductivity of Negative Figures (I—II) and of Positive Figures (III—IV).

The strong field at b and later at c will cause ionization by collision, by which means the conductivity between c and the electrode B will be comparatively high, even though the negative charge between these two points will be comparatively small. From what is stated here, it appears that there will be a strong tendency to spark-formation between A and B, and the spark will begin at B. This is verified by our experiments, see chapt. III 4 (a).

(b) Conductivity of Positive Spreaders.

Here we find entirely different conditions. The spreader *a b* in III fig. 29 has a positive charge, the number of positive ions being greater than the sum of electrons and negative ions. The electrons are mainly present in the upper part of the spreader.

When the spreader *a b* gets near to *B* there will arise a strong field at *b*, which will drive the positive ions into the electrode *B*. Owing, however, to the low mobility of the ions they will move comparatively very slowly, and we will therefore consider them as insignificant at present. We might perhaps assume that the strong field at *b* would drive the electrons with great velocity towards *A*. Such a movement would, however, very soon be stopped. If f. inst. most of the electrons in the space *b d* in IV are carried over to the space *d e* then this would give a charge distribution as indicated in the figure. But in such a case those of the electrons within the space *d e* which were near *d* would be carried back again with great force toward *B*, while those near *e* would be carried along in the direction of *A*, but with a smaller force. An "electron-wave" such as *d e* will therefore be dispersed and cannot travel from *B* to *A*. Consequently no spark can be formed.

A flow of electrons may, however, very well occur from *B* to *A* i. e. an electric current passing from *A* to *B*, provided only that a suitable number of electrons are set free at *B*, and such a current may be quite strong without actually forming a spark. The strong field at *b* will cause the positive ions to impinge on *B* with great velocity and thus set free a sufficient supply of electrons. These will then, as shown in V of fig. 29, drift to the surface of the spreader and here be carried toward the electrode *A*.

If the *p. d.* between *A* and *B* is not of sufficient value to cause this current to ionize by collision, no light is emitted and no spark is initiated.

If, finally, we maintain the *p. d.* between *A* and *B* so long that the positive ions get time to move sufficient distances, then we get a discharge analogous to the one shown in fig. 28 II, except that here the moving particles are positive ions instead of electrons and in consequence the discharge moves relatively slowly.

The features explained here are in full agreement with the experimental results set forth in chapt. III 4.

A theoretical treatment of the discharge conditions in the case when the potential is maintained long enough to allow the heavier ions to play a deciding rôle in the formation of the discharge, f. inst. the question of spark formation, will be taken up elsewhere. We have mentioned these conditions in chap. III 4 (b) and (c) only to emphasize that they have nothing to do with the formation of the regular Lichtenberg figures.

5. Various Circumstances in Connection with the Formation of the Lichtenberg Figures.

(a) No Influence of an Initial Ionization.

It will be understood without further explanation that an initial ionization has no appreciable influence on the spreading-out of the positive figures as long as this ionization is not strong enough to give rise to a conductivity which will materially alter the electric field active in the formation of the figure. The hypothesis thus gives a perfectly satisfactory explanation of the features mentioned in chap. III 5 (a).

(b) The Bright Border-Line.

The bright rim-formations mentioned in chap. III 5 (b) are easily explained by means of what is set forth in 1 (d) above and in Appendix 2.

In the case of negative figures the electrons will, as is mentioned above, continue their outward movements even after the field at the edge has decreased so much that no ionization by collision takes place. The said electrons will then, however, generally combine with neutral molecules into heavier negative ions, and the outermost edge of this ion-stream will, as pointed out in Appendix 2, become comparatively sharp.

If now, simultaneously with this outward movement, a great number of electrons are set free over the area in question, then ionization by collision will occur at the sharp edge of the figure, and this ionization will cause the emission of light a little outside the edge which, owing to the former mentioned reasons, is situated a little outside the boundary of the photographic image.

Somewhat similar conditions occur in the case of positive figures, except that here the positive ions are so firmly placed that no appreciable spreading out of the figure will occur after the formation of the photographic image has stopped. A rim-formation will in this case occur close to the edge of the photographic image.

Concluding Remarks.

A theory of the formation of negative figures, which is in the main satisfactory has — as stated in the introduction — already been given earlier. When we have also treated of negative figures in the investigations described here our object has been to throw further light upon certain features of the theory already set forth.

The theory developed above of the formation of positive Lichtenberg figures, namely, that formation of positive spreaders is due to protons which the strong field at the tip of the spreaders drive outwards with great velocity, by which means electrons are set free in sufficient number to initiate a sudden and strong ionization by collision which again sets free electrons in sufficient numbers necessary to carry away the charge towards the electrode is found to explain throughout in a satisfactory manner the many peculiar features presented by the positive spreaders.

The investigations here described further indicate that protons play an important rôle not only in the formation of positive Lichtenberg figures, but that their importance in connection with spark formation is much greater than hitherto assumed. None of the theories so far proposed for spark formation offer a satisfactory explanation of a number of peculiar spark phenomena which have been observed and pointed out during recent years[1]. The theory of the formation of the positive figures given here will presumably also be useful for the solution of those problems, but this question will be treated at a later occasion.

Presumably, also, protons play a more important rôle than has hitherto been assumed in a number of other discharge phenomena.

Finally I wish to express my cordial thanks to Carlsberg Fondet for its valuable support of these investig-

[1] P. O. Pedersen: The papers (a)—(e) mentioned p. 4.

ations. To the Rask-Ørsted Fond for contributing to the publication. And also to the various cooperators who have kindly assisted me in carrying out the experimental investigations: namely, Mr. J. P. Christensen, Mr. Chr. Nyholm and Mr. B. B. Rud who have taken most of the photographic Lichtenberg figures here used. A few of the pictures were taken by Mr. C. Schou and Mr. N. E. Holmblad while Mr. F. Nielsen has carried out the photographic enlargements. Mr. J. P. Christensen has further assisted me in preparing this publication for the press.

APPENDIX I

On Positive Particles.

Within the velocity range — $1 \cdot 10^7$ to $10 \cdot 10^7$ cm sec^{-1} — which is of particular interest for positive Lichtenberg figures only very little has been published about investigations of how the range, the ionization, and the charge etc. etc. of the positive particles depend upon the velocity.

For particles at somewhat higher velocities a number of investigations have been carried out and these we have mentioned below under sections 1—5 in so far as they are judged to be of interest for the understanding of the positive Lichtenberg figures. Finally, A. J. Dempster has recently carried out investigations with H^+ particles in hydrogen and helium at velocities within the said range, and these and a few other investigations we will discuss under section 6.

1. Range, Ionization and Velocity of Positive Particles.

The range R of high-speed α-particles is known[1] to be proportional to the third power of the velocity U that is

$$U^3 = K \cdot R = 1.076 \cdot 10^{27} R \tag{1}$$

This relation holds good as long as the range is more than 2 cm *i. e.* at velocities above $1.3 \cdot 10^9$ cm sec.$^{-1}$.

At small ranges and velocities the range is, on the con-

[1] H. Geiger: Proc. Roy. Soc. (A) Vol. 83, p. 505—515, 1910. E. Marsden and T. S. Taylor: Proc. Roy. Soc. (A) Vol. 88, p. 443—454. 1913. Geiger und Scheel: Handbuch d. Phys. Bd. 24, p. 152, 1927.

trary, proportional to the velocity in the power $\frac{3}{2}$ as shown f. inst. by P. M. S. BLACKETT[1]. That is

$$U^{\frac{3}{2}} \doteq aR, \tag{2}$$

which holds good with sufficient accuracy for ranges from 1 cm down to somewhat less than 1 mm.

The equation (2) may also be written as

$$R = b \cdot V_0^{\frac{3}{4}}, \tag{2'}$$

where V_0 is the *p. d.* through which the particle must fall to obtain the velocity U. The constant b has the value

$$b = \frac{1}{a}\left(2\frac{e'}{m}\right)^{\frac{3}{4}}, \tag{2''}$$

where m is the mass of the particle and e' its charge.

Equation (2) holds good — as shown by BLACKETT — not only for α-particles but also for positive atomic ions of hydrogen, argon and "atmospheric air". The corresponding values of a and some other factors of importance for the following — all based on the BLACKETTS measurements — are set up in the following table A.

Table A. Various properties of positive particles.

Particle	a	$\frac{a}{a_H}$	Atomic weight m	$\sqrt{m}$	Average loss of energy per unit length of track $\frac{\frac{m}{R}}{\frac{m_H}{R_H}}$	Relative charges $\eta = \frac{\sqrt{\frac{m}{R}}}{\sqrt{\frac{m_H}{R_H}}}$
H	$6.2 \cdot 10^{13}$	1.0	1.0	1.0	1.0	1.0
He (α-Part)	$3.3 \cdot 10^{13}$	1.9	4.0	2.8	2.1	1.5
"Air"	$1.94 \cdot 10^{13}$	3.2	14.4	3.8	4.5	2.1
A	$1.21 \cdot 10^{13}$	5.1	40	6.3	7.8	2.8

[1] P. M. S. BLACKETT: Proc. Roy. Soc. (A) Vol. 103, p. 62—78, 1923.

The table contains also the average value of the relative average loss of energy suffered by the various particles per cm of the track, which loss BLACKETT takes to be proportional to $\frac{m}{R}$. Further BLACKETT assumes this loss to be proportional to the number of pairs of ions set free per cm, and this number again with approximation proportional to the square of the effective charge of the particle. The latter he therefore takes to be proportionat to $\sqrt{\frac{m}{R}}$, and the value of this quantity is given in the last column of the table. As the charge of the particles is by no means constant over the entire track this assumption may not be quite justified but will nevertheless presumably lead to an approximately correct result.

If the average energy neccessary to set free one pair of ions is w ergs, and if the entire kinetic energy of the particle is spent in the formation of ions, then — denoting the number of pairs of ions set free per cm by I — we have the following relations:

$$wIdR = \frac{1}{2} m \left[U^2 - (U - dU)^2\right] = mUdU.$$

or

$$wI = mU \frac{dU}{dR}, \tag{3}$$

but according to (2) we have

$$\frac{dU}{dR} = \frac{2a}{3U^{\frac{1}{2}}}. \tag{4}$$

From the equations (2)—(4) we get

$$I = \frac{2ma}{3w} \cdot U^{\frac{1}{2}} = \frac{2mU^2}{3wR} = \frac{4}{3} \cdot \frac{V_0}{RV'} = \frac{4}{3} \cdot \frac{V_0^{\frac{1}{4}}}{b \cdot V'}, \tag{5}$$

where V' is the ionization-voltage of the gas in which the particle moves.

The measurements carried out by BLACKETT unfortunately do not go down to such small velocities and ranges as those in which we are directly interested, but until further we will assume that the law expressed by equation (2) is also applicable to velocities from $2 \cdot 10^7$ to $10 \cdot 10^7$ cm sec.$^{-1}$. In order to give an idea of the magnitude of

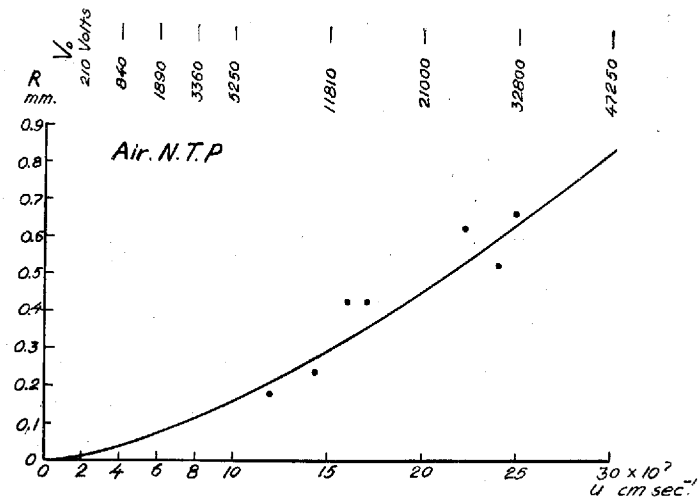


Fig. 1. Range of H^+ particles in Air N. T. P. according to BLACKETT.

the extrapolation thereby performed we have in fig. 1 marked by dots the corresponding values of velocity and range found by BLACKETT for H^+ particles in atmospheric air, while the curve shown has the equation

$$U^{\frac{3}{2}} = 6.2 \cdot 10^{13} R .$$

2. Velocity in Strong Fields.

Further we have

$$U = \sqrt{2 \frac{e'}{m} V_0} = c \sqrt{V_0} . \tag{6}$$

From this expression and from equation (2) follows

$$\frac{dV_0}{dR} = X = \frac{2\,V_0^{\frac{1}{2}}}{c} \cdot \frac{dU}{dR} = \frac{4\,a\,V_0^{\frac{1}{2}}}{3\,c\,U^{\frac{1}{2}}} = \frac{4\,a}{3\,c^2} \cdot U^{\frac{1}{2}}. \qquad (7)$$

Here X is the intensity of the electric field neccessary and sufficient to maintain the velocity U of the particle, assuming that the particle all the time has the charge e'.

In the following table B is shown among other things the value of c in equation (6) when V_0 is measured in volts for the particles investigated by BLACKETT. The figures given for α-particles refer actually to a helium atom with one positive charge.

Table B. Various properties of positive particles in air.

Particle	c	c^2	a	$\frac{a}{c^2}$	Relative Charge η	$\xi = \frac{4}{3\eta} \cdot \frac{a}{c^2}$	X for $U = 2 \cdot 10^7$ cm sec^{-1}
$H+$	$13.8 \cdot 10^5$	$190 \cdot 10^{10}$	$6.2 \cdot 10^{13}$	32.6	1.0	43.5	$195 \cdot 10^3$ volt cm^{-1}
α-Part (H_e^+)	$6.9 \cdot 10^5$	$47.5 \cdot 10^{10}$	$3.3 \cdot 10^{13}$	69.6	1.5	61.8	$259 \cdot 10^3$ volt cm^{-1}
(Air)+	$3.6 \cdot 10^5$	$12.9 \cdot 10^{10}$	$1.9 \cdot 10^{13}$	148	2.1	94.2	$442 \cdot 10^3$ volt cm^{-1}
$A+$	$2.2 \cdot 10^5$	$4.6 \cdot 10^{10}$	$1.2 \cdot 10^{13}$	263	2.8	125	$559 \cdot 10^3$ volt cm^{-1}

ξ indicates the factor by which $U^{\frac{1}{2}}$ is to be multiplied in order to obtain the intensity X of the electric field necessary to maintain the velocity of the particle in question in the gas in question, which latter in the above table is air at N. T. P. It must, however, be remembered that for this calculation of X it is assumed that the charge of the H-particle is equal to $+e$ over the entire track.

3. The Mean Value of the Charge of the Hydrogen Atom.

If l_1 is the average length of those parts of the track (in the direction of the force) along which the H-particle has the charge $+e$, i. e. is a proton, while l_2 is the corresponding average length of track along which the H-particle is a neutral hydrogen atom — and its charge consequently is zero — then the values found for X must evidently be multiplied by the factor

$$\mu = \frac{l_1 + l_2}{l_1} = 1 + \frac{l_2}{l_1}. \tag{8}$$

For the value of $\frac{l_1}{l_2}$ there exist a number of rather contradictory determinations. In the following we will apply only those given by E. RÜCHARDT[1]. Fig. 2 shows RÜCHARDT's

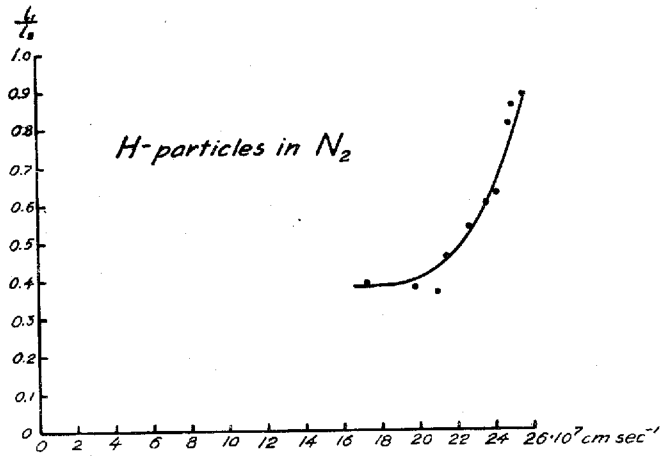


Fig. 2. Value of $\frac{l_1}{l_2}$ for H-particles in N_2 according to RÜCHARDT.

results with H-rays in N_2. For H-particles in O_2 it seems that $\frac{l_1}{l_2}$ is somewhat smaller. Unfortunately none of the measurements deal with such relatively small velocities as those with which we have to do in the case of the Lichtenberg figures. An extrapolation must therefore be very uncertain. But RÜCHARDT's results seem to indicate

[1] E. RÜCHARDT: Ann. d. Phys. (IV). Bd. 71, p. 377—423, 1923.

that for H-particles in both H_2 and N_2 $\frac{l_1}{l_2}$ has decreased to its minimum value at velocities about $1.7 \cdot 10^8$ cm sec.$^{-1}$. Since no other information on this point is available, we will for the present assume that $\frac{l_1}{l_2}$ is constant at velocities from 10^7 to 10^8 cm sec.$^{-1}$ in which we are here interested, and we will put, for H-particles in atmospheric air

$$\frac{l_1}{l_2} = 0.4. \tag{9}$$

Although this value for $\frac{l_1}{l_2}$ is a little higher than the one found for N_2 and still more so in comparison to the one determined by Rüchardt for O_2, we have selected it because in all the cases in which we are particularly interested the particle will move in an exceedingly strong electric field which will tend to increase the relative velocity between the H-particle and the electrons set free by its collisions. This increase in relative velocity will decrease the probability that an electron is again captured by an H^+-particle, and consequently l_1 will be greater than it is without such a strong field, compare Rüchardt's work dealing with this question.[1]

4. The Value of the Ratio $\frac{l_1}{l_2}$ depends upon the Intensity of the Electric Field.

An exact treatment of this question is not possible at present owing to insufficient knowledge of the ionization — and recombination — processes. But in order to form an idea of the influence of a strong field on these pro-

[1] E. Rüchardt: Zeitschr. f. Phys. Bd. 15, p. 164—171, 1923. E. Rüchardt: Ann. d. Phys. (IV). Bd. 73, p. 228—236, 1924.

cesses we will subject the matter to some simple considerations based upon the ideas outlined by RÜCHARDT.

