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# $C^{12}(C^{12},\alpha)Ne^{20}$ CROSS SECTION MEASUREMENTS

BY

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## Synopsis

The reaction  $C^{12}(C^{12},\alpha)Ne^{20}$  has been investigated from 9 to 16 MeV center-of-mass energies. The alpha particles were detected with semiconductor detectors in a geometry which resolved the individual alpha-particle groups feeding the five lowest states in  $Ne^{20}$ . Excitation functions of the differential cross sections at center-of-mass angles of  $0^\circ$ ,  $37.3^\circ$ , and  $90^\circ$  showed strong fluctuations in the yields with no definite cross correlation between the positions of the maxima for different alpha-particle groups.

In most cases, the angular distributions of the alpha-particles to the ground state contained a dominant contribution from a squared Legendre polynomial of one definite order, usually 8 or 10. Angular distributions for each of the alpha-particle groups populating the excited states were more complex.

The general structure of the yield curves and angular distributions is in agreement with the statistical theory of overlapping resonances of the compound nucleus.

## 1. Introduction

The fluctuations in the cross sections of the reactions from  $C^{12}$  bombardment of  $C^{12}$  have generally been interpreted<sup>1) 2)</sup> in terms of "quasi-molecular" resonances and clustering in the compound  $Mg^{24}$ . At energies up to about 8 MeV center-of-mass, ALMQVIST et al.<sup>1)</sup> indeed observed cross correlations of the cross sections (cross sections for different emitted particles having maxima at the same energy), and thus isolated levels of some type apparently were observed. However, at higher energies extending to about 13 MeV, cross correlations in these cross sections were not apparent, nor were they observed by KUEHNER et al.<sup>2)</sup> for the reaction  $C^{12}(C^{12},\alpha)Ne^{20}$  to the ground and first excited state of  $Ne^{20}$ . This lack of cross correlation argues against the "quasi-molecular" resonance and  $Mg^{24}$  clustering interpretations of the fluctuations in these measured excitation functions at the higher energies.

LASSEN and OLSEN<sup>3)</sup> observed the inverse reaction  $Ne^{20}(\alpha,C^{12})C^{12}$  for a wider span of  $Mg^{24}$  excitation energy, but only to the ground state of  $C^{12}$ . As in the measurements of KUEHNER et al.<sup>2)</sup>, both fluctuations in the excitation functions and unusually well defined angular distributions were observed. A particularly interesting angular distribution for the ground-state reaction observed in both experiments was at 25.3 MeV excitation energy of the compound nucleus, at which energy a maximum in the cross section occurs. This angular distribution was reasonably well fitted<sup>3)</sup> by the square of a Legendre polynomial of a single order, in this case 8. These observations led to the present investigation of more extensive excitation functions and angular distribution of the  $C^{12}(C^{12},\alpha)Ne^{20}$  reaction to the ground state and to the lowest excited states of  $Ne^{20}$ .

In the present experiment, excitation functions and angular distribution of alpha-particle groups have been measured with instrumental resolutions less than the average width  $\Gamma$  of the compound nucleus levels and at energies such that  $\Gamma$  was greater than the average level spacing  $D$ , i. e.  $\Gamma > D$ . These are the conditions for observing statistical fluctuations in nuclear cross sections<sup>4) 5)</sup>.

The statistical theory for the present reaction between identical bosons is particularly simple since only even values of the compound nucleus angular momentum  $J$  are possible. Cross sections at selected angles and for final nucleus states of selected spins and parities have further simplicity. These are discussed by BONDORF and LEACHMAN<sup>6)</sup> in the following paper. In the present measurements, advantage is taken of these simplicities to increase the sensitivity of testing the statistical theory of nuclear reactions. These tests, to establish that a statistical combination of incoherent and coherent reaction amplitudes from overlapping resonances lead to the observed cross sections, are

1) No cross correlations between excitation functions for the alpha-particle groups to the ground state and the various excited states in  $\text{Ne}^{20}$  should exist. This test is most meaningful for the cross section integrated over all angles, rather than for the differential cross section.

2) Even for the  $0+$  ground-state reaction (for which the orbital angular momentum  $l$  equals  $J$ ) the angular distribution is not expected to be a pure Legendre polynomial of one single order, but the polynomial of the usually dominant spin is expected to be statistically combined with admixtures of polynomials of different orders.

3) Angular distributions with orders of the dominant Legendre polynomials that are exceptions to the usually dominant order are expected.

4) Probability distributions of the cross sections are expected to follow  $\chi^2$  distributions with the number of degrees of freedom in the distributions predicted by theory.

BONDORF and LEACHMAN<sup>6)</sup> have analysed some of the present data in terms of the last two tests.

## 2. Experiment

The  $\text{C}^{12}$  beam for the reaction  $\text{C}^{12}(\text{C}^{12},\alpha)\text{Ne}^{20}$  was obtained from the tandem Van de Graaff accelerator at the Institute for Theoretical Physics. After separation in the analysing magnet, the ions had a charge of  $4+$ . After passing through the target, the  $\text{C}^{12}$  ions have a larger effective charge according to the semi-empirical equation of NORTHCLIFFE<sup>7)</sup>. Over the energy range of this experiment the effective ionic charge is calculated to vary less than 2 % from an average value of 96 % of the nuclear charge 6. Therefore, the beam current could be read directly from the Faraday cup stopping the beam. The beam intensities obtained were between  $0.3 \times 10^{-7}$  and  $1.5 \times 10^{-7}$  A.

