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SOME STUDIES ON HEAVY MESON EVENTS IN STRIPPED EMULSIONS

BY

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An account is given of 17 K-meson events found in half a stack of stripped emulsions exposed during the Sardinia expedition of 1953. Evidence is presented for the mode of decay $K_\mu \rightarrow \mu + \nu$, where the μ-meson is emitted with $p\beta c =$ (224 ± 7) MeV., and the best mass of the K_μ-meson is (990 ± 16) m_e. An example of χ-decay is also reported and discussed.

I. Introduction.

In the course of a special scan of 106 cm³. of emulsion for stopped K-mesons, 17 examples have been found in which a stopped K-particle emits a fast singly charged secondary, together with one or more neutral particles. In addition, one example of the capture of a negative K-meson, two examples of τ-decay, one probable and one certain excited fragment, 1054 examples of π-μ decay, and 1540 σ-stars were found in the same volume of emulsion. The emulsions searched formed half of a stack of 40 stripped emulsions, each $150 \times 100 \times 0.6$ mm³., exposed by the Sardinia expedition in the summer of 1953. (S 17; exposed on flight 20). The plates reached a maximum altitude of 85,000 feet, and remained above 62,500 feet for seven hours, at which latter altitude the cut-off operated. Full details of the flight data, etc., have been published by Davies and Franzinetti (1954). The plates were later developed in Bristol.

It is difficult to estimate the efficiency of detection of the stopping K-particles which decay with the emission of only a fast charged particle at or near minimum ionization. The grain density in our emulsions is considerably lower than normal (plateau $\approx$ 9 grains per 50 μ) and the grains themselves are small. A guide to the efficiency of observation of the decay tracks from the stopped particles may be had from the ratio of the numbers of ϱ-meson events with and without observed decay electrons.

The observed ratio is 1.15, compared with an expected value ≈ 1. It must be noted, however, that the comparison between the ϱ-mesons and the K-mesons is by no means exact; it is easier to distinguish ϱ-mesons from stopping protons, so that the search for a minimum secondary will be made more carefully.

The method of scanning adopted probably leads to some bias against negative K-events, for, when a negative K-meson is stopped and captured shortly after entering a particular emulsion sheet, the event is not easily distinguished from an ordinary small star. On the other hand, a stopping track with which was associated a lightly ionizing secondary would be traced back into the next emulsion if there were any doubt at all about its nature. Tracing back has been made very simple and quick by the use of frames in which the individual plates are mounted, so that the scanners are always able to follow doubtful examples back quickly.

Table I includes all the data which have been obtained on the examples of K-mesons. Preliminary data (Bøggild et al., 1954) on some of the events have already been included in the table published by Dilworth et al. (1954), but there are changes in the present table due to more detailed measurements which have been made more recently.

II. Mass Measurements on the Primary Particle Tracks.

(a) *Constant Sagitta Method.*

Measurements by the constant sagitta scattering method have been carried out on all the suspected K-particles to rule out the possibility that any of them are examples of hyperons decaying at rest. The cell scheme used was that published by Fay et al., (1954; table 4) which extends from 40 to 10,300 μ and which was designed for a mean sagitta, $\bar{D}$, of 0.500 μ for π-mesons. Taking into account the variation of the scattering constant and $p\beta c$ along the track, a particle of mass $M = 963\ m_e$ would be expected to yield a value of $\bar{D} = 0.288\ \mu$, while one of mass $M = 2330\ m_e$ would give $\bar{D} = 0.193\ \mu$. When the full set of cells is not used, $\bar{D}$ must be corrected by a small factor which is a function of the mass. The scattering was measured using cells,