At first we assume the electric field to be zero. The particle having the charge $+e'$ moves with the velocity U_0 while the electrons are assumed to be at rest. With regard to the question of recombination we may as well consider the particle to be at rest and the electrons to move with the velocity U_0 relatively to the particle. The electrons that in a given moment are set free with the velocity zero — relative to the particle consequently with the velocity U_0 — and that are located around the particle within a sphere having the radius ϱ_0, will all move in eliptical orbits around the particle provided that ϱ_0 satisfies the following conditions.

$$\frac{1}{2} m U_0^2 = \frac{e \cdot e'}{\varrho_0}, \tag{10}$$

m denoting the mass of the electrons and $-e$ their charge.

Electrons set free outside this sphere will move in hyperbolic orbits and will consequently not be captured by the particle, while all those set free inside the sphere will be captured.

If we put $e = e' = 4.774 \cdot 10^{-10}$ E. S. E. and $m = 9 \cdot 10^{-28}\,g$ then

$$\varrho_0 = \frac{5.05 \cdot 10^8}{U_0^2}. \tag{10'}$$

For

$U_0 = 1 \cdot 10^7 \quad 2 \cdot 10^7 \quad 3 \cdot 10^7 \quad 4 \cdot 10^7 \quad 5 \cdot 10^7\,\text{cm}\,\text{sec}^{-1}$

we get

$\varrho_0 = 5.05 \cdot 10^{-6} \quad 1.26 \cdot 10^{-6} \quad 5.6 \cdot 10^{-7} \quad 3.16 \cdot 10^{-7} \quad 2.02 \cdot 10^{-7}$ cm.

We will next consider the case where a constant field X (E. S. E.) acts parallel to the velocity U_0. Here an exact treatment is very difficult, we will therefore take only a simple approximation.

Referring to figure 3, the particle is at a given moment located at P and an electron is set free at A. To begin with the electron has — in relation to the particle — the velocity U_0 in the positive direction of the X-axis. It is further acted upon by the electric field X which tends to

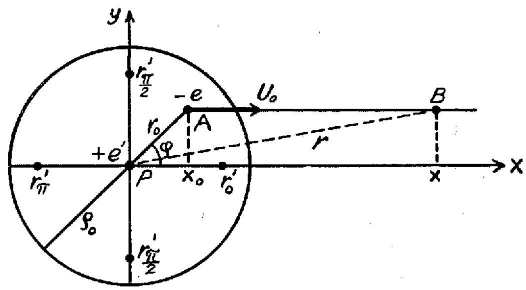


Fig. 3. Recapture of Electrons in Strong Electric Field.

increase the relative velocity. We now assume that the electron is continually moving out along the line AB which is parallel to the X-axis. This is of course not correct, since the track will be curved. But we are here not interested in the shape of the track itself but only whether the electron will infinitely continue its movement away from the particle, or return to its vicinity. In the latter case we consider it to be captured, in the first case not.

If the starting point A were located on the x-axis then the electron would remain there and would only be captured if at some distance or other its velocity decreases to zero. If this occurs then the electron will return to the particle, in the reverse case it would travel away. The conditions are quite analogous if the electron is bound to move along the line AB.

At the point B the velocity U_x of the electron is determined by

$$\begin{aligned}\frac{1}{2}\, m\, U_0^2 &= \frac{1}{2}\, m\, U_0^2 - \frac{ee'}{r_0} + \frac{ee'}{r} + eX\,(x - x_0)\\ &\cong \frac{1}{2}\, m\, U_0^2 - \frac{ee'}{r_0} + \frac{ee'}{x} + eX\,(x - x_0)\\ &= ee'\left(\frac{1}{\varrho_0} - \frac{1}{r_0} + \frac{1}{x}\right) + eX\,(x - x_0)\,. \qquad (11)\end{aligned}$$

The velocity U_x will be zero for $x = x'$ if

$$e'\left(\frac{1}{\varrho_0} - \frac{1}{r_0} + \frac{1}{x'}\right) + X\,(x' - x_0) = 0 \qquad (12)$$

or when

$$x'^2 - x'\left[x_0 + \frac{e'}{X}\left(\frac{1}{r_0} - \frac{1}{\varrho_0}\right)\right] + \frac{e'}{X} = 0 \qquad (13)$$

and consequently for

$$x' = \frac{1}{2}\left[x_0 + \frac{e'}{X}\left(\frac{1}{r_0} - \frac{1}{\varrho_0}\right)\right] \pm \sqrt{\frac{1}{4}\left[x_0 + \frac{e'}{X}\left(\frac{1}{r_0} - \frac{1}{\varrho_0}\right)\right]^2 - \frac{e'}{X}}\,. \qquad (14)$$

A necessary condition for the velocity to become zero at all is that the value of x' as determined by (14) is real, that is, we must have

$$x_0 + \frac{e'}{X}\left(\frac{1}{r_0} - \frac{1}{\varrho_0}\right) \geqq 2\sqrt{\frac{e'}{X}}\,. \qquad (15)$$

In the limiting case, to which the lowest sign of equation (15) applies, we have

$$x' = \sqrt{\frac{e'}{X}} \quad \text{or} \quad X = \frac{e'}{X'^2}, \qquad (16)$$

so that the electric force in the point x' is zero. If the velocity has not gone quite to zero when the electron has attained this point then the velocity can never be zero.

Since $x_0 = r_0 \cos\varphi$ then the equality in (15) gives the following equation for the determination of the maximum value r' of r_0, for which the electron will return to the vicinity of the particle:

$$r'^2 - r' \frac{2}{\cos\varphi}\left(\sqrt{\frac{e'}{X}} + \frac{e'}{2\varrho_0 X}\right) + \frac{1}{\cos\varphi} \cdot \frac{e'}{X} = 0 \tag{17}$$

and therefore

$$\begin{aligned} r' = \Bigg[& \frac{1}{\cos\varphi} \cdot \left(1 + \frac{1}{2\varrho_0}\sqrt{\frac{e'}{X}}\right) \\ & \pm \sqrt{\frac{1}{\cos^2\varphi}\left(1 + \frac{1}{2\varrho_0}\sqrt{\frac{e'}{X}}\right)^2 - \frac{1}{\cos\varphi}}\,\Bigg] \cdot \sqrt{\frac{e'}{X}}. \end{aligned} \tag{18}$$

$$\left.\begin{aligned} & \text{For } \varphi = 0 \text{ we have} \\ & r' = \left[1 + \frac{1}{2\varrho_0}\sqrt{\frac{e'}{X}} - \sqrt{\frac{1}{\varrho_0}\sqrt{\frac{e'}{X}} + \frac{1}{4\varrho_0^2} \cdot \frac{e'}{X}}\,\right] \cdot \sqrt{\frac{e'}{X}} = r'_0, \\ & \text{and for } \varphi = \frac{\pi}{2} \text{ we have} \\ & r' = \frac{\frac{1}{2}\sqrt{\frac{e'}{X}}}{1 + \frac{1}{2\varrho_0}\sqrt{\frac{e'}{X}}} = r'_{\frac{\pi}{2}}, \\ & \text{and for } \varphi = \pi \text{ we have} \\ & r' = \left[-\left(1 + \frac{1}{2\varrho_0}\sqrt{\frac{e'}{X}}\right) + \sqrt{2 + \frac{1}{\varrho_0}\sqrt{\frac{e'}{X}} + \frac{1}{4\varrho_0^2} \cdot \frac{e'}{X}}\,\right] \cdot \sqrt{\frac{e'}{X}} = r'_\pi. \end{aligned}\right\} \tag{19}$$

The space within which the electrons return and are captured may then with approximation be reckoned to be a sphere having the radius ϱ' determined by

$$\varrho' = \frac{1}{4}\left(r'_0 + 2\, r'_{\frac{\pi}{2}} + r'_\pi\right). \tag{20}$$

The electric field has thus decreased the space within which capturing occurs in the ratio

$$\nu = \left(\frac{\varrho'}{\varrho_0}\right)^3. \tag{21}$$

In table C below are given values of ν calculated for a series of velocities from $1 \cdot 10^7$ to $1 \cdot 10^8$ cm sec.$^{-1}$ assuming there is a field intensity $X = 10^3$ E. S. U. $= 3 \cdot 10^5$ Volt cm^{-1}.

Table C. Values of ν for various velocities and for $X = 3 \cdot 10^5$ volts cm^{-1}; $\sqrt{\frac{e'}{X}} = 6.9 \cdot 10^{-7}$ cm.

U_0	ϱ_0	r'_0	$r'_{\frac{\pi}{2}}$	r'_{π}	ϱ'	$\nu = \left(\frac{\varrho'}{\varrho_0}\right)^3$
	cm	cm	cm	cm	cm	
$1 \cdot 10^7$	$5.05 \cdot 10^{-6}$	$4.82 \cdot 10^{-7}$	$3.23 \cdot 10^{-7}$	$2.70 \cdot 10^{-7}$	$3.50 \cdot 10^{-7}$	0.00033
$2 \cdot 10^7$	$1.26 \cdot 10^{-6}$	$3.34 \cdot 10^{-7}$	$2.71 \cdot 10^{-7}$	$2.39 \cdot 10^{-7}$	$2.79 \cdot 10^{-7}$	0.011
$3 \cdot 10^7$	$5.6 \cdot 10^{-7}$	$2.38 \cdot 10^{-7}$	$2.14 \cdot 10^{-7}$	$1.96 \cdot 10^{-7}$	$2.15 \cdot 10^{-7}$	0.057
$4 \cdot 10^7$	$3.13 \cdot 10^{-7}$	$1.72 \cdot 10^{-7}$	$1.63 \cdot 10^{-7}$	$1.52 \cdot 10^{-7}$	$1.62 \cdot 10^{-7}$	0.137
$5 \cdot 10^7$	$2.02 \cdot 10^{-7}$	$1.31 \cdot 10^{-7}$	$1.27 \cdot 10^{-7}$	$1.24 \cdot 10^{-7}$	$1.27 \cdot 10^{-7}$	0.262
$1 \cdot 10^8$	$5.05 \cdot 10^{-8}$	$4.83 \cdot 10^{-8}$	$4.43 \cdot 10^{-8}$	$4.14 \cdot 10^{-8}$	$4.14 \cdot 10^{-8}$	0.69

For small velocities, l_1 must accordingly be many times smaller with the strong field than without such a field, while l_2 will presumably be nearly independent of the field. This relation will, however, hardly be so pronounced as appears from the above table, because the density of the electrons will presumably be the greatest near the particle. In the table we have reckoned with a uniform density of released electrons.

A H^+-particle will presumably need less energy in order to penetrate the air than a H-atom. Since the strong field increases the value of $\frac{l_1}{l_2}$ it tends to reduce the energy necessary to maintain a certain velocity for a H-particle as compared to the value arrived at by calculations based upon BLACKETT's investigations where no field was employed.

5. Relation between the Intensity of the Electric Field and the Velocity of Positive Particles.

According to table B and the formulae (8) and (9) the field intensity necessary to maintain the velocity $U_0 = 2 \cdot 10^7$ cm sec.$^{-1}$ should be

$$X = 195\,000 \left(1 + \frac{1}{0.4}\right) = 683\,000 \text{ volts cm}^{-1}. \quad (22)$$

Considering the influence of the strong field on the ability of the H^+-particle to capture electrons it will presumably be reasonable to reduce the value given for the necessary field intensity, and as a rough estimate we put it at

$$X = 300\,000 \text{ volts cm}^{-1} \text{ for } U_0 = 2 \cdot 10^7 \text{ cm sec.}^{-1}. \quad (23)$$

Although this value is only based upon a rough estimate it may presumably be of the right order of magnitude.

6. Collisions between Slow Positive Particles and neutral Molecules.

It may certainly be considered doubtful whether positive ions having a velocity less than that corresponding to 20—30 volts are able to ionize common gases at all. The ionizing ability of these slow positive ions is at all events extremely small[1]. W. J. HOOPER (l. c.) is furthermore of the opinion that even at velocities corresponding

[1] J. FRANCK und P. JORDAN: Handb. d. Phys. Bd. 23, p. 730—733, 1926. W. J. HOOPER: Phys. Rev. (II) vol. 27, p. 109, 1926.

to 925 volts, positive ions in hydrogen at low pressures (0.012 mm) produce very little or no ionization at all, while at this velocity the ionization seems to be quite considerable at higher pressures.

From experiments carried out by W. Aich[1] it appears that the cross-sectional area of hydrogen molecules, determined for movements of protons in hydrogen, is very nearly the same as the gas-kinetic cross-sectional area if the proton is considered to have infinitesimal dimensions.

Similar relations presumably exist in connection with slow protons in other gases.

From investigations with protons having a velocity corresponding to about 900 volts A. J. Dempster[2] draws the conclusion that the effective cross-sectional area of hydrogen- and helium-molecules in case of collisions with protons of this velocity is very nearly equal to zero, and the protons therefore — just as electrons — should show the Ramsauer-effect if their velocity is about $4 \cdot 10^7$ cm sec.$^{-1}$. In that case also, no alternations should occur in the charge of the proton.

It will hardly be possible to take a definite stand-point with regard to these questions at the present time.[3] There is at all events hardly any reason to assume that nitrogen, oxygen, carbonic acid, and other gases that do not show the Ramsauer-effect in connection with electrons, should do so in connection with protons, and what we have set forth above in Chapt. IV indicates, in our opinion definitely, that this will not be the case.

[1] W. Aich: Zeitschr. f. Phys. Bd. 9, p. 372—378, 1922.

[2] A. J. Dempster: Proc. Nat. Acad. Sc. Amer. vol. 11, p. 552—554, 1925; vol. 12, p. 96—98, 1926.

[3] Compare for example: E. Rüchardt in Handb. d. Phys. Bd. 24, p. 99—101, 1927 and W. Wien: Handb. d. Experimentalphysik Bd. 14, p. 527, 1927.

The conditions are different with the noble gases which we have had no opportunity to investigate in a perfectly pure state. Here our investigations are not decisive, but some observations in connection with helium indicate that the results found for this gas by DEMPSTER are correct.

7. Resume.

Even though the material of experimental results at hand is very incomplete we are, no doubt, on the safe side when we assume that:

At velocities which are smaller than that corresponding to 20—30 volts, the positive ions — including the protons — will only in an extremely small degree have an ionizing effect on those gases referred to in the investigations here discussed, while the molecules of those gases, for collisions with positive ions, show nearly the same cross-sectional area as they should have according to the kinetic gas-theory.

At velocities corresponding to more than 10 000 volts, any kind of positive ions have a strongly ionizing effect in any kind of gas.

At velocities corresponding to more than a few hundred volts all positive ions — except the protons — will have a strongly ionizing effect in any kind of gas.

At velocities corresponding to more than a few hundred volts the protons will also have a strongly ionizing effect in all gases except the noble ones and possibly hydrogen. In the noble gases — and possibly in hydrogen — the protons do not cause ionization up to velocities corresponding to about 900 volts.

APPENDIX II

1. Dispersion of Positive or Negative Charges.

We put the velocity v of the ions equal to

$$v = kE, \tag{1}$$

where E is the electric field intensity and where k may be considered to be constant within very wide limits; for example for air at N. T. P. from $E = 0.1$ to $E = 20\,000$ volts $\mathrm{cm.}^{-1}$.[1] Within very wide limits, k is inversely proportional to the air pressure p, so that we may write

$$k = \frac{k_0}{p}, \tag{1'}$$

where k_0 is a constant independent of the air pressure.