The targets were evaporated, self-supporting carbon films with thicknesses from 20 to 70  $\mu\text{g}/\text{cm}^2$ ; for some targets, thicknesses were determined either by Mott scattering or by weighing, or both. For other targets, the thicknesses were determined by reaction yield intercomparisons. The beam had an energy spread of about  $\pm 50$  keV and the target thicknesses correspond to energy losses<sup>8)</sup> of approximately 80 to 240 keV for  $\text{C}^{12}$  ions at 25 MeV in the laboratory system. This resulted in an instrumental resolution within the limits  $65 \text{ keV} < \Delta E < 130 \text{ keV}$  in the center-of-mass system for the compound nucleus formation.

The alpha particles from the disintegration of the  $\text{Mg}^{24}$  compound nuclei were observed by one or two  $10^4$   $\Omega$ -cm *p*-type semiconductor detectors which could be rotated around the target from the outside. The beam was defined by 3-mm-diameter collimators, but burn spots on the target indicated that the beam size at the target was in general smaller than this. The counter apertures were 3 mm in diameter. The distances between target and counter were 65 mm for some measurements and 70 mm for others, and so the angular resolutions were approximately  $\pm 1.3^\circ$  (Lab.) from a point beam spot on the target. Uncertainties in the angular alignment were about  $\pm 0.3^\circ$ .

For  $0^\circ$  measurements, a 24.2  $\text{mg}/\text{cm}^2$  gold absorber was mounted 10 mm in front of the counter to absorb the beam of  $\text{C}^{12}$  particles but transmit the reaction alpha particles. A measurement of the beam current was provided by  $\text{C}^{12}$  particles elastically scattered from the  $\text{C}^{12}$  target and detected by another counter at  $20^\circ$  (Lab.). The special simplicity of the distribution in the cross section expected for  $0^\circ$  has been calculated by GIBBS<sup>9)</sup> to become rapidly more complicated with angle; already at  $4^\circ$  considerable change in the cross section distribution is expected. Thus, the reaction alpha particles leaving the target at  $4^\circ$  or more, and undergoing Rutherford scattering from the absorber into the counter centered at  $0^\circ$ , is of concern\*. Approximate calculations indicate that the number of such scattered alpha particles detected is approximately a few percent of the reaction alpha particles leaving the target at angles in the admittance cone of the  $0^\circ$  detector. However, this scattering, the possibility of a full 3-mm-diameter beam on the target increasing the resolution to about  $\pm 3^\circ$ , and alignment errors can all combine to cause an appreciable yield of approximately  $4^\circ$  reaction products to be observed by the  $0^\circ$  centered counter.

A run at each energy amounted to an irradiation corresponding to about  $2 \times 10^{-5}$  coulomb which required typically 3 min. at the beam currents used. For the targets and solid angles used, the conversion between the differential

\* We are grateful to E. ALMQVIST for suggesting this effect.

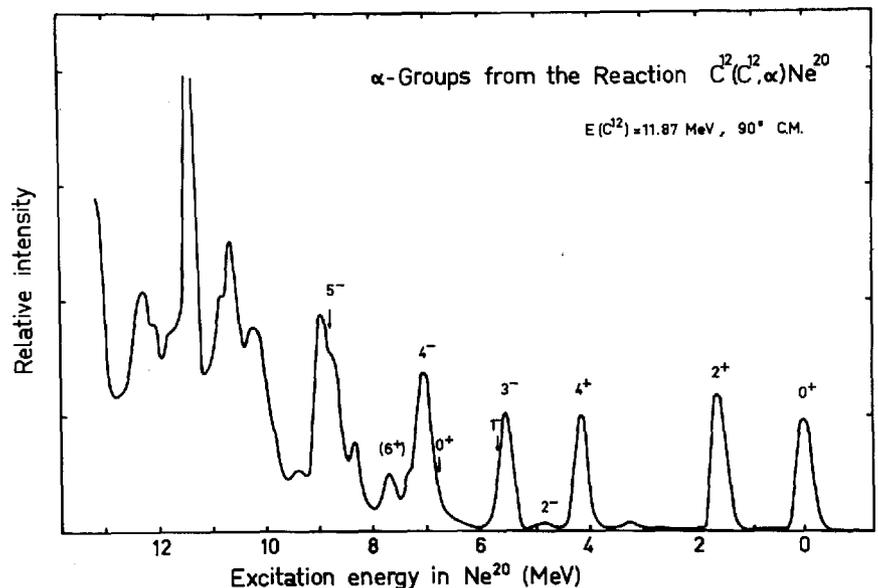


Fig. 1. Partial spectrum of alpha particles populating the states indicated in  $Ne^{20}$ . The absolute energy of the ground-state alpha-particle group is 15.3 MeV.

cross section and the number of alpha particles in the group being measured was typically 1 mb/ster for about 50 alpha particles detected. Effects of the statistical uncertainties in these small numbers have not been included in the analyses.

A typical spectrum of observed alpha particles is shown in fig. 1. As seen from the figure, only the alpha-particle groups to the five lowest states in  $Ne^{20}$  were resolved. The observed resolution resulted from the combination of the angular resolution and the reaction kinematics; the inherent resolutions of the counters were considerably better than the observed resolution. The small peak at 3.3 MeV on the energy scale is due to alpha particles from the reaction  $O^{16}(C^{12},\alpha)Mg^{24}$ , where the oxygen presumably is a contamination from pump oil buildup on the target. Under the worst conditions, the buildup on the target also increased the  $C^{12}$  thickness by  $1 \mu\text{g}/\text{cm}^2/\text{h}$ . Corrections to the cross-section measurements were made for this buildup.