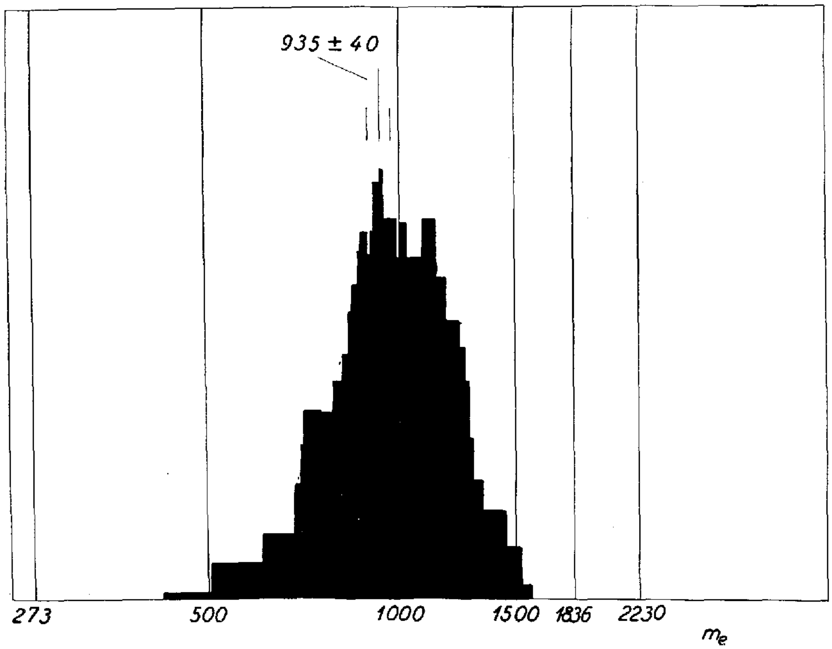

Fig. 1. A weighted distribution of the masses of the 17 K-mesons and one τ-meson measured by the constant sagitta method. Each track is represented by a rectangle whose width is the standard deviation, and whose area corresponds to the statistical weight.

each of which was half the length given in the table. A $4 \times \overline{D}$ cut-off was used and the noise was eliminated between these cells and those of full size, on the assumption that it is independent of cell-size. For steep tracks, the zero point of the cell set was frequently adjusted, so that, on the average, the assumed range was equal to the true range. The value of $\overline{D}$, measured on a track of angle of dip Φ in the unshrunk emulsion, was corrected by a factor $(\cos \Phi)^{3/2}$. This factor was 0.8—0.9 for events $K4$, $K13$, and $K17$, and between 0.6 and 0.8 for events $K8$, $K9$, and $K18$. For all the other events $(\cos \Phi)^{3/2} > 0.9$. The value of $\overline{D}$ from six tracks which were shorter than the full set of cells was corrected on the assumption that the mass was 963 m_e. This correction never exceeded 1 %.

The masses quoted in Table I were calculated, using the relation

$$M = (963\ m_e)\ (0.288\ \mu/\overline{D})^{2.32},$$

while the errors were derived from the number of cells, taking into account the effect of noise. The probability that any of these particles has a mass outside the interval 273 m_e to 2340 m_e is less than 2 %. Taking the measurements of all the particles together, the mean value of $\overline{D} = (0.291 \pm 0.005)\ \mu$, corresponding to a mean mass value of $M = (935 \pm 40)\ m_e$. The weighted distribution of the individual mass values is shown in Fig. 1.

(b) *Mean Gap-Length vs. Range Method.*

In view of the large statistical error inherent in the measurement of mass by the constant sagitta method, we have, in addition, measured, by the mean gap-length vs. range method, the masses of six primary particles whose tracks were flat in the emulsion. As is well known (O'Ceallaigh, 1954; Della Corte et al., 1953, 1954), the mean gap-length is a particularly good measure of ionization, since it is very little dependent on small fluctuations of development in the plates. In our experiment, no corrections for variation of this quantity with depth in the emulsion have been found to be necessary, provided that track within 30 μ of either surface of the shrunk emulsion is not measured. As the greatest fluctuations in the mean gap-length differed by only a few per cent between different parts of the same emulsion, and between different emulsions of this batch, it has only been necessary to make a detailed calibration curve for one of the emulsions of the batch. The curve was then fitted to points corresponding to specific proton ranges obtained from the other emulsion sheets. Of the plates which have been examined in detail, three yield mean gap-lengths as a function of range which are identical to within about 1 % with those obtained from the calibration plate; the mean gap-lengths from three other plates are very close to that of the calibration plate for black tracks, but deviate from it when the mean gap-length becomes larger. The deviations are about 4 % when the ionization is that corresponding to a proton of 5 cm. residual range. As the deviations are small, it is sufficient to assume that the percentage correction of the mean gap-length is proportional to the mean gap-length.