If we are not taking into account the mutual repulsion between the single particles, then the density of these will not be varied by their movement in an outer electric field. To understand this, we may just consider a space-element, the outer boundaries of which are a thin tube of lines of force representing the outer field, and two equipotential surfaces (1) and (2) having the areas f_1 and f_2 (see fig. 1). The electric field at these surfaces are respectively E_1 and E_2. We then have:

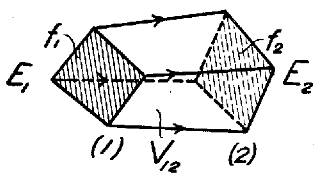


Fig. 1. Element of space having for boundaries a tube of lines of force as shown, and the two equipotential surfaces having the areas f_1 and f_2.

[1] See L. B. LOEB: "Kinetic Theory of Gases", p. 440, 1927.

$$f_1 E_1 = f_2 E_2. \tag{2}$$

During the time dt the volume of the element of charge at the area (1) will, owing to displacement of the charge be reduced by $f_1 v_1 dt = f_1 k E_1 dt$, and at the area (2) be increased by $f_2 v_2 dt = f_2 k E_2 dt$, but since according to equation (2) $f_1 v_1 = f_2 v_2$ the volume of the element of charge will remain unaltered. Variations in the volume — and consequently also in the density — can therefore be due only to the repulsive forces within the element of charge in question, and we shall therefore proceed to discuss the effect of this repulsion.

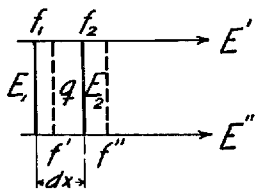


Fig. 2. E' and E'' denotes lines of force; f_1 and f_2 equipotential surfaces.

The two equipotential surfaces of which f_1 and f_2 are sections, are separated by the infinitesimal distance dx, and for the areas f_1 and f_2, circumscribed by the tube of lines of force, we therefore have that $f_1 = f_2$. During the time dt, f_1 covers the volume $f_1 v_1 dt$ while f_2 covers the volume $f_2 v_2 dt = f_1 v_2 dt$.

The element of space is consequently increased from $f_1 \cdot dx$ to $\left(f_1 \ (dx + \frac{v_2 - v_1}{dx} \cdot dx\, dt\right)$.

The relative increase in volume is consequently

$$\frac{dV}{V} = \frac{v_2 - v_1}{dx} \cdot dt, \tag{3}$$

where V is the volume of the space-element.

If the electric density at the point in question is q, then we have

$$\frac{v_2 - v_1}{dx} = k \frac{E_2 - E_1}{dx} = 4 \pi q k. \tag{4}$$

We consequently get

$$\frac{1}{V}\cdot\frac{dV}{dt} = 4\pi q k, \tag{I}$$

or

$$\frac{dV}{dt} = 4\pi k Q, \tag{II}$$

where $Q = q \cdot V$ is the total charge of the space-element V.

Since $Vdq + qdV = 0$ then (I) may be written

$$-\frac{dq}{dt} = 4\pi k q^2. \tag{III}$$

From this equation we see that if the density was originally constant, $q = q_0$, then the density will always have the same value all over but this value itself will decrease at the rate

$$\frac{1}{q} - \frac{1}{q_0} = 4\pi k t. \tag{IV}$$

For an infinitely long cylinder, no movement occurs in the direction of the axis of the cylinder and consequently in (II) we can replace V by the cross-sectional area A of the cylinder, if at the same time instead of Q we insert the quantity of charge per cm length of the cylinder, Q_1.

$$\frac{dA}{dt} = 4\pi k Q_1. \tag{II'}$$

If we put $A = \pi R^2$ then equation (II′) will be

$$R\frac{dR}{dt} = 2 k Q_1 \tag{II''}$$

or $R^2 - R_0^2 = 4 k Q_1 t$, where R_0 is the radius for $t = 0$.

If mn (fig. 3) is a plane surface on which the charge q is distributed, and if the ordinates to the curve $abcd$ are equal to the corresponding values of q, then the electric strength acting out along the surface will be smaller at the point d than at the point c' at which latter point the charge-curve is assumed to have a point of inflection. If we draw a curve $d'c$ symmetrical to cd with regard to $c'c$ then the outwardly directed field in c' will originate

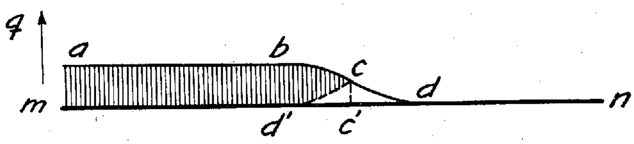


Fig. 3. The spreading out of a charge q along a plane surface mn.

from the shaded part of the front of the charge. The electric strength will therefore have very nearly its highest value at the point of inflection c'. The charge at c' will consequently move outward with greater velocity than will the charge outside this point. The steepness of the outermost front will consequently gradually increase as the charge is spreading out.

Near the centre of a large, plane charge — see fig. 4 — we shall have

$$D - D_0 = 4\pi k q_1 t. \tag{V}$$

At the edge, the charge will disperse with a somewhat smaller velocity, so that after a while its outer boundary will have the form shown by the dotted line, and then it will gradually approach a spherical form.

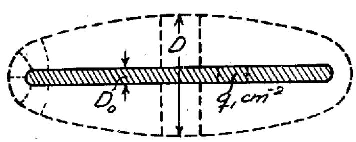


Fig 4. The spreading out of a plane electric charge.

In order to form an idea of the tendency of an originally flat front of an electric charge to assume a steeper form we will treat a couple of geometrically simple cases where the calculations offer no difficulty. The first one is a spherically distributed charge in which the density q

depends only on the time and on the radius r from a fixed centre. In the second case — a charge having cylindrical form — q depends only on the distance r from the straight line representing the axis of the cylinder.

According to (III) the charge-density q at a distance r decreases during the time dt from q to $q - 4\pi k q^2 dt$ while the charge-density $\left(q + \frac{dq}{dr} \cdot dr\right)$ at a distance $r + dr$ during the same time interval decreases to $(q+dq) - 4\pi k (q+dq)^2 \cdot dt$.

The difference Δq between the charge-densities, (which originally was $\Delta q_{(t=0)} = dq$), is after the time dt.

$$\Delta q_{(t=dt)} = dq\,(1 - 8\pi k q\, dt). \tag{5}$$

In case of the spherical charge we have according to (1)

$$v = k \cdot \frac{Q_r}{r^2}, \tag{6}$$

where Q_r is the total charge within the sphere of radius r.

During the time dt the distance $\Delta r_{(t=0)} = dr$ between the two charge-particles will change to

$$\Delta r_{(t=dt)} = dr + \frac{dv}{dr} \cdot dt = dr\left(1 + 2k\,\frac{2\pi q r^3 - Q_r}{r^3} \cdot dt\right). \tag{7}$$

From (5) and (7) we get

$$\left(\frac{\Delta q}{\Delta r}\right)_{(t=dt)} = \frac{dq}{dr} \cdot \left[1 - 2k\left(6\pi q - \frac{Q_r}{r^3}\right) dt\right]. \tag{8}$$

In the preceding the products $(k \cdot Q_r)$ and $(k \cdot q)$ are always positive (compare equation (1)), and from equation (8) it therefore appears that the steepness of the front will remain unaltered for

$$6\pi q = \frac{Q_r}{r^3} = \frac{4}{3}\pi \cdot q_0 \quad \text{or for} \quad q = \frac{2}{9} \cdot q_0, \tag{9}$$

where q_0 is the mean density of the charge within the sphere of radius r.

From (8) and (9) appears that the steepness of the charge-curve increases for $q < \frac{2}{9} \cdot q_0$ and decreases for $q > \frac{2}{9} \cdot q_0$.

For the cylindrical charge distribution we get, in a corresponding manner,

$$\left(\frac{\varDelta q}{\varDelta r}\right)_{(t=dt)} = \frac{dq}{dr} \cdot \left[1 - 2k\left(6\pi q - \frac{Q_r}{r^3}\right) \cdot dt\right], \qquad (10)$$

so that the steepness of the charge-curve increases when

$$6\pi q < \frac{Q_r}{r^2} = \pi q_0 \quad \text{or} \quad q < \frac{1}{6} q_0, \qquad (11)$$

while in the opposite case it decreases.

These calculations confirm the fact that the moving front of an electric charge will have a tendency to increase in steepness.

We shall further give some examples where the results from the foregoing may be applied.

Examples:

(1) A homogeneous sphere has a radius R and the total charge Q. We have

$$\frac{dR}{dt} = k \cdot E = k \frac{Q}{R^2} \qquad (12)$$

and

$$\frac{dV}{dt} = \frac{d\left(\frac{4\pi}{3} R^3\right)}{dt} = 4\pi k Q,$$

which agrees with equation (II).

(2) For a homogeneous circular infinitely long cylinder having the radius R and the charge Q_1 per cm of length we have

$$\frac{dR}{dt} = k \cdot \frac{2\,Q_1}{R}$$

and

$$\frac{dV}{dt} = \frac{d(\pi R^2)}{dt} = 4\,\pi\,k\,Q_1. \qquad (13)$$

2. Dispersion of Positive and Negative Charges.

If there are ions of both signs present, and if the charge-densities are respectively q_+ and q_- then the resultant density will be $q = q_+ - q_-$.

In the preceding, we have assumed that only ions of the one sign were present, but the results found are of course also valid with approximation in the case when ions of both signs are present, provided that those of the one sign are extremely few compared to those of the op posite sign.

Finally, if q_+ and q_- have such great values that the ionized area may be considered to have infinitely good conductivity, then the surplus of charge Q will collect on the surface of the area, and a further extension of the area can then be determined by means of simple electrostatic considerations.

A sphere or a cylinder of such a conductivity will thus expand in accordance with the formulae (5) and (6) above.

Also in this case, the outer boundary of the charge will have a tendency to become sharp, since stray ions found outside the charge, but being of the same sign as this, will move more slowly than the surface of the charge.

APPENDIX III

1. The Shape of the Positive and Negative Spreaders at the Meeting Point.

There is still a single phenomenon — appearing quite peculiar at first sight — which we will just touch upon, although it does not directly belong to the regular Lichtenberg figures the theory of which has been our sole object in the preceding. Our purpose is to prevent that some readers should find the phenomenon in question to be contradictory to the above evolved theory of the formation of the positive Lichtenberg figures.

If, in the arrangement outlined in fig. 23, the conditions are so selected that the positive and the negative spreaders only just reach each other, very peculiar features are often observed at the place where the two figures meet. An example is shown in L. F. I, fig. 63 and enlarged reproductions of others are shown on plate 24. Of these latter, we will first discuss the upper meeting point in part II.

From the end of the positive spreader, a strongly luminous thread projects over to the edge of the negative figure, where it ends in a strongly luminous spot at the outer end of a negative spreader. This luminous thread has the same characteristics as the subsequent negative discharges in — or along — positive spreaders. (See for example plates 12 VIII, 18 I and 20 I). A closer inspection of the examples reproduced in plate 24 shows that those negative spreaders which have come into contact with the positive ones are more luminous than those that have no connection. There can be no doubt that in the cases considered negative electricity flows over to the end of the positive spreaders. So far the phenomenon is clear enough.

A number of peculiar features may, however, be observed on this link between the positive and the negative

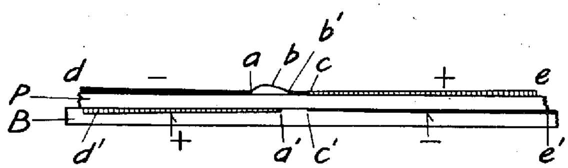


Fig. 1.

spreaders. All of the four junctions on plate 24 show such a strongly luminous spot at the end of the respective negative spreaders, but in all of the four cases the luminosity is comparatively faint in the immediate vicinity of the spot and this is so on both the positive and the negative side. Something similar is apparent on the other figures shown in plate 24.

In order to explain the faintness in luminosity on the positive side of the spot we may, by referring to fig. 1, propose the following. The regular negative figure *da* has just reached the point *a* at the same moment when the regular positive figure has reached the point *c*, and would normally have stopped here but for the negative figure which exerts an attraction on it. Under this influence, the positive spreader proceeds to b'. The electric field extending from the negative charge is here still stronger than at *c* and the positive spreader will proceed further, but the field has here a slightly different direction because the induced positive charge $d'a'$ on the metal plate B at a' projects a little beyond the edge a' of the negative discharge. This positive charge at a' changes the direction of the electric field at the tip of the positive spreader from the usual downward to a somewhat upward direction at the point b. A little further forward near the point a the electric field from the negative spreaders will predominate and the positive spreader will "strike down" in the end-point a of the negative spreader.

In the case where the positive and negative figures are placed nearer together, so that near the meeting line both of the figures are still moving on at a considerable velocity, the positive spreaders will be held down against the photographic plate all the way until where they join with the negative ones. Examples of this are shown plate 24 I. The positive spreaders are in this case of no greater luminosity than usual although the corresponding negative spreaders show that a flow of negative electricity has taken place in the direction of the positive electrode, but this circumstance is quite analogous to what is mentioned in chap. III 4 (a) (see also fig. 20).

This terminates our discussion of the relations shown by positive figures where they join with negative ones.

2. Distribution of the Photographic Intensity at the Meeting Point.

In this connection also we observe certain peculiarities, as for instance the formation of luminous wings, stretching out from the point of junction. A satisfactory and thorough explanation of this light distribution can hardly be given at present, although the main features may be explained without difficulty.

The luminous effect is in the main due to three different causes. It is partly due to the comparatively strong light emitted from the luminous thread *abb′c* (fig. 1) and especially from the part *abb′* which is raised a little above the photographic film; and partly to the light emitted by the recombination of the positive and negative ions which are present immediately at the surface of the film. The most active factor here is presumably the recombination of electrons with positive ions, while the recombination between positive and negative atomic or molecular ions seems to occur without the emission of any great amount of actinic light.

We shall next discuss some particulars concerning the distribution of the light. In those cases where the positive and negative spreaders just get into contact with each other, a strongly luminous junction-point is visible in the outer faint part of the negative spreader; see for example the upper point of junction in part II and the two middle ones in part III plate 24. This feature is less pronounced in the cases where the positive and negative eleetrodes are placed nearer to each other; see for example the lower points of junction in II and III and all of the points of junction in IV plate 24.

In the first mentioned cases the formation of the negative figure is completed before the positive figure reaches over to it. The junction will then occur at the edge of that area outside the strongly luminous negative figure to which the charge has further moved after the photographic figure was in reality completed, and at a moment when the electrons have already combined with oxygen-, water- or other molecules to form molecular ions. In this case the point of junction will be located outside the strongly luminous part of the negative spreaders.

If, on the contrary, the electrodes are placed so near each other that the positive spreaders reach the negative ones before the formation of the latter is completed, then the point of junction will be located in that part of the negative figure where ionization by collision occurs and where light is consequently produced.

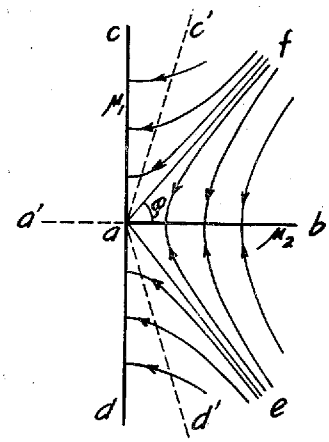


Fig. 2. Charge- and field-distribution after the joining of a positive spreader *ba* with a negative spreader *a'a*.

At the moment the positive spreader has joined with the negative one at *a*, electrons will start out along the positively charged spreader where they will cause ionization by collision, which is followed by recombinations which will produce the strongly luminous tracks visible in the figures, while the charge of these tracks will simultaneously change from positive to negative.

The charge- and the field-distribution thus obtained are outlined in fig. 2. The line *cad* marks the boundary of the negative charge at the moment when the positive spreader reaches the point *a*. The track *ab* will then, as mentioned, rapidly become negatively charged, whereupon we shall have the field-distribution shown.

We may with approximation assume the field from the negative figure to be produced by a charged line *cad* having the charge density $-\mu_1$, since the influence of that part of the charge which is located to the left of *cad* is mainly compensated by the influence of the induced positive charge on the plate B below (see fig. 1). We will further assume the charge density along *ab* to be equal to μ_2. For the sake of simplicity we will further assume that the charge along *ba* projects to the left from *a* as indicated by the dotted line *aa'*.

Under these assumptions the lines of force will take the form of hyperbolae with the axis *ab*, *ac* and *ad* while the asymptotes *af* and *ae* with *ab* form the angle φ determined by

$$tg\ \varphi = \sqrt{\frac{\mu_2}{\mu_1}}.$$

The negative charge at great distances from *a* will move on with practically unaltered velocity, while its spreading out will be completely stopped at *a*. The boundary line of the negative charge will consequently acquire a bend at *a* and after a while it will follow the dotted line *c'ad'*. If this line is reached by the negative charge at the moment when the light from the luminous track *abb'* (fig. 1) is releasing a considerable number of electrons by photoelectric effect, then a strong ionization by collision will occur along the line *c'ad'* since here, according to appendix 2, the charge-density changes quite suddenly and therefore a strong field will exist. The subsequent recombination will then produce the luminous "wings".