To anticipate such unresolved alpha-particle groups, we have calculated the relative energies of the groups from the oxygen and carbon reactions as functions of bombarding energy at the three different laboratory angles. These curves are shown on figs. 2-4 with the experimental energy resolution indicated so that the identity of possible unresolved alpha-particle groups

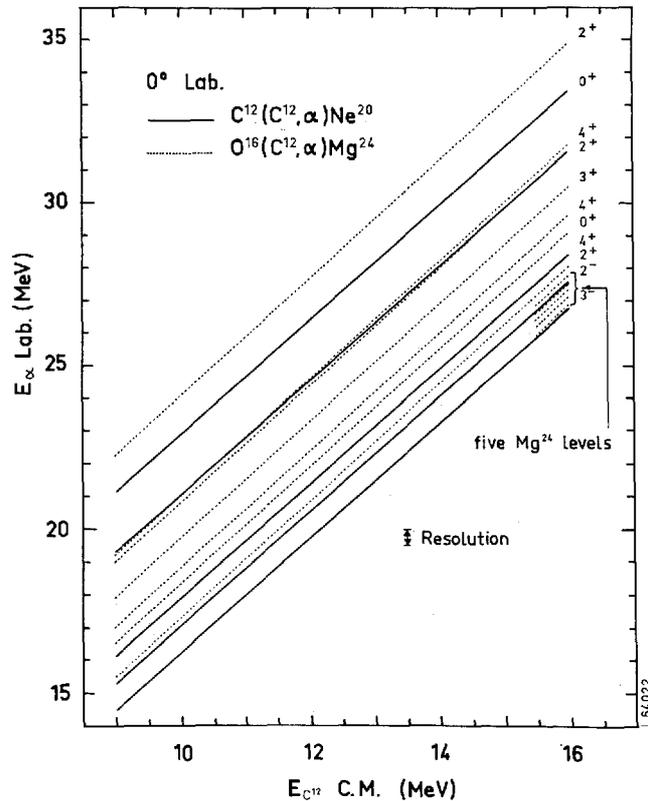


Fig. 2. Calculated energies of the alpha-particle groups from the reactions  $C^{12}(C^{12},\alpha)Ne^{20}$  and  $O^{16}(C^{12},\alpha)Mg^{24}$  as a function of the bombarding energy measured in the center-of-mass system of the carbon-carbon reaction. The observed energy resolution of the alpha-particle counter is indicated.

can be established. Unresolved alpha particles from the different reactions from oxygen contribute an incoherent amplitude to the cross section and thus increase the apparent number of degrees of freedom in the distribution of the observed cross section. The number of these other alpha particles is determined by the amount of  $O^{16}$  on the target and the cross section, which increases with increasing spin of the final state in  $Mg^{24}$ .

### 3. Angular Distributions

The angular distributions of the ground-state alpha particles from the  $C^{12}(C^{12},\alpha)Ne^{20}$  reaction are expected to be relatively simple for the following reasons.

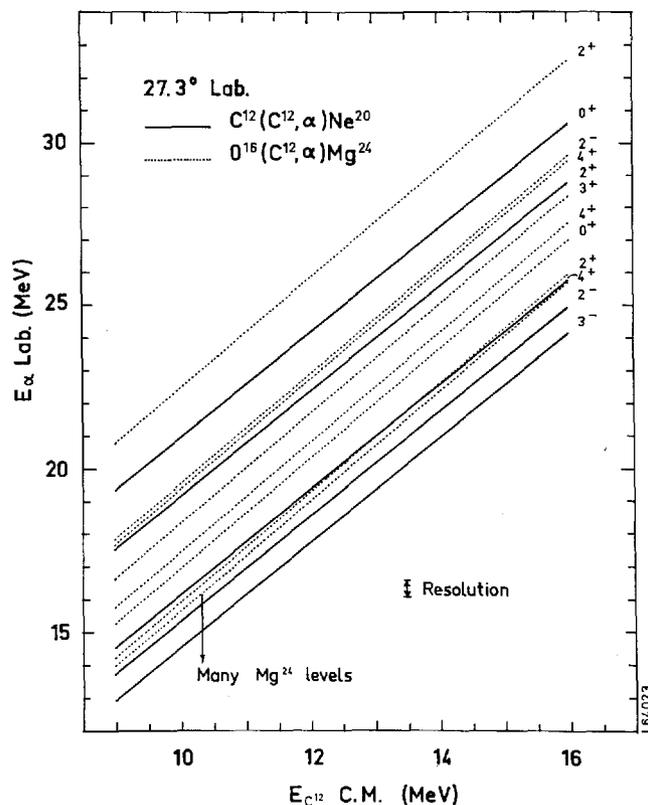


Fig. 3. Calculated energies for  $27.3^\circ$  lab., which corresponds to  $37.3^\circ$  C.M. in the  $C^{12} + C^{12}$  system. See caption to Fig. 2.

1) The identity of the projectile and target nuclei makes the angular dependence of the alpha particle symmetric about  $90^\circ$  in the center-of-mass system independent of the type of reaction.

2) Since both of the colliding particles are bosons, the reaction is described by a symmetric wave function. Thus, the parity is even, and only states with even  $J$  in the compound nucleus are formed.

3) If the compound states are isolated resonances with angular momentum  $J$  or are overlapping resonance of essentially only one angular momentum  $J$ , then the alpha particles to the ground state in the  $Ne^{20}$  nuclei must have an angular momentum  $l$  equal to  $J$ , and the distribution of the alphas is the square of a Legendre polynomial of order  $l$ , i. e.  $[P_l(\cos\theta)]^2$ .

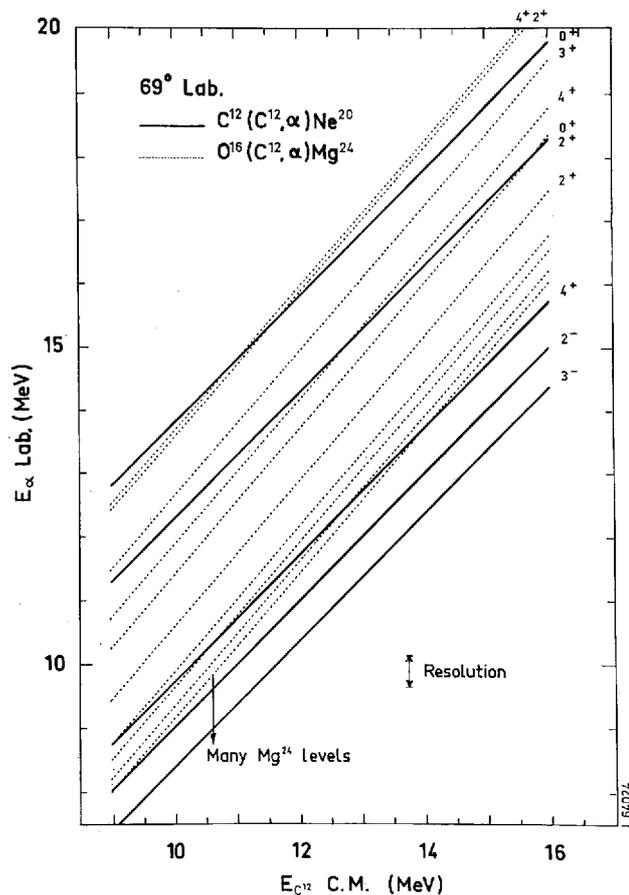


Fig. 4. Calculated energies for  $69^\circ$  lab., which corresponds to  $90^\circ$  C.M. in the  $C^{12} + C^{12}$  system. See caption to Fig. 2.

The distribution of ground-state alpha particles at a bombarding energy of 11.5 MeV is shown in fig. 5 in the center-of-mass system; parts of this distribution have been reported earlier<sup>2) 3)</sup>. The positions of the peaks and valleys are rather well reproduced by the square of the eighth order Legendre polynomial  $[P_8(\cos\theta)]^2$ , but the curve itself is only moderately well fitted by this function. This confirms the expectation from the statistical theory for overlapping resonances that reaction amplitudes of more than one angular momentum  $J$  in general contribute to the observed cross section.

A survey of most of the ground-state distributions measured in this experiment is shown in fig. 6. It is evident that the angular distributions at

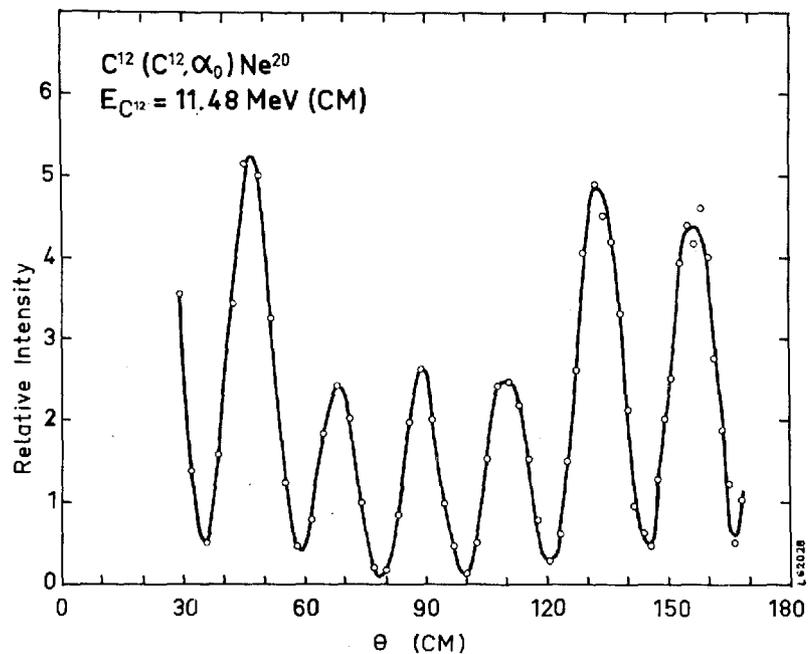


Fig. 5. Angular distribution of alpha particles to the ground state in  $\text{Ne}^{20}$  for 11.5 MeV C.M.  $\text{C}^{12} + \text{C}^{12}$ .

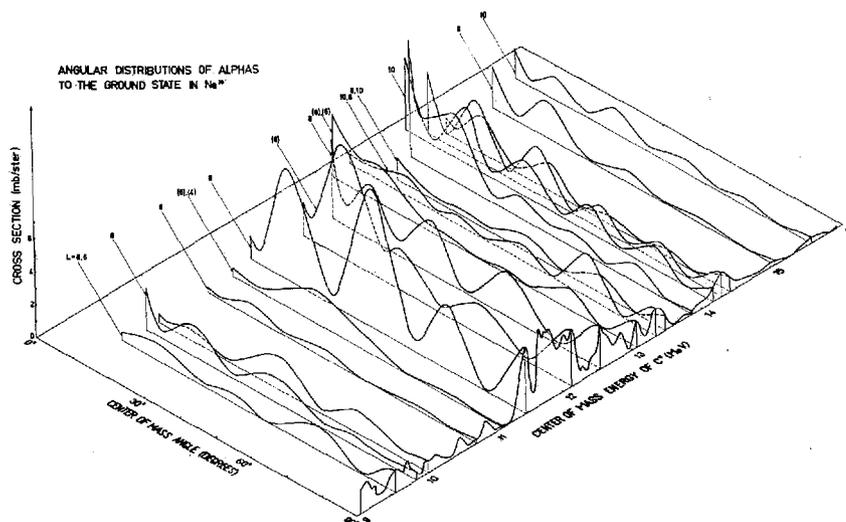


Fig. 6. Survey of angular distribution of alpha particles to the ground state in  $\text{Ne}^{20}$ . The  $l$  values are assigned according to the simple rules discussed in the text; where two values are given, the first refers to the main contribution. Parenthesis indicates a less certain determination of  $l$ .