CONTENTS

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CHAPTER II

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The Properties of the Positive Figures.

CHAPTER IV
The Formation of the Positive Figures.

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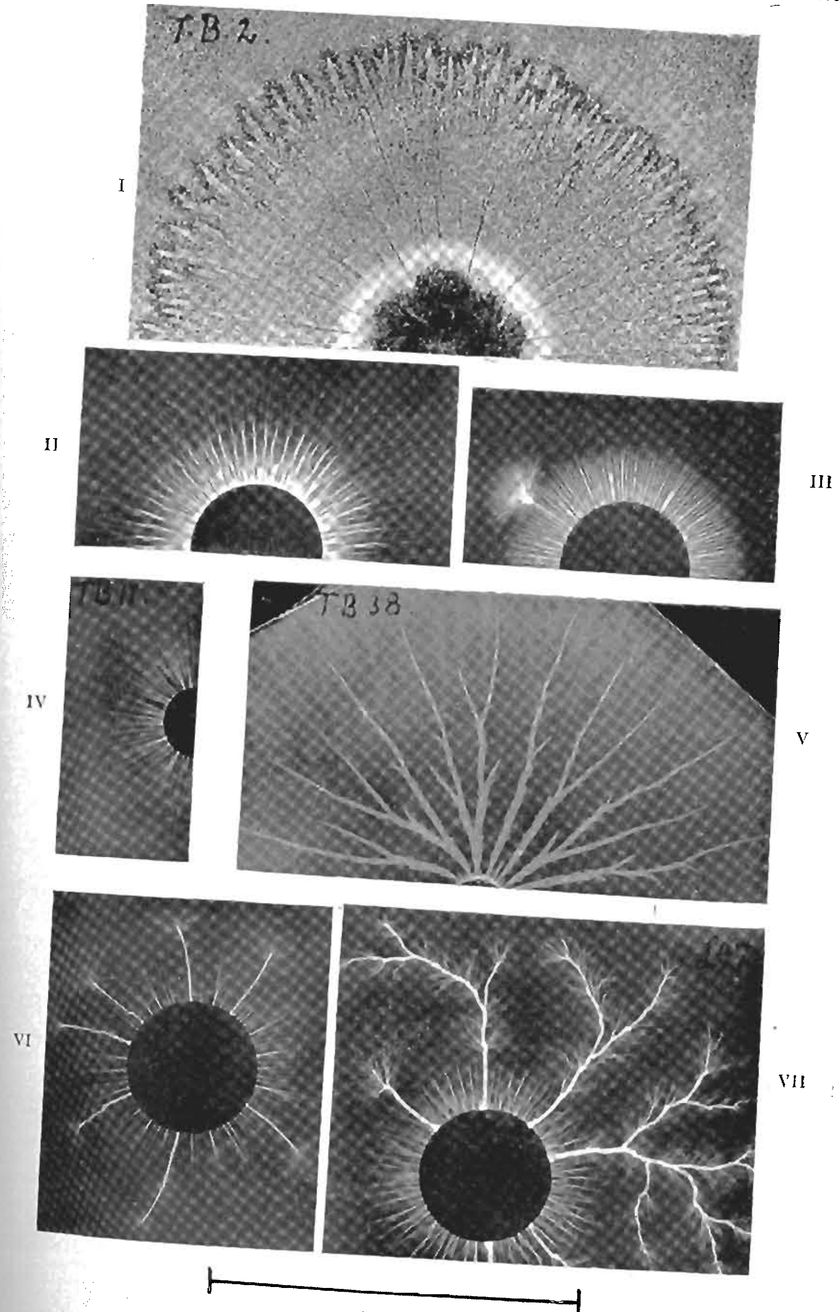
On Positive Particles.

APPENDIX II

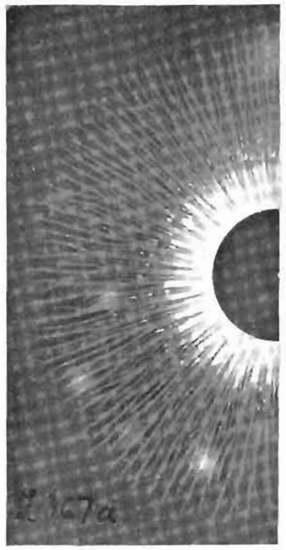
APPENDIX III

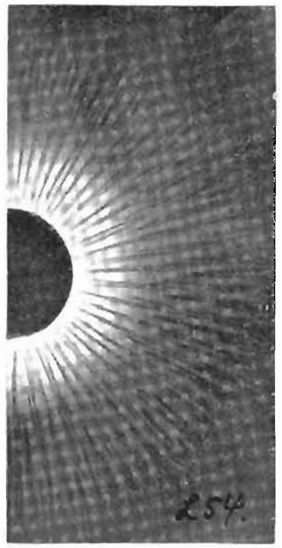
Forelagt paa Mødet den 20. Januar 1928.
Færdig fra Trykkeriet den 28. Maj 1929.



I. O_2; $l = (+)\,3$ mm; $p = 350$ mm Hg. Dust Figure.
II. O_2 (+ Air); $l = (+)\,5$ mm; $p = 750$ mm Hg.
III. O_2 (+ Air); $l = (-)\,5$ mm; $p = 750$,, ,,
IV. O_2; $l = (-)\,3$ mm; $p = 350$,, ,,
V. He (+ Air); $l = (+)\,2$ mm; $p = 600$,, ,,
VI. O_2; $l = (-)\,5$ mm; $p = 750$,, ,,
VII. O_2; $l = (+)\,5$ mm; $p = 750$,, ,,

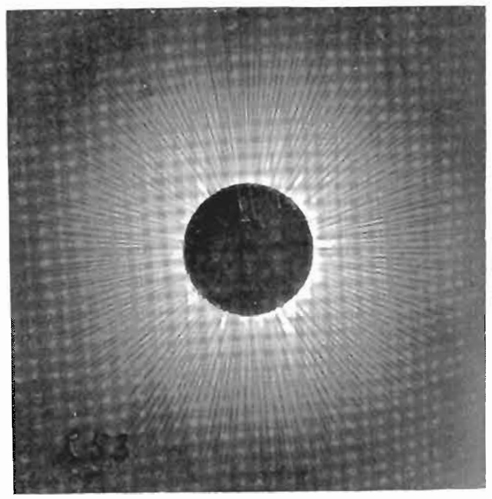
I





II

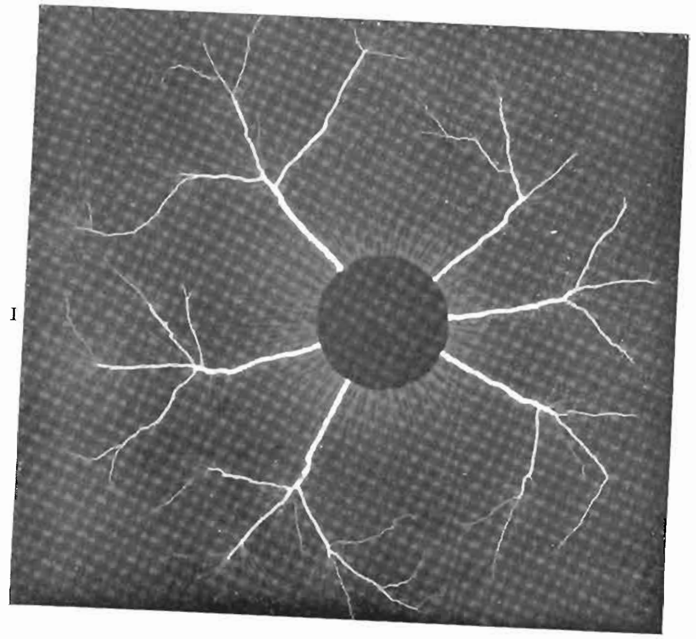
50 mm



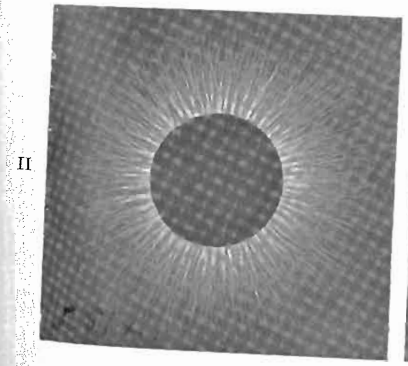
III

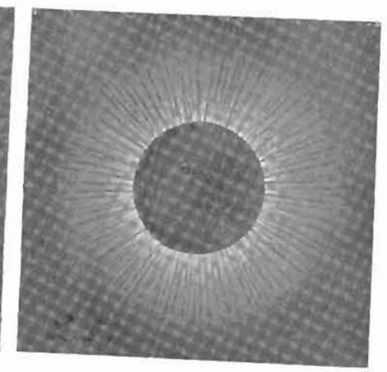
I. H_2 (impure); $l = (+) 5$ mm; $p = 750$ mm *Hg*.
II. H_2; $l = (+) 5$ mm; $p = 750$ mm *Hg*.
III. H_2; $l = (-) 5$ mm; $p = 750$ mm *Hg*.

I



II





III

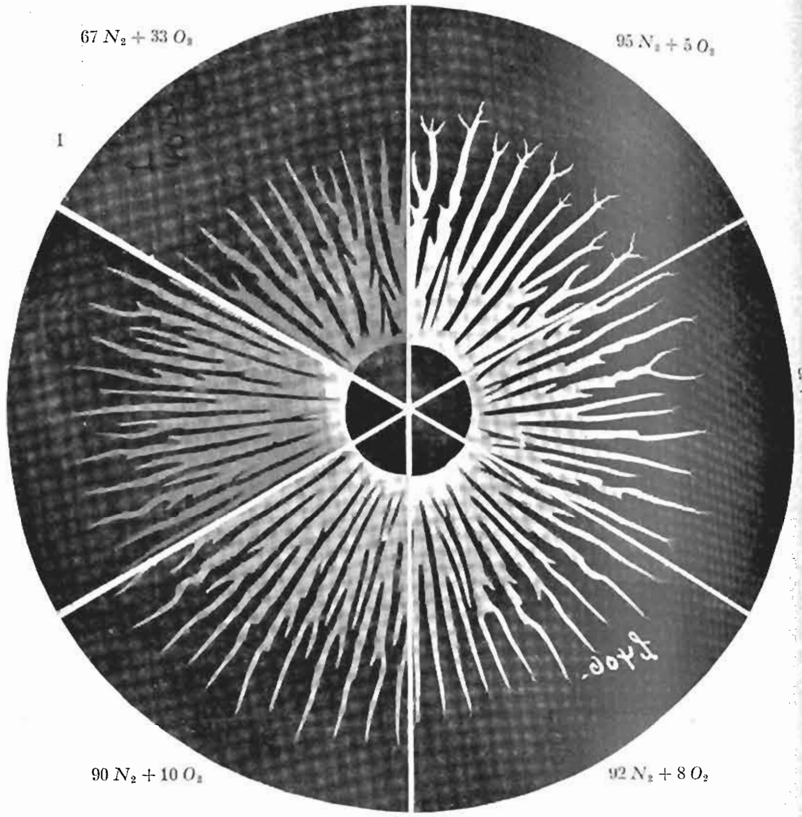
100 mm

I. CO_2; $l = (+) 7{,}5$ mm; $p = 760$ mm *Hg*.
II. CO_2; $l = (+) 6$ „ $p = 760$ „ „
III. CO_2; $l = (-) 6$ „ $p = 760$ „ „

Plate 4.

$l = 0.5$ mm; $p = 150$ mm Hg.

I



$67\,N_2 + 33\,O_2$

$95\,N_2 + 5\,O_2$

$85\,N_2 + 15\,O_2$

$93\,N_2 + 7\,O_2$

$90\,N_2 + 10\,O_2$

$92\,N_2 + 8\,O_2$

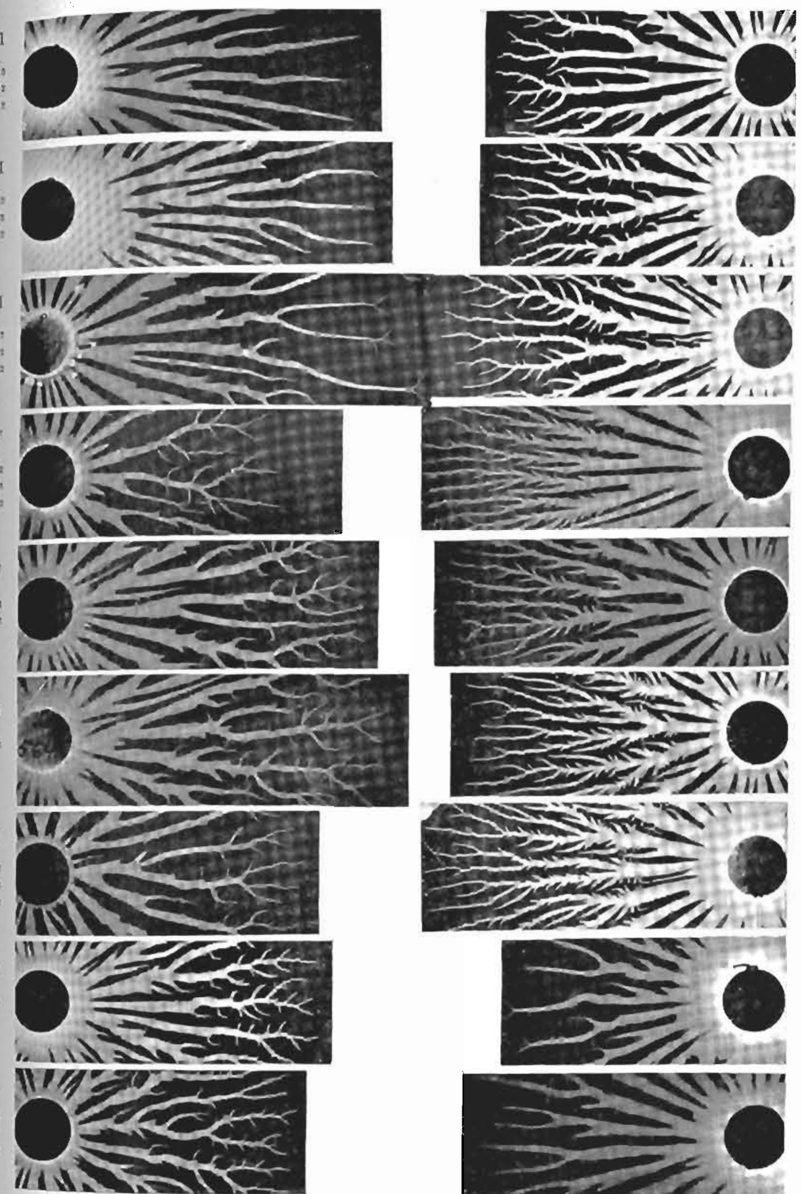
II

Argon + Air; $l = 0.5$ mm
$p = 150$ mm Hg.

50 mm

Plate 5.

$l = 0.5$ mm; $p = 150$ mm Hg.



I $78\,N_2 + 20\,O_2 + 2\,H_2$

II $79\,N_2 + 20\,O_2 + 1\,H_2$

III $89\,N_2 + 10\,O_2 + 1\,H_2$

IV $97.5\,N_2 + 1.5\,O_2 + 1\,H_2$

V $99\,N_2 + 1\,O_2$

VI $100\,N_2$

VII $100\,N_2 + ?\,O_2 + ?\,H_2$

VIII $100\,N_2$

IX $100\,N_2 + ?\,O_2 + ?\,H_2$

X $100\,N_2 + ?\,H_2$

XI $99\,N_2 + 1\,H_2$

XII $98\,N_2 + 2\,H_2$

XIII $50\,N_2 + 50\,H_2$

XIV $95\,N_2 + 5\,H_2$

XV $90\,N_2 + 10\,H_2$

XVI $80\,N_2 + 20\,H_2$

XVII 80 Air $+ 20\,H_2$

XVIII $45\,N_2 + 45\,O_2 + 10\,H_2$

50 mm

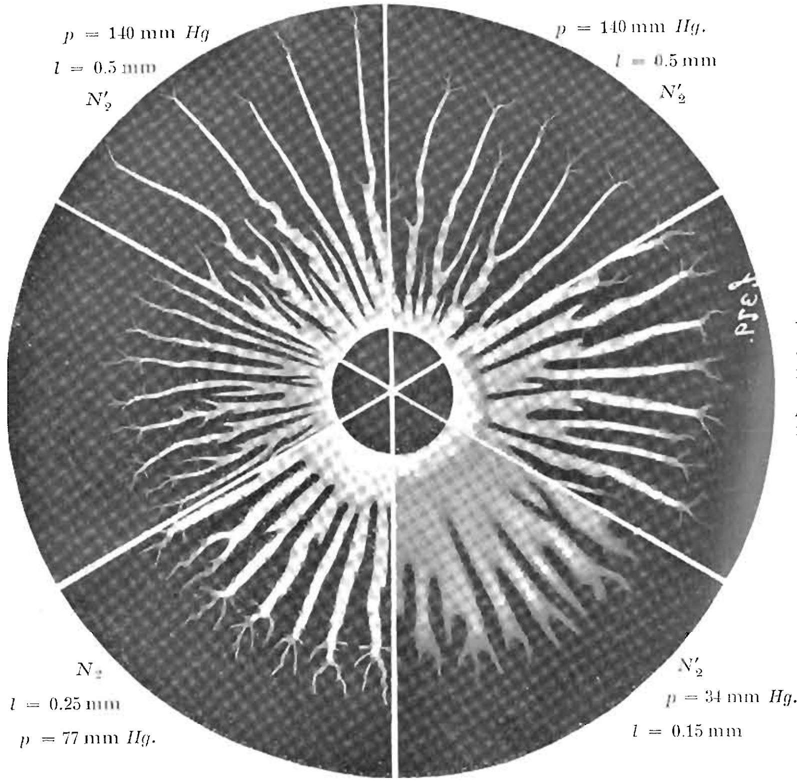
Plate 6.