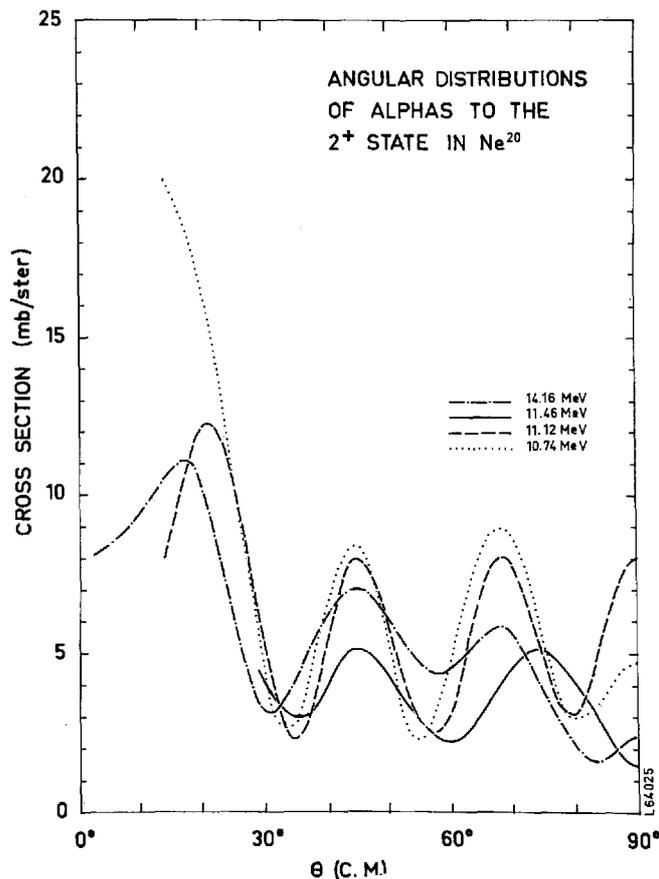


Fig. 7. Typical angular distributions of alpha particles to the  $2^+$  state in  $\text{Ne}^{20}$  for different carbon bombarding energies (C.M.).

11.5 MeV and at 14.2 MeV each have an exceptionally dominant  $J$  value resulting in nearly pure  $[P_8(\cos\theta)]^2$  and  $[P_{10}(\cos\theta)]^2$  angular distributions, respectively. All the other angular distributions are more complex.

The angular distributions are expected<sup>6)</sup> to be of the form

$$|P_l + (\alpha + i\beta)P_{l'}|^2 = (P_l)^2 + (\alpha^2 + \beta^2)(P_{l'})^2 + 2\alpha P_l P_{l'}.$$

In cases where the admixture of the angular momentum  $l' = l \pm 2$  is relatively small, it is in general possible to determine the  $l$  value of the main contribution just from the number of maxima or minima in the experimental distribution between  $0^\circ$  and  $90^\circ$ . The sign of  $\alpha$  has a marked effect on the envelope of the angular distribution. One notes that, if  $P_l(\cos 90^\circ)$  is po-

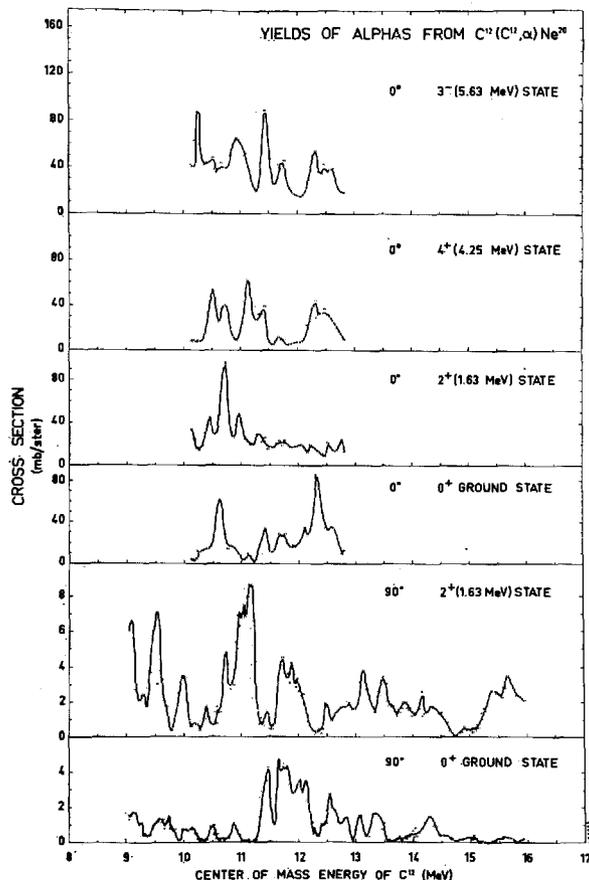


Fig. 8. Excitation functions for different alpha-particle groups at  $0^\circ$  and  $90^\circ$  C.M. The energy axis corresponds to zero target thickness. At  $0^\circ$  there is no yield of alphas to the  $2-$  state in  $\text{Ne}^{20}$ . The  $3-$  state at  $0^\circ$  was only poorly resolved from the neighbouring  $1-$  state of  $\text{Ne}^{20}$ .

sitive, then  $P_l \pm_2(\cos 90^\circ)$  is negative, and vice versa, and that  $P_l(\cos 0^\circ)$  is always positive. A positive  $\alpha$  will therefore decrease the yield at  $90^\circ$ ; a negative  $\alpha$  will decrease the yield at  $0^\circ$  and increase the yield at  $90^\circ$ . In the latter case, the envelope of the angular distribution can be almost constant with angle. (See the distribution at 13.48 MeV in fig. 6).

One can determine the value of  $l'$  by observing the direction of the shift of the positions of the minima from those of the distribution of the main component. With a positive  $\alpha$ ,  $l' = l + 2$  and  $l - 2$  will cause a shift towards  $0^\circ$  and  $90^\circ$ , respectively; the opposite is true for a negative  $\alpha$ .

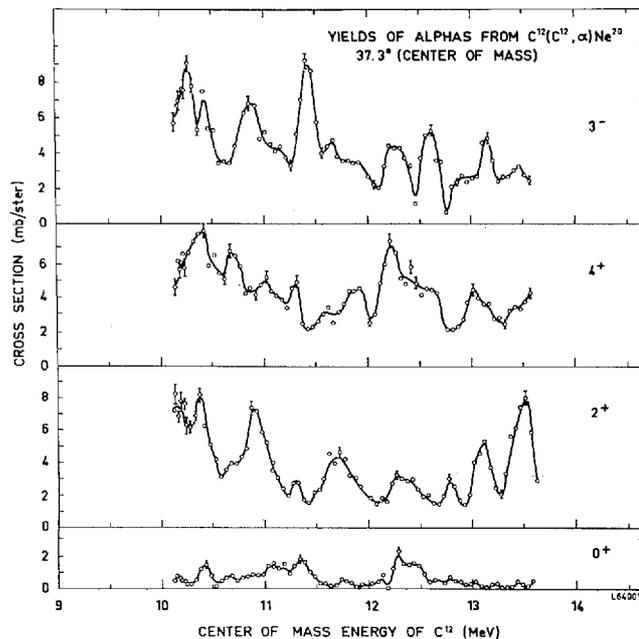


Fig. 9. Excitation functions at  $37.3^\circ$  C.M. The energy axis corresponds to zero target thickness. The 3- state was only poorly resolved from the two neighbouring 2- and 1- states of  $\text{Ne}^{20}$ .

Values of  $l$  and  $l'$  determined by the simple method discussed above are indicated in fig. 6.

In fig. 7 are shown some angular distributions of alpha particles populating the  $2+$  first excited state of  $\text{Ne}^{20}$ . The more complex structures of these angular distributions follow from the number of associated Legendre polynomials  $P_l^M(\cos\theta)$  that combine for the various orbital angular momenta  $l$  and substates  $M$  allowed for each  $J$ . The allowed values are  $J+2 \geq l \geq J-2$  and  $2 \geq |M| \geq 0$ .

#### 4. Excitation Functions

The measured excitation functions of the differential cross sections at  $0^\circ$  and  $90^\circ$  (C.M.) for the clearly resolved groups are shown in fig. 8. The yield curves in fig. 9 were obtained at  $37.3^\circ$  (C.M.), this angle being a null angle for the Legendre polynomial of order eight. Both results are for a larger span of  $\text{Mg}^{24}$  compound nucleus energy than that observed by KUEHNER et al.<sup>2)</sup> or by LASSEN and OLSEN<sup>3)</sup>, and are with considerably better energy resolution than the latter.

The following points regarding the present excitation functions are of significance.

1) The use of thinner carbon targets did not change the excitation functions appreciably. This indicates that the instrumental resolution is smaller than the level width in the compound nucleus in this region of excitation.

2) The yield of the ground-state alpha particles at  $37.3^\circ$  in fig. 9 is unusually low. The expectation from BONDORF and LEACHMAN<sup>6)</sup> and from fig. 6 that the dominant  $J$  value over most of this energy region is  $J = 8$  is thus confirmed by the low cross section resulting from this suppressed major component. The cross section to the excited states is of approximately normal amplitude because the reaction amplitudes from all but ( $l = 8$ ,  $M = 0$ ) contribute to the observed cross section.

3) There is a complete lack of cross correlation between peaks in the yield curves for the various alpha-particle groups. This cross correlation behaviour is difficult to understand if the peaks are interpreted as originating in single, isolated resonances in the compound nucleus. The differences between the yield curves for the various alpha-particle groups exceed what can be expected from differences in barrier penetrabilities<sup>6)</sup> for different angular momenta resulting from different spins of the final nuclei and cannot be related in a systematical way to such differences.

The theoretical cross sections are shown in fig. 4 of BONDORF and LEACHMAN<sup>6)</sup> for the partial cross sections  $\langle\sigma_J\rangle$  (averaged over overlapping levels of the compound nucleus) integrated over angle for reactions going through compound states of spin  $J = l$ , and also for the average cross section  $\langle\sigma\rangle$  over all spins.

The theory indicates that one should be able to observe energy regions with usually a single dominant  $J$  value. This is indeed confirmed by the experimental results in fig. 6. We emphasize that the cross sections are expected<sup>6)</sup> to fluctuate around the calculated averages. Thus, we should expect occasionally to observe for the  $0+$  ground state case an angular distribution characterized by an  $l$  value other than the calculated usually dominant  $J$  for that region. Such exceptions are an  $l = 6$  distribution at 11.12 MeV in the usually  $J = 8$  region and an  $l = 8$  distribution at 15.05 MeV in the usually  $J = 10$  region. This probability of exceptions is expected by theory<sup>6)</sup>.