I

$p = 140$ mm Hg
$l = 0.5$ mm
N_2'

$p = 140$ mm Hg.
$l = 0.5$ mm
N_2'

N_2
$l = 0.4$ mm
$p = 157$ mm Hg.

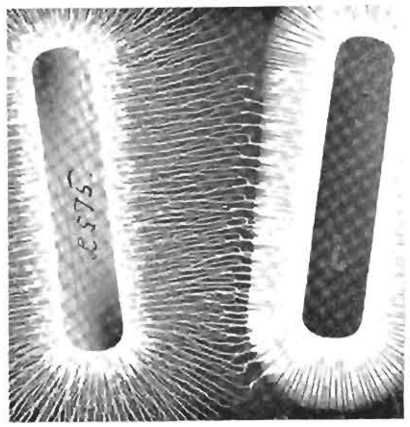


N_2'
$l = 0.25$ mm
$p = 75$ mm Hg

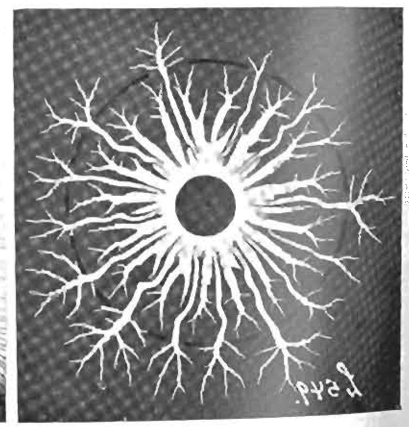
N_2
$l = 0.25$ mm
$p = 77$ mm Hg.

N_2'
$p = 34$ mm Hg.
$l = 0.15$ mm

II



Air; $l = 8$ mm; $p = 760$ mm Hg.



III

N_2; $l = 0.5$ mm; $p = 150$ mm Hg.

50 mm

(For diagram of connections see Fig. 23 in text.)

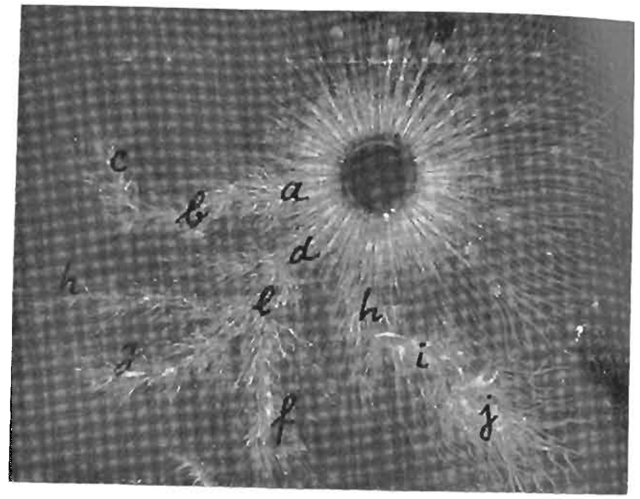
(Black circle indicates the approximate range of the initial figure.)

Plate 7.

I

m n

m n the upper limit of the mica plate



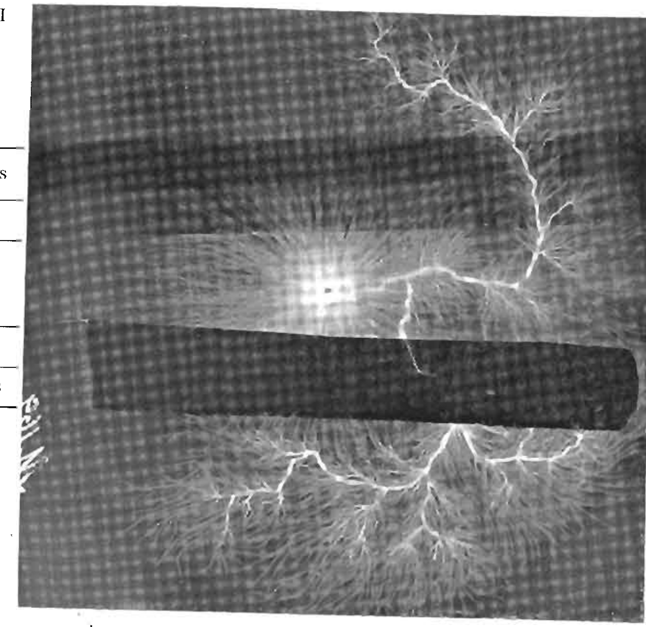
Mica plate 0.05 mm thick coated with a layer of red ink (dry)

$p = 760$ mm Hg.

II

V. P. 2 layers
1 layer

B. P. 1 layer
2 layers



2 Violet
1 Paper

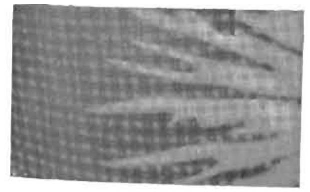
1 Black
2 Paper

$l = 7.5$ mm

50 mm

III

P.



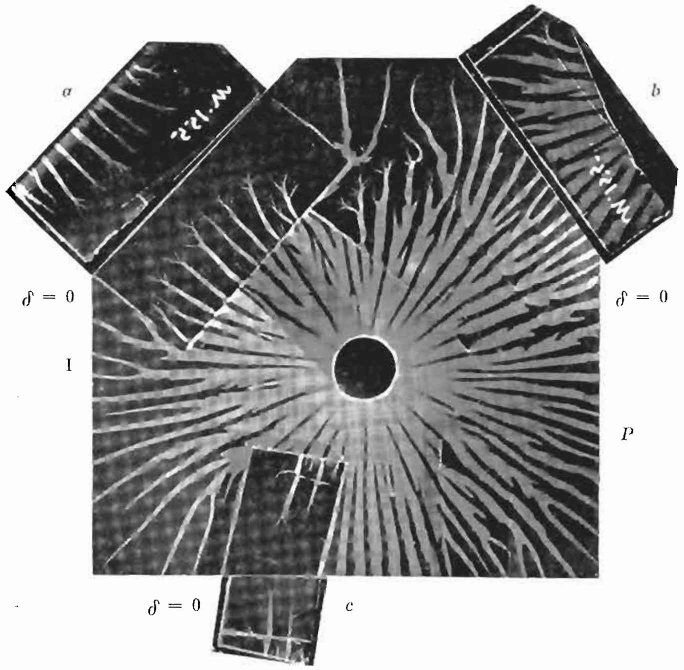
P'

$p = 70$ mm Hg; $l = 0.75$ mm; $\delta = 0.25$ mm (see Fig. 4).

a b

$\delta = 0$ $\delta = 0$

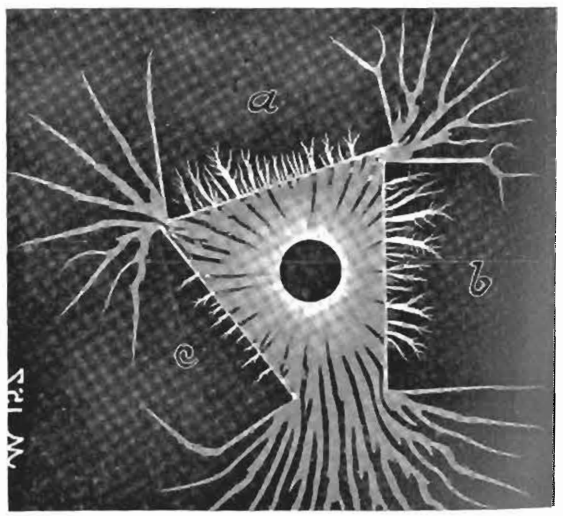
I



P

$\delta = 0$ c

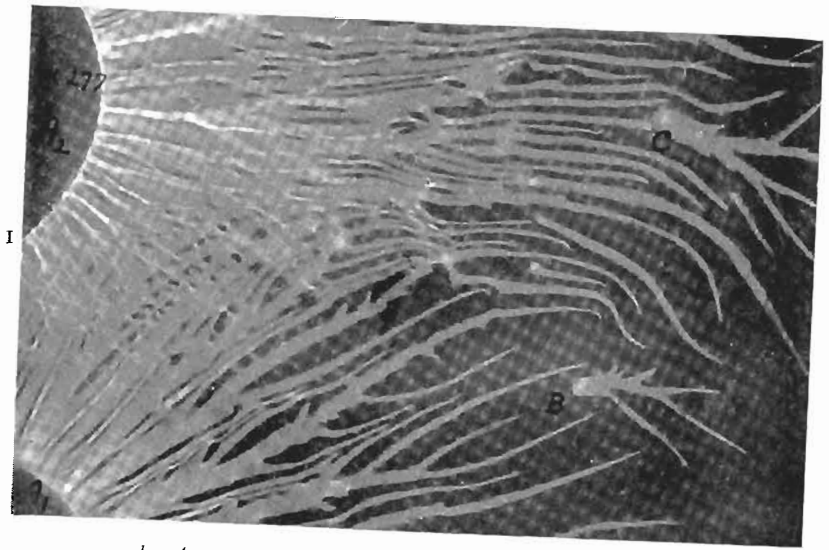
II $\delta = 0$



$l = 1$ mm; $p = 150$ mm Hg; Air.

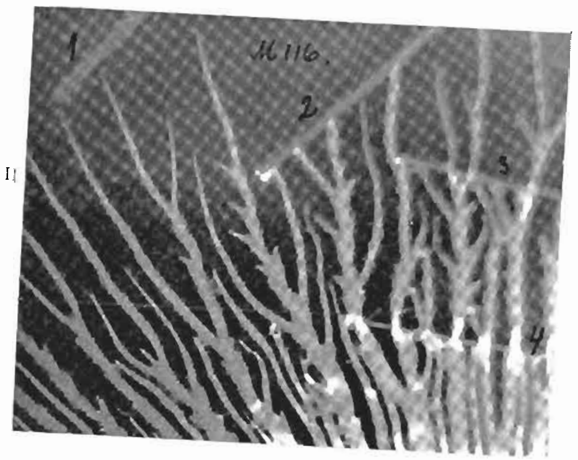
50 mm

I



$l = 4$ mm; $p = 760$ mm Hg; $L_2 - L_1 = 5$ m; $d = 1.4$ mm.

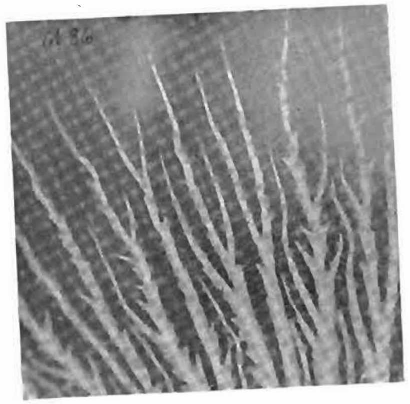
II



$l = 0.75$ mm; $p = 100$ mm Hg; Pencil Marks.

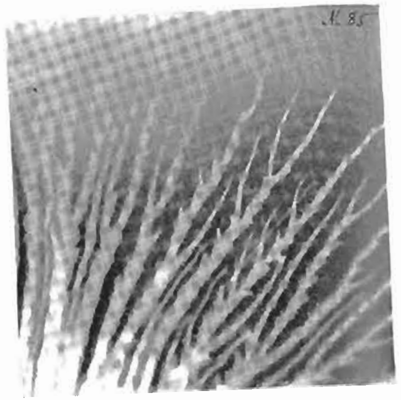
10 mm

I



l = 6 mm; p = 760 mm Hg.
Radium 2 mm from photografic Film.

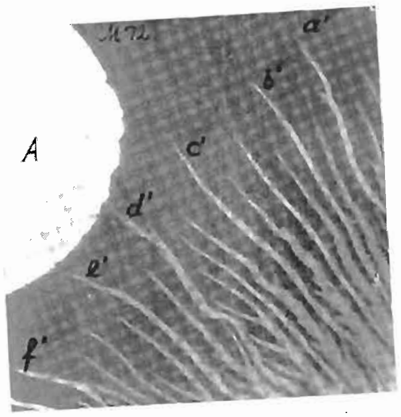
II



l = 6 mm; p = 760 mm Hg.
Radium 1 mm from photographic Film.

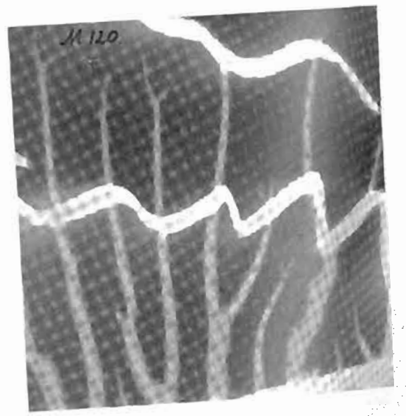
10 mm

III



l = 2 mm; p = 760 mm Hg.
Part A exposed to the light from a spark immediately before the Lichtenberg Discharge occurs.

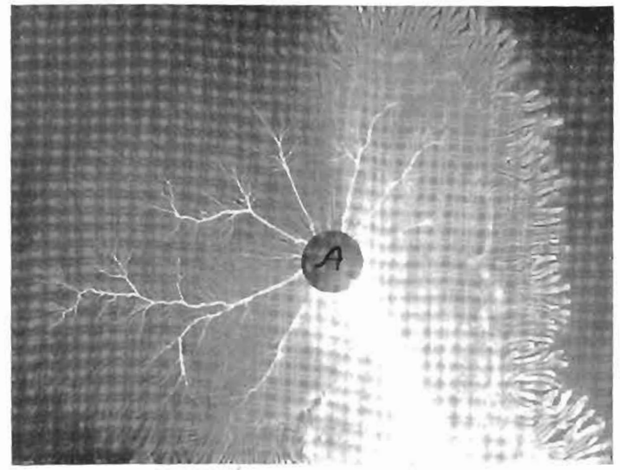
IV



l = 1 mm; N_2 (+ 7 per cent Air).
Ink Marks; p = 300 mm Hg.

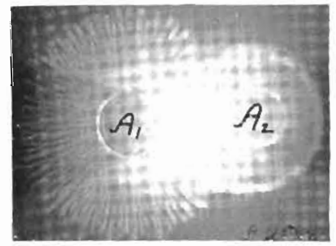
Air; l = 7 mm; p = 760 mm Hg.

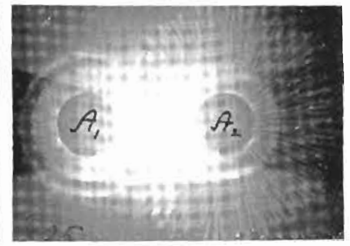
I



Air
l = 5 mm

II

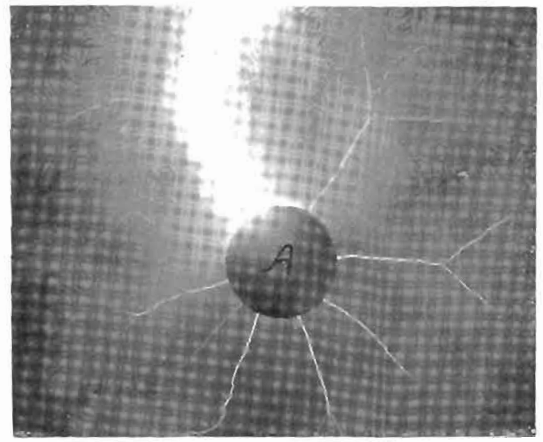




Air
l = 5 mm

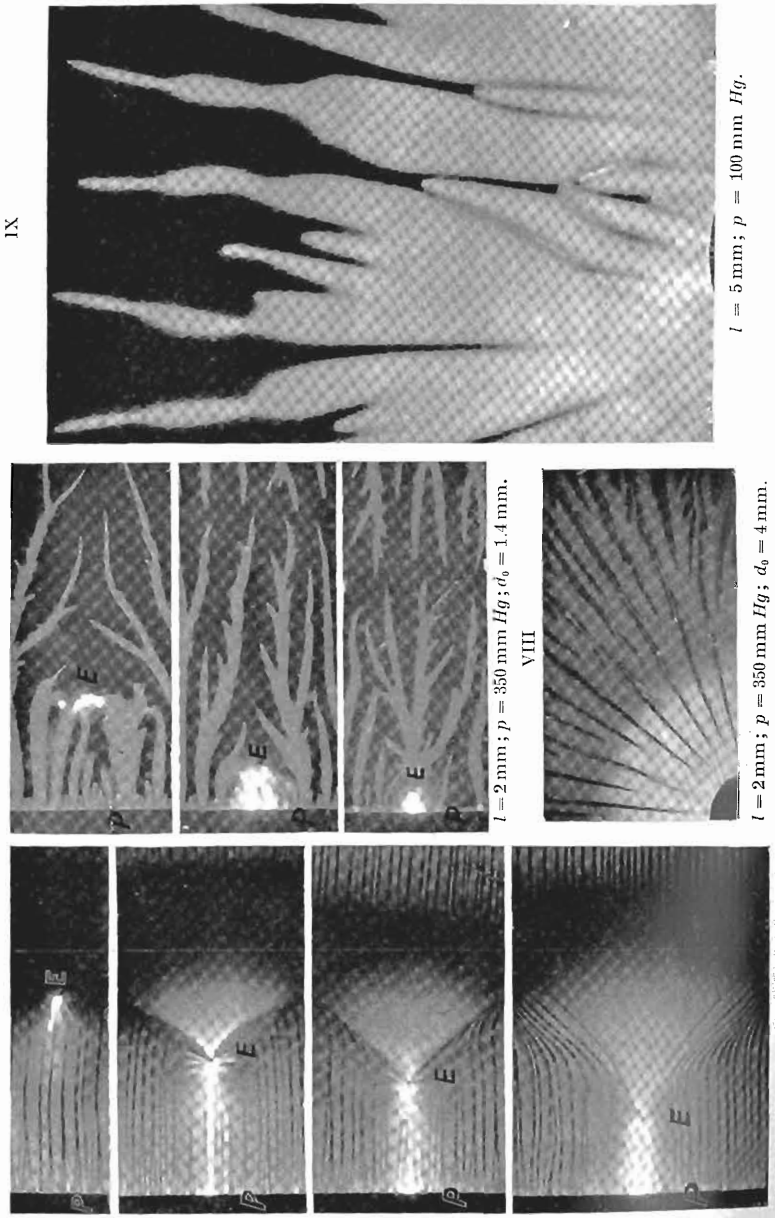
III

IV



O_2; l = 6 mm; p = 750 mm Hg.

50 mm



IX. $l = 5$ mm; $p = 100$ mm Hg.

V–VII. $l = 2$ mm; $p = 350$ mm Hg; $d_0 = 1.4$ mm.

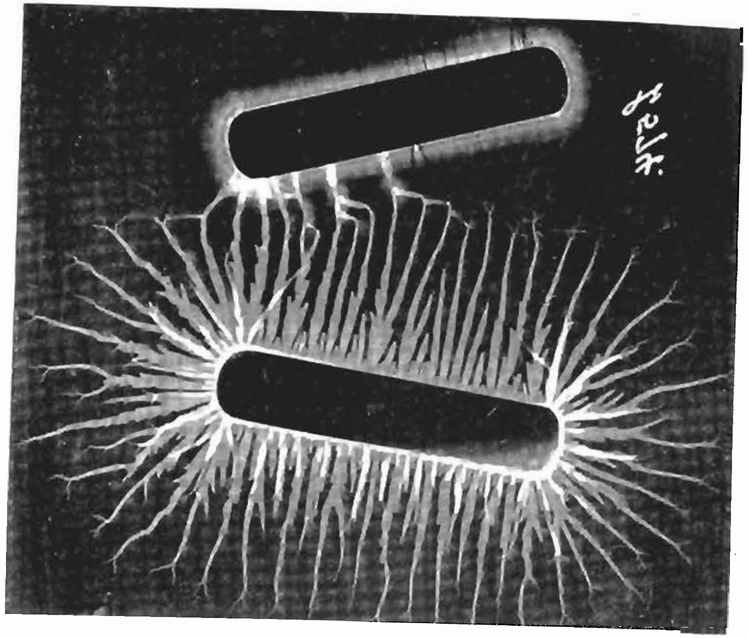
VIII. $l = 2$ mm; $p = 350$ mm Hg; $d_0 = 4$ mm.

20 mm

I II III IV

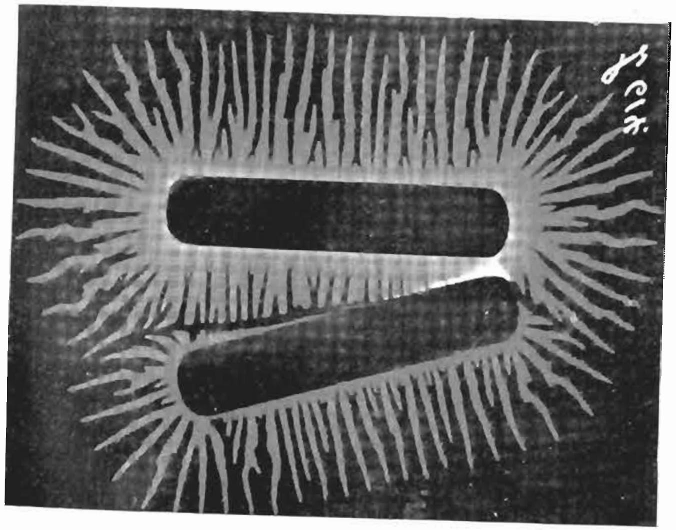
For parts I–VII see Fig. 19 in the text.

I



N'_2. $l = 1.0$ mm. $p = 225$ mm Hg.

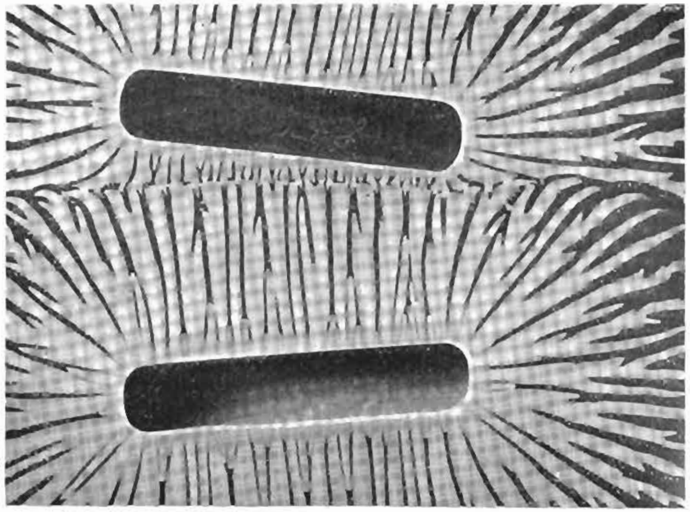
II



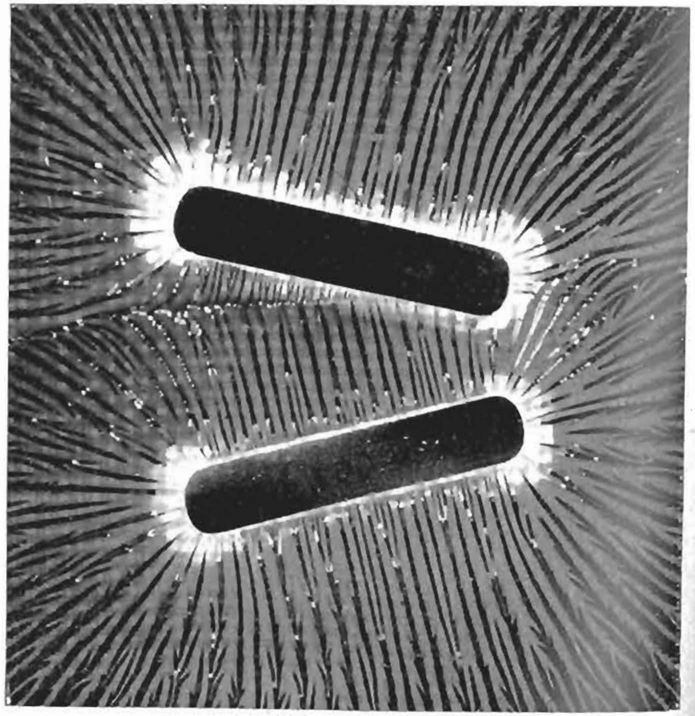
Air; $l = 0.3$ mm; $p = 150$ mm Hg; $L_0 = 6$ m.

50 mm

I



II

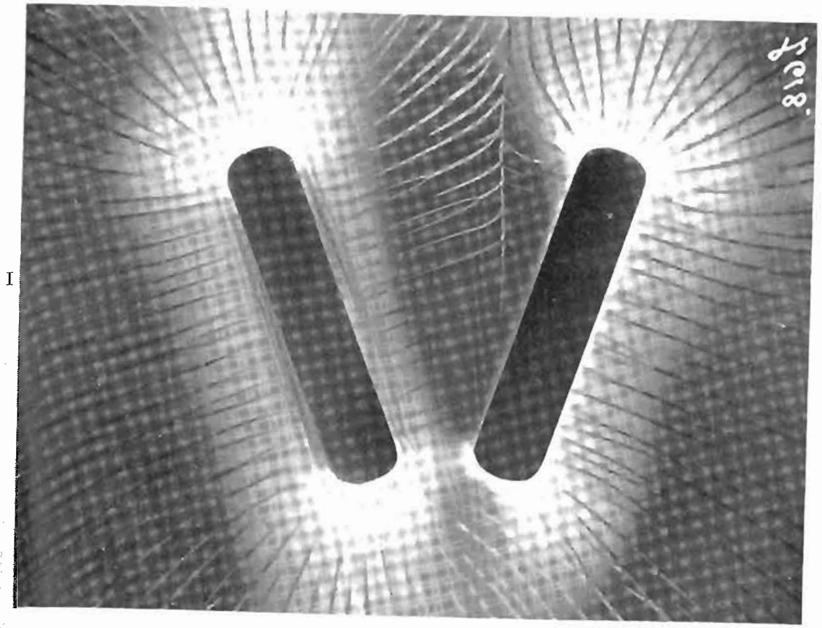


50 mm

I. l = 2.5 mm; p = 300 mm Hg; L_0 = 6 m.
II. l = 0.5 ,, p = 150 ,, ,, L_0 = 14 ,,
For diagram of connections see Fig. 24 in the text.

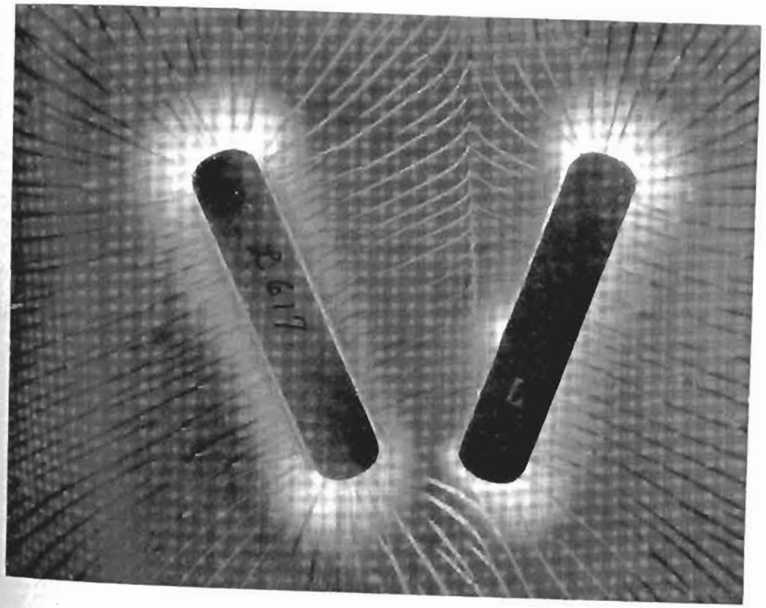
Air. p = 150 mm Hg. L_0 = 6 m.

I



l = 4 mm.

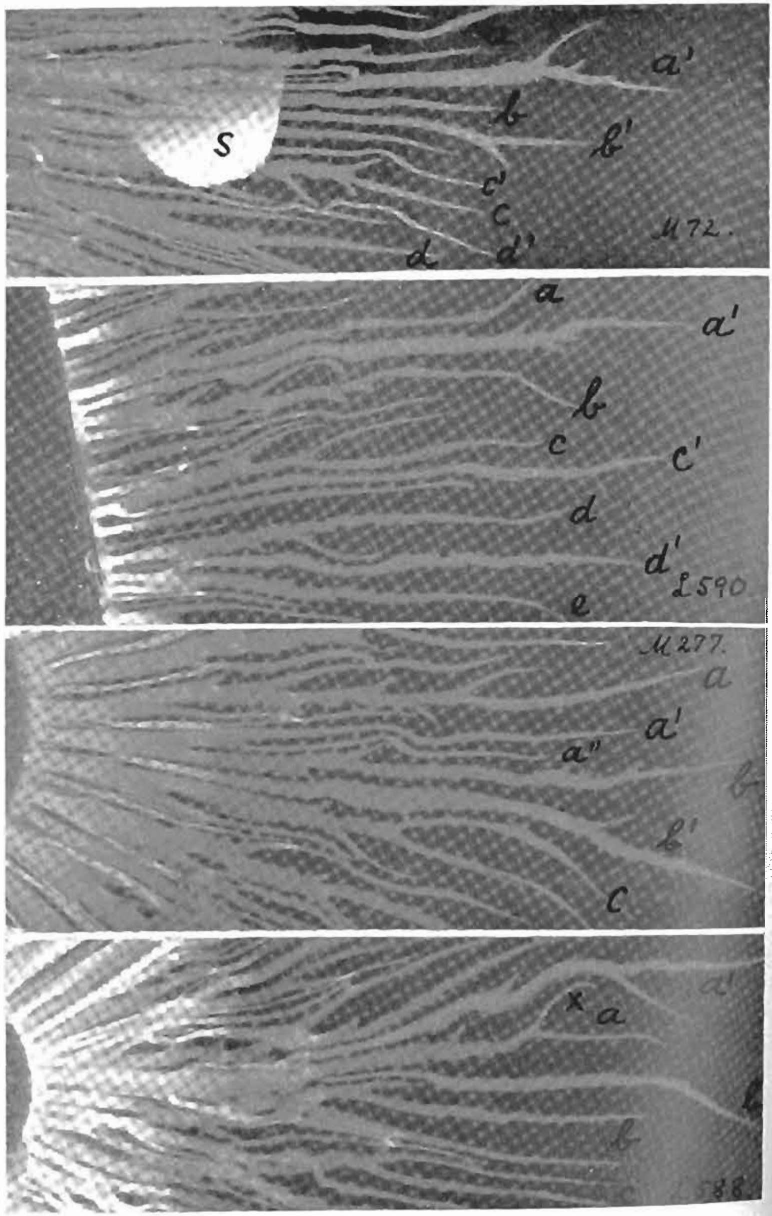
II



l = 2 mm.

50 mm

For diagram of connections see Fig. 24 in the text.



I — $l = 2$ mm.