## 5. Discussion

The regions of excitation in the compound system covered by the yield curves in figs. 8 and 9 range from 23 MeV to 30 MeV. In this region of

excitation, the level density in the compound nucleus is of course very high. However, one must remember that the  $C^{12}(C^{12},\alpha)Ne^{20}$  reaction is only one of many open channels because the compound state can decay by many channels of protons, neutrons, and low energy alpha particles in competition with alpha particles going directly to the ground state or the other low-lying states of interest. For states of large angular momentum, the emission of alpha particles populating the low-lying levels of  $Ne^{20}$  will be enhanced compared with the other decay modes because of the larger transmission coefficients  $T_l$  for large orbital momentum  $l$  of alpha particles than of neutrons and protons<sup>10</sup>).

In an excited nucleus with high angular momentum, part of the excitation energy is taken by the nuclear rotation. This effectively reduces the energy entering in the exponential of the statistical expression for the level density so that the density of nuclear states  $\varrho_J$  with angular momentum  $J$  is given by

$$\varrho_J = (2J + 1) \exp \left[ -\frac{J(J + 1)}{2S^2} \right] \varrho_0$$

relative to the density  $\varrho_0$  of states with angular momentum zero. The quantity  $S^2$  is related to the nuclear moment of inertia  $\mathfrak{J}$  and the nuclear temperature  $t$  through

$$S^2 = t\mathfrak{J}/\hbar^2.$$

From an analysis of experimental data, the nuclear temperature is expected to be around 2.1 MeV (see BONDORF and LEACHMAN<sup>6</sup>). The value of  $\hbar^2/(2\mathfrak{J})$  for the highly excited system must be close to the rigid value. The latter is 150 keV for a nuclear radius of  $1.25 A^{1/3}$  fermi and a deformation parameter  $\beta = 0.35$ . With these values the average level spacing  $1/\varrho_8$  of spin 8 states at an excitation energy of 25 MeV is calculated to be about 100 keV. The level spacings calculated in this manner are of course extremely dependent on the values of  $\mathfrak{J}$  and  $t$  entering in the exponent. Reasonable changes in these two parameters cause variations in  $1/\varrho_8$  from  $\sim 20$  to  $\sim 500$  keV.

Although it is thus possible by a suitable choice of parameters to calculate a level density of the compound nucleus resonances that would result in resonances spaced at the observed energy spacings of enhanced cross sections, there are two important arguments against an interpretation in terms of individual resonances. One is the lack of cross correlation between

the maxima in the yields of the different alpha groups discussed above. The other argument is based on the relation

$$\frac{\Gamma}{D} = \frac{\Sigma T}{2\pi}$$

between the average width  $\Gamma$  and spacing  $D$  of compound resonances. The sums  $\Sigma T$  over the transmission coefficients for all open channels are  $\gg 2\pi$  (cf. table V of BONDORF and LEACHMAN<sup>6)</sup>) in the cases considered here, with the result that the average width is considerably larger than the average spacing of levels. Under such circumstances, the observation of individual resonances is highly unlikely, and the fluctuations in the yield functions must be ascribed to the random, but coherent, combination of the reaction amplitudes from overlapping resonances of various  $J$  values. The data also confirm<sup>6)</sup> that fluctuations are reduced by the incoherent combination of reaction amplitudes characterized by different values of the magnetic quantum numbers  $M$ .

The analysis in the following paper shows that such a picture explains the fluctuations in our observed yield curves not only in a qualitative way, but also in a quite detailed manner accounts for the statistical behaviour of the distribution in the differential cross sections measured at different angles and in the measured cross section integrated over angle<sup>2)</sup>.

Some qualitative features of the cross-section probability distributions bear on questions of angular resolution and on  $O^{16}$  contamination of the target. The effects of these experimental difficulties are particularly easy to observe for the excitation functions for which  $\nu$ , the number of degrees of freedom in the probability distribution, is expected to be equal to two from the "simple theory" of BONDORF and LEACHMAN<sup>6)</sup>. For two degrees of freedom, the cross-section probability follows

$$P(\sigma) = \frac{\exp\left(-\frac{\sigma}{2}\right)}{2},$$

and so the cross-section probability decreases from a maximum at zero cross section. All observations of alpha particles to the ground state and all observations at  $0^\circ$  are expected to exhibit this property. These are shown in fig. 10 in comparison with the theoretical  $\chi^2$  distribution of two degrees of freedom.

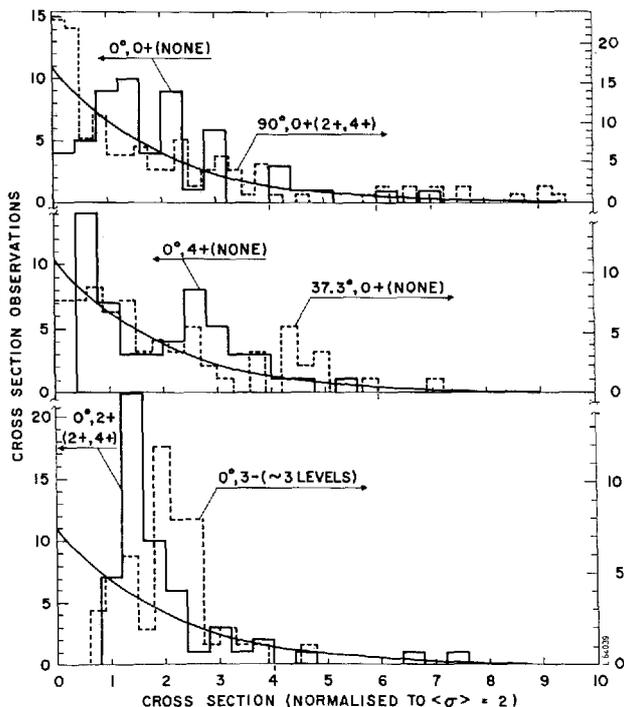


Fig. 10. Histograms of differential cross section data from Figs. 8 and 9 compared to normalized  $\chi^2$  curves with two degrees of freedom. Numbers in parentheses indicate possible alpha groups to states in  $Mg^{24}$  which could not be resolved from the alpha-particle group to the  $Ne^{20}$  state considered. At  $0^\circ$  the poor resolution of the 3— state from the neighbouring 1— state of  $Ne^{20}$  contributed additional incoherent amplitudes.