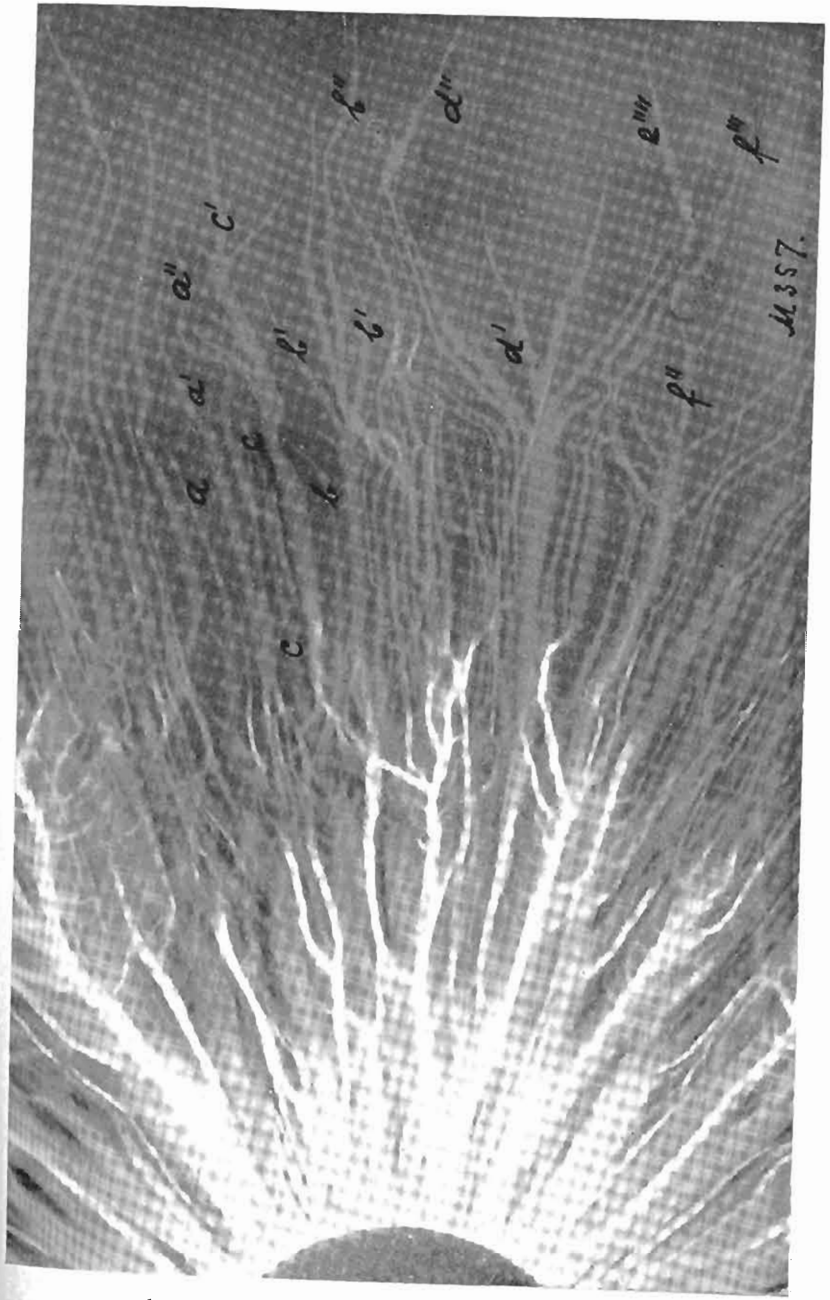
II — $l = 2{,}5$ mm. $L = 14$ m.

III — $l = 4$ mm. $L = 5$ m.

IV — $l = 4$ mm. $L = 14$ m.

$p = 760$ mm Hg. $d_0 = 1{,}4$ mm.

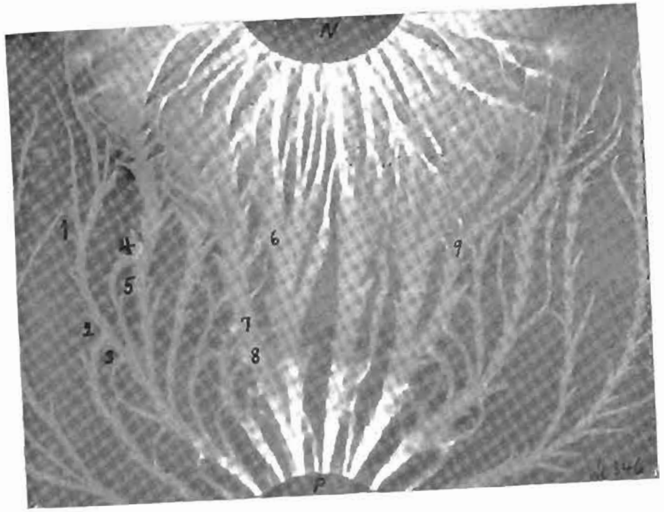
10 mm.



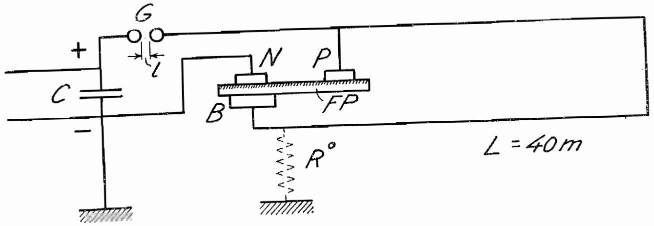
$l = 2{,}5$ mm; $p = 760$ mm Hg; $L = 29$ m; $d_0 = 1{,}4$ mm

10 mm

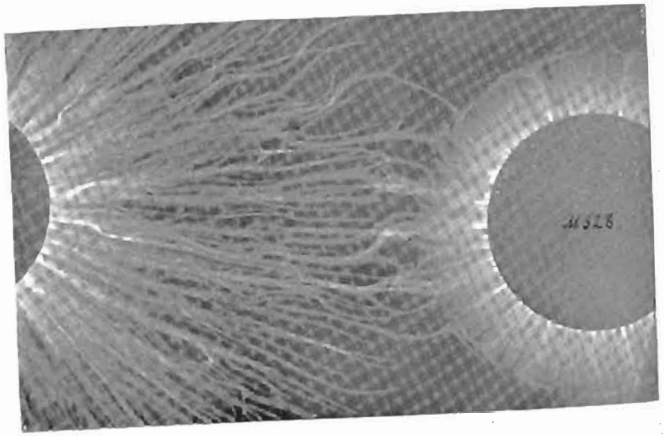
I



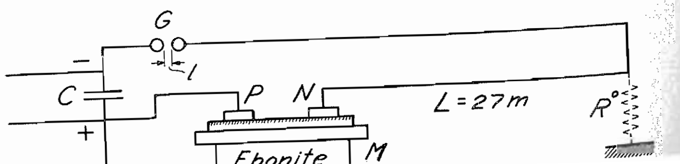
$l = 7\,mm$ $p = 760\,mm\,Hg$ 10 mm



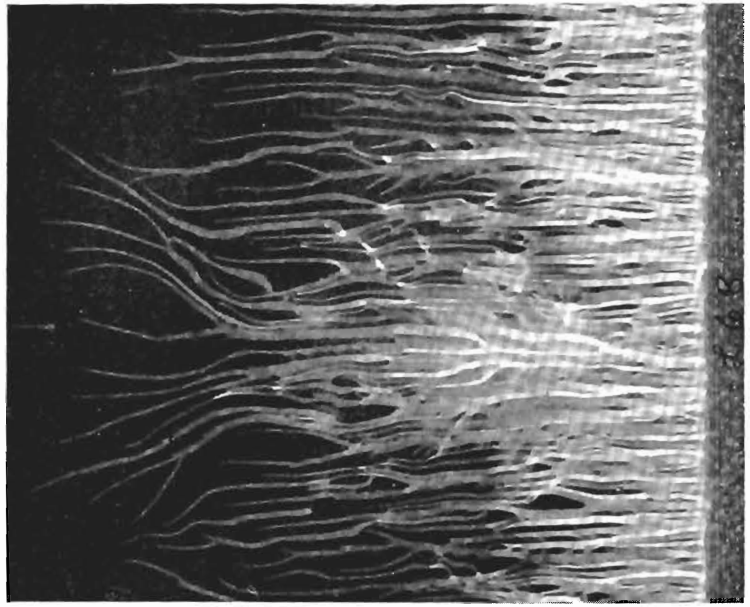
II



$l = 4\,mm$ $p = 760\,mm\,Hg$ 10 mm

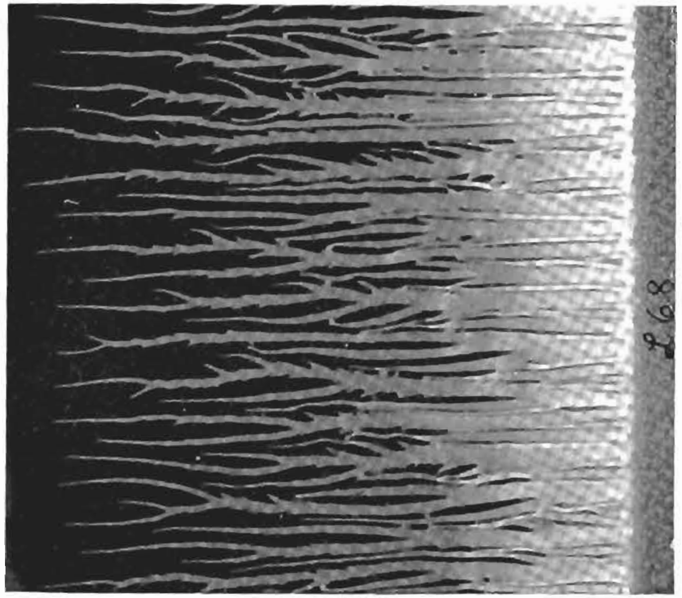


II



$L_0 = 15$ m.

Air. $l = 6$ mm. $p = 760$ mm Hg.

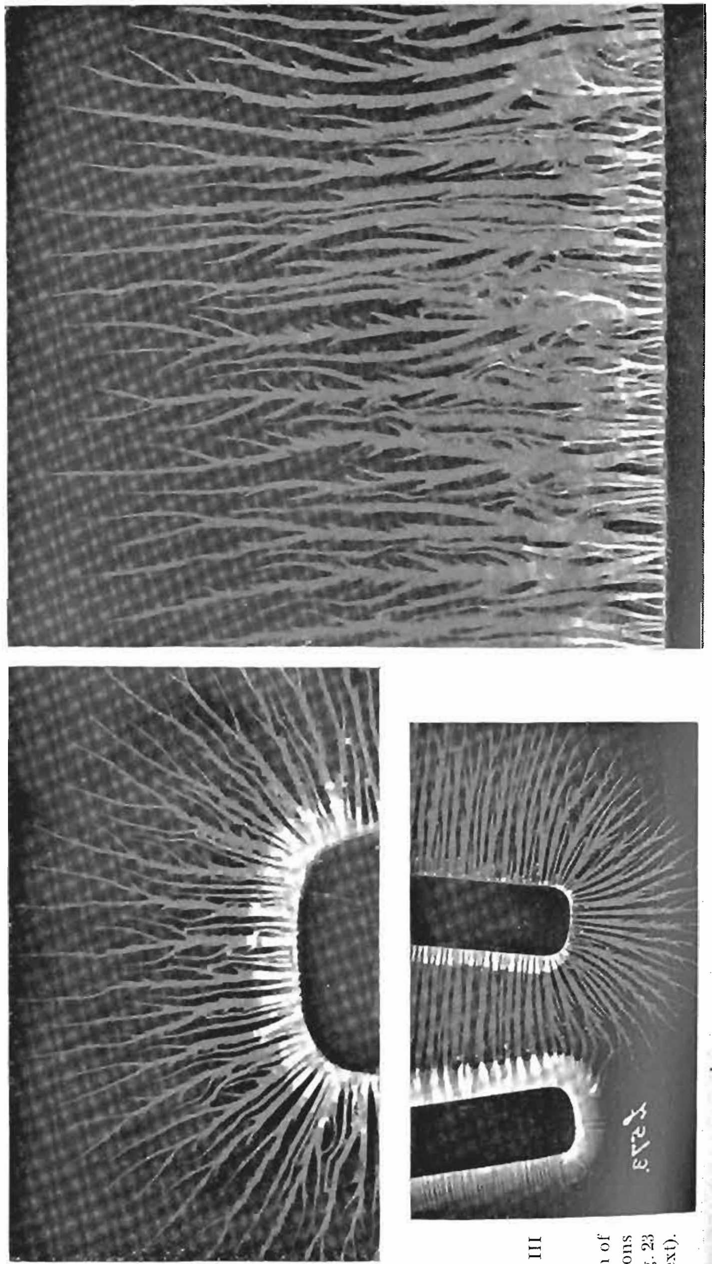


$L_0 = 2$ m.

50 mm

I

Plate 20.



II

$l = 5$ mm; $L_0 = 23$ m; $p = 760$ mm *Hg*.

I

$l = 3$ mm; $L_0 = 2$ m; $p = 760$ mm *Hg*.

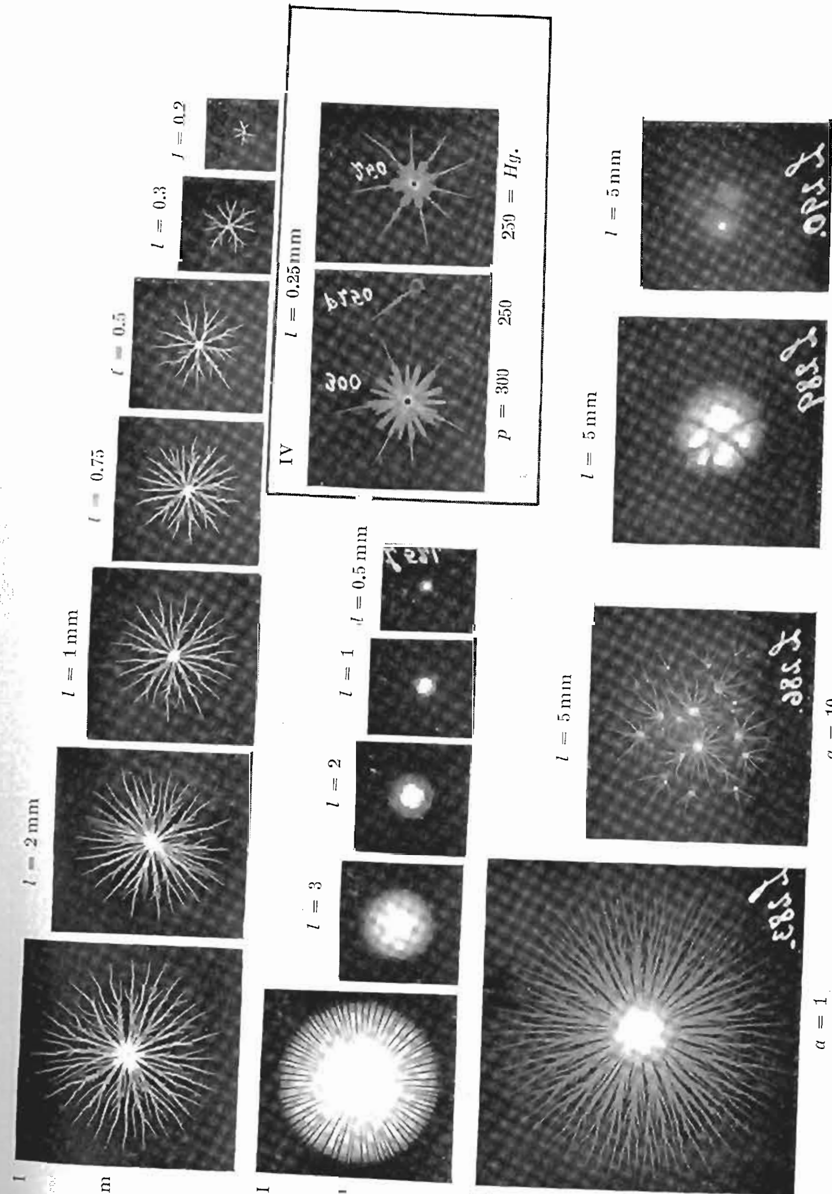
III

$l = 3$ mm; $p = 400$ mm *Hg*.

(Diagram of connections see Fig. 23 in Text).

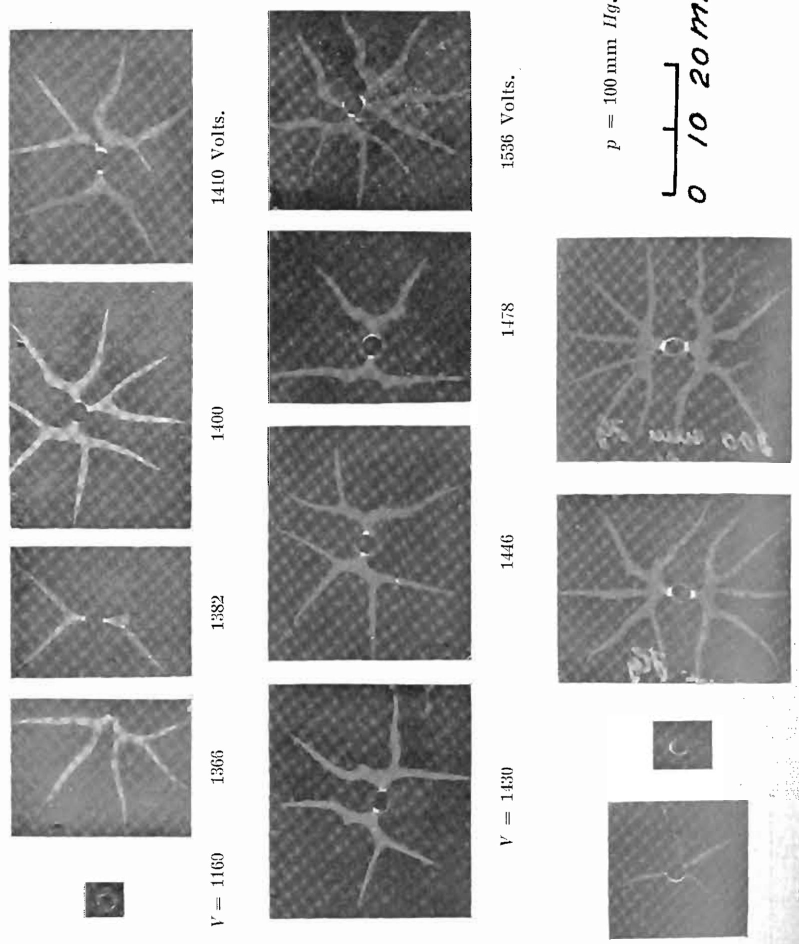
Air.