This expectation for the  $0+$  ground state cases is qualitatively confirmed by the excitation functions at  $0^\circ$ ,  $37.3^\circ$  and  $90^\circ$  in figs. 8 and 9 and their histograms in fig. 10, which have the nearly zero cross sections being the most probable. No unresolved alpha particles from  $O^{16}(C^{12}, \alpha)Mg^{24}$  levels are expected with the ground-state alpha particles at  $0^\circ$  and  $37.3^\circ$ , as is seen from figs. 2 and 3, respectively. For the  $90^\circ$  case, fig. 4 shows that alpha groups to two  $O^{16}(C^{12}, \alpha)Mg^{24}$  levels are unresolved from the ground-state alpha particles. The observed large probability for nearly zero cross sections implies that the  $O^{16}$  contamination in this run was negligible in terms of the cross-section probability distribution. The small buildup of  $C^{12}$  on the target during the  $90^\circ$  runs agrees with this.

On the other hand, the histograms in fig. 10 show that only the alpha particles for the  $0+$  ground state cases (for any angle) qualitatively have

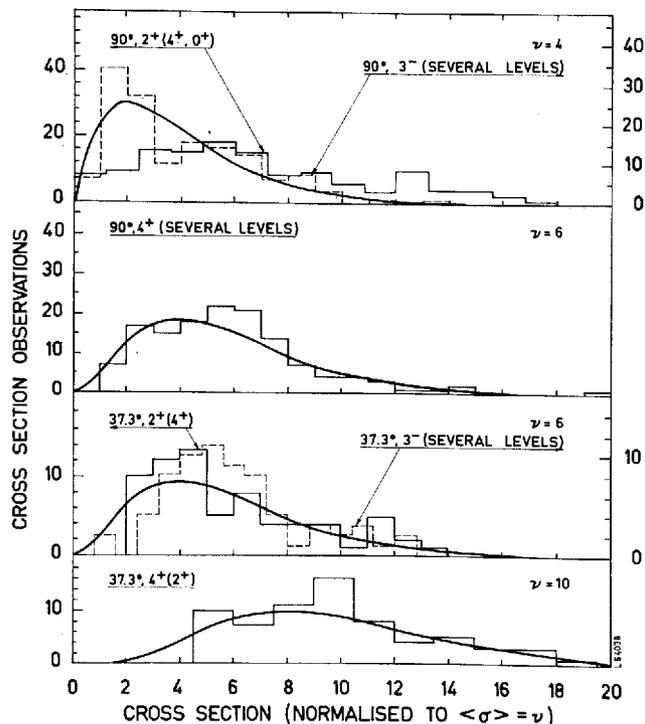


Fig. 11. Histograms of differential cross section data from Figs. 8 and 9 and ref.<sup>10</sup> compared to  $\chi^2$  curves with various numbers of degrees of freedom,  $\nu$  predicted by the "simple theory" of BONDORF and LEACHMAN<sup>6</sup>. For numbers in parentheses, see caption to Fig. 10. The poor resolution of the 3—state from the neighbouring 2— and 1—states of Ne<sup>20</sup> contributed additional incoherent amplitudes. All the 90° curves contain data between 9 and 16 MeV although not the full energy span is published in ref.<sup>10</sup>.

the most probable cross section being nearly zero. The combination of ground state and 0° is surely expected to result in two degrees of freedom, because fig. 2 shows no interfering levels and the angular resolution is of less consequence for this ground state case, which results in two degrees of freedom for any angle (provided the angular resolution is smaller than the coherence angle). We emphasize that the excitation function for alpha particles to the 4+ state at 0° is not affected by unresolved groups from the O<sup>16</sup> reaction (see fig. 2), but yet the histogram in fig. 10 (from the excitation function in fig. 8) shows that the nearly zero cross sections are clearly not the most probable. This gives a strong indication that the angular resolution effects discussed in section 2 are playing a role. Even greater deviations from the expectation of the nearly zero cross sections being the most pro-

bable are observed from the histograms in fig. 10 (again obtained from the excitation functions of fig. 8) for the observation at  $0^\circ$  to the  $2+$  and  $3-$  states. However, unresolved levels seen in fig. 2 are an additional factor in these cases, and the target in this case was known experimentally to have a large buildup, as indicated by the growth of  $C^{12}$  on the target during the runs.

Thus, in a qualitative manner, we can account for the agreement and disagreement with the two degrees of freedom expectation for the excitation functions in figs. 8 and 9. Another possible cause for disagreement is the finite size of the sample of cross sections analysed<sup>11)</sup>, particularly for  $0^\circ$  and  $37.3^\circ$  data.

Shown in fig. 11 are histograms of data from figs. 8 and 9 and ref.<sup>10)</sup> for which larger numbers of degrees of freedom are expected. It is seen that generally good agreement with the  $\chi^2$  distributions is obtained with the number of degrees of freedom from the "simple theory" of BONDORF and LEACHMAN<sup>6)</sup>. However, these comparisons with the larger number of degrees of freedom  $\nu$  in fig. 11 do not provide as sensitive a test of the agreement between theory and data as do the comparisons for two degrees of freedom in fig. 10.

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