Plate 21.



100 mm

$p = 760$ mm *Hg*.



V = 1169, 1366, 1382, 1400, 1410 Volts.

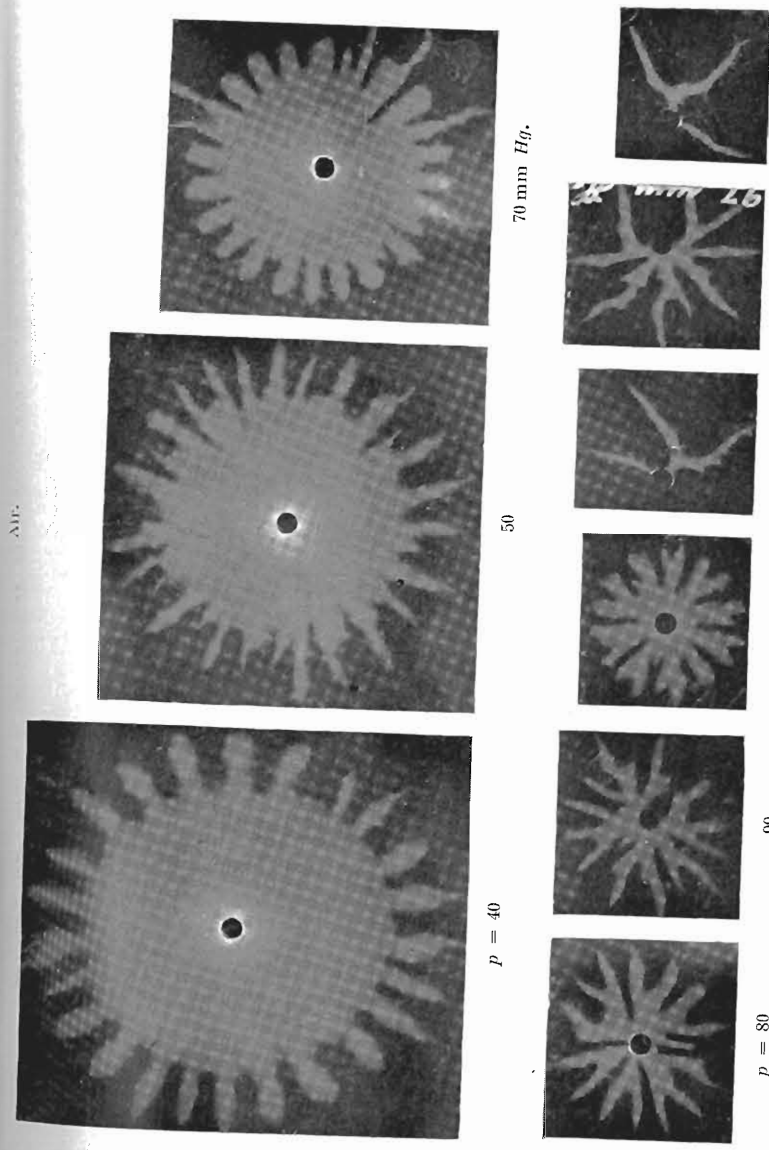
V = 1430, 1446, 1478, 1536 Volts.

V = 1541, 1556, 1581, 1593 Volts.

p = 100 mm Hg.

0 10 20 mm

Air.



p = 40, 50, 70 mm Hg.

p = 80, 90, 91, 95, 97, 99 mm Hg.

p = 100, 101 mm Hg.

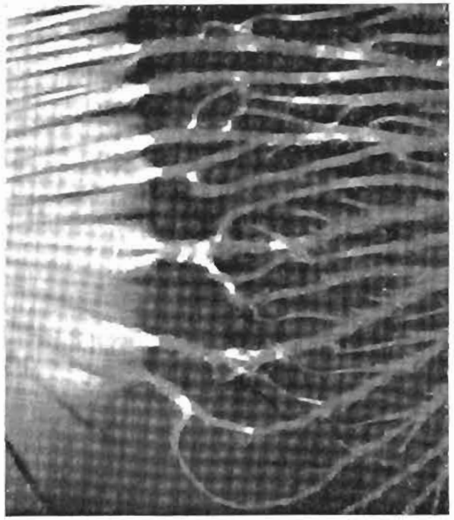
V = 1190 Volts.

0 10 20 30 40 mm

Air.

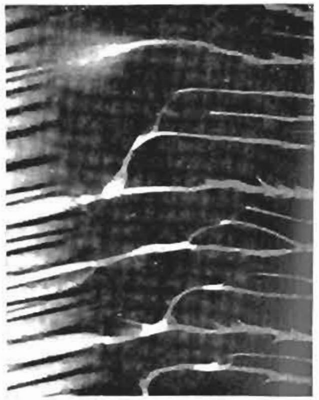
Air; p = 760 mm Hg.

I



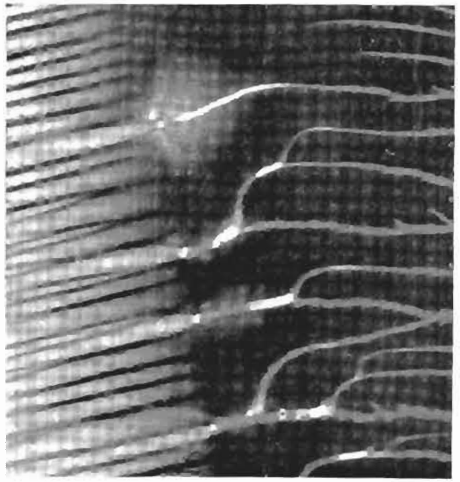
l = 9 mm

III



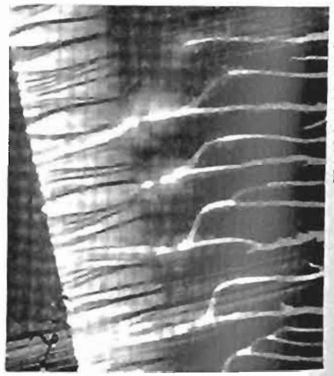
l = 8.5 mm

II



l = 9 mm

IV



l = 5 mm

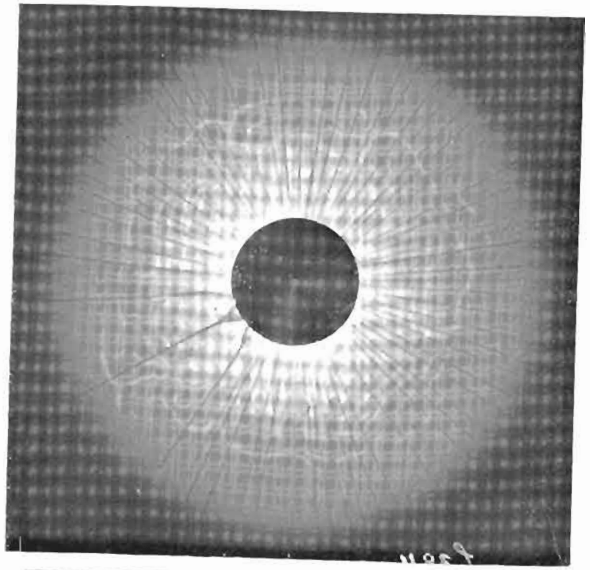
10 mm

(For Diagram of connections see Fig. 23 in the Text).

I

N'_2

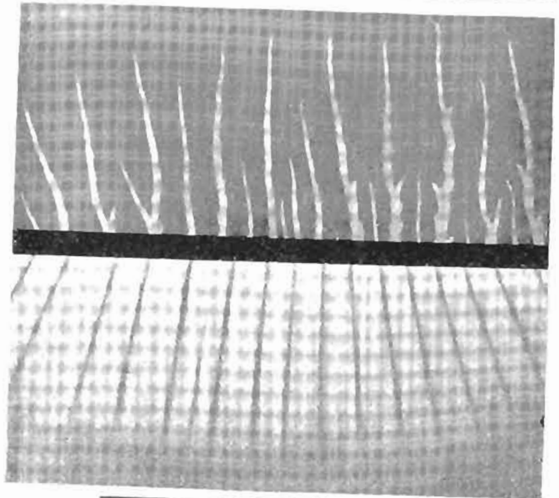
p = 150 mm



II

Air.

p = 760 mm

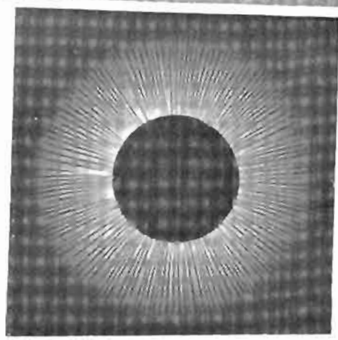


10 mm

III

N'_2

p = 760 mm



Parts I and III

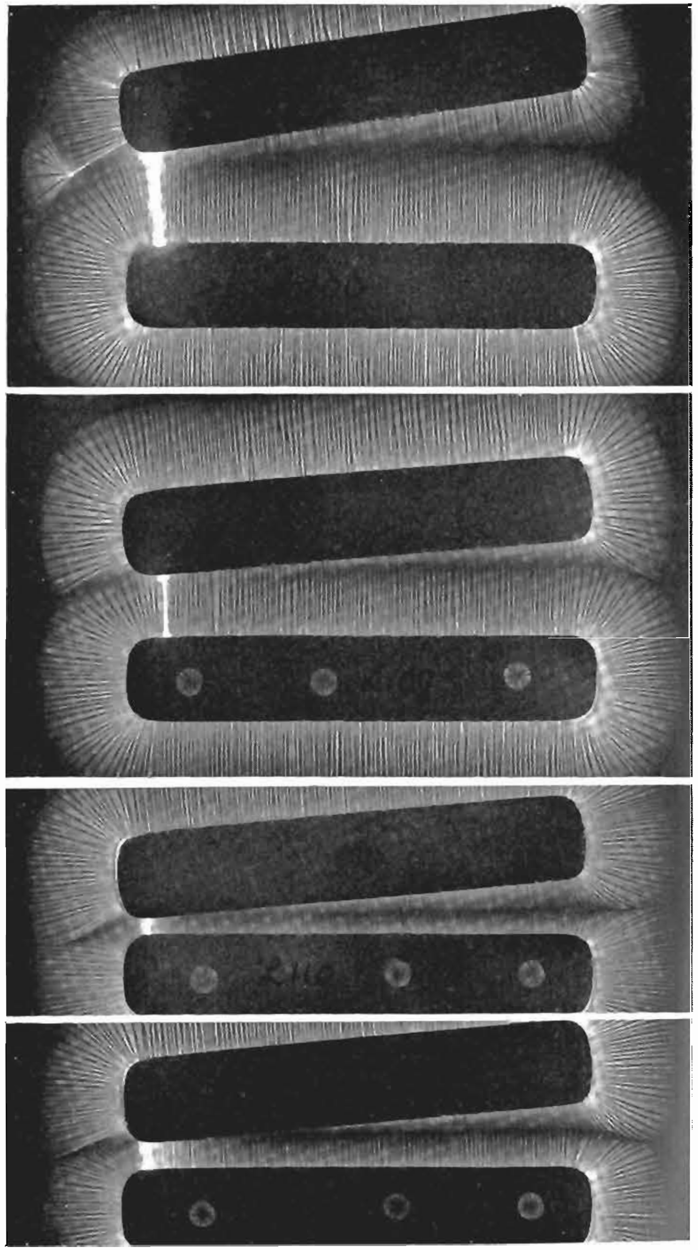
50 mm

I

II

III

IV



Air; p = 760 mm Hg; l = 6 mm.

50 mm

(For Diagram of connections see Fig. 1 in *L. F.* II and in Ann. d. Phys. Bd. 69, p. 207).

I

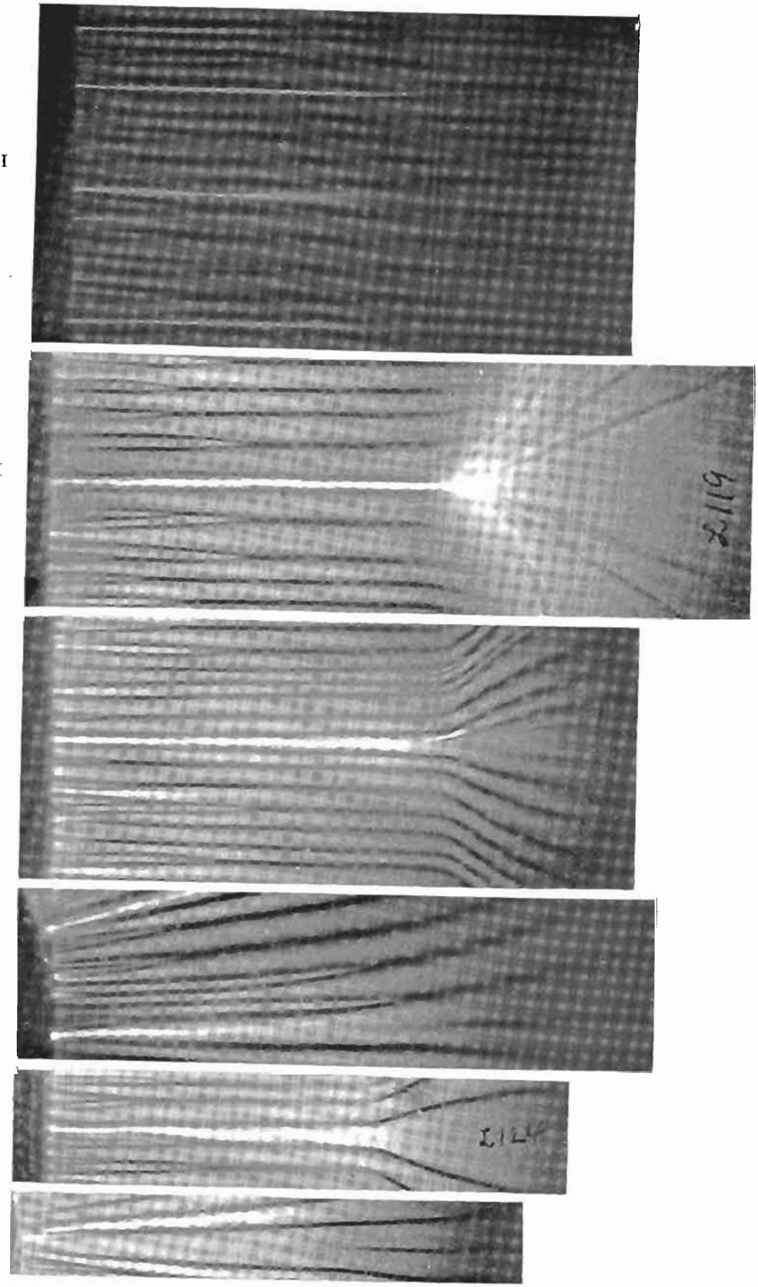
II

III

IV

V

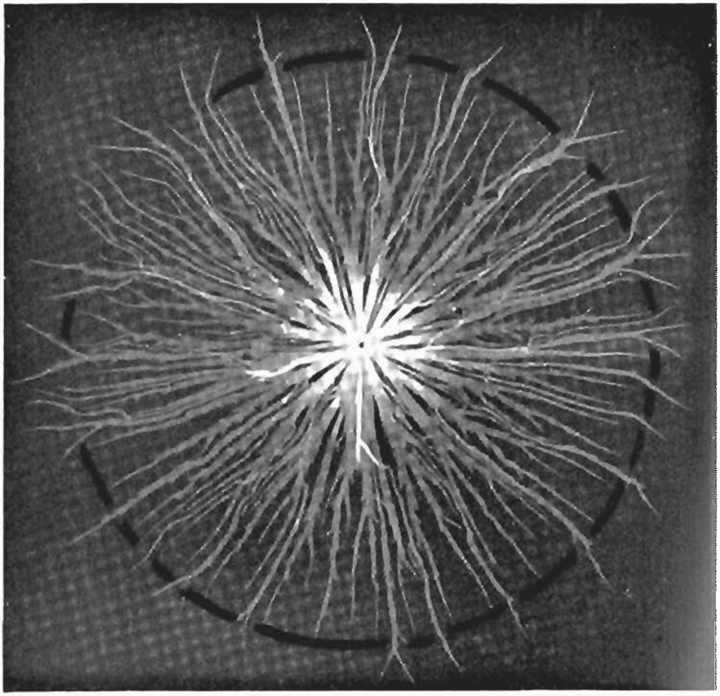
VI



Air; p = 760 mm Hg; l = 6 mm.

10 mm

I



II