In view of the arguments put forward by Della Corte and his coworkers (1954), such deviations might, at first sight, seem surprising, but it must be remembered that their arguments only apply to "fully developed" emulsions. Though it is difficult to define in a practical way what exactly is meant by "fully developed", the present stack of emulsions is, by any standards, rather lightly developed. When the mean gap-length in a plate was found to deviate from that in the calibration plate, it was always larger, indicating lighter development. In such plates, the fluctuations of the mean gap-length along a track were found to be very much larger than for those plates in which it did not deviate, and this effect was found to be due to "islands" of low development, in which the mean gap-length was sometimes as much as 10 % above that of the calibration plate. No evidence was found for "islands" in those plates which did not deviate appreciably from the primary calibration. In order to correct for the effects of the "islands", special calibration tracks close to those of the K-mesons were used in those plates which deviated. Some confidence can be felt that all the "islands" which could have led to appreciable errors in the mass determination have been found, for their effects would be expected to show up as large deviations in the apparent mass between different plates. In only one of the six measured examples, $K6$, did the whole of the track lie within the same emulsion sheet. Only two sections of K-track, both on $K7$, were found to pass through the "islands".

A simple cosine correction was applied to the mean gap-length whenever the dip of the track exceeded one in thirty-five in the shrunk emulsion, but as measurements by this method were only made on tracks which were relatively flat, corrections were only used on a few short sections, and they never exceeded 3 %.

In order to test the method for any possible systematic error when applied to particles of K-meson mass, measurements were made, using exactly the same experimental techniques, on the track of a τ-meson. The value obtained for its mass was $(979 \pm 43)\ m_e$, in excellent agreement with the accepted value (e. g., see Amaldi et al., 1954). It should be noted that the error quoted does not take any account of possible errors arising from the calibration.

The errors were found by splitting all the tracks into sections, 1.74 mm long, from each of which a value of the mass was found.

The errors could then be calculated from the internal consistency of each track. From the calibration tracks, the standard deviation of the individual sections from their means were $8.2\,\%$, $8.8\,\%$, and $8.9\,\%$ on μ-mesons, π-mesons, and protons, respectively. The standard deviations found from the three longest K-meson tracks were $9.4\,\%$, $5.6\,\%$, and $7.5\,\%$ for individual 1.74 mm lengths. Taking all the measured sections of K-meson track together, one finds that the standard deviation is $8.6\,\%$, in good agreement with the above figures. This is consistent with the assumption that all the K-particles have the same mass.

29 individual sections of K-track were measured. If the measurements were completely independent, the final error on the mean would be $(8.6/\sqrt{29})\,\% = 1.6\,\%$. In point of fact, the error calculated from the final masses was only $0.6\,\%$. Though the statistical weight of this figure is small, it appears that the true standard deviation taken from the final mass values cannot be larger than that obtained from the individual sections. This confirms that all the masses are consistent with an unique value, and at the same time shows that any effects due to energy straggling are already included in the error which is deduced from the individual sections. A narrow distribution of the masses determined from long tracks might be a consequence of correlation between energy-loss fluctuations and fluctuations in the grain-density.

The errors quoted for the individual particles in Table I are calculated from the relation s. d. $= (8.6/\sqrt{n})\,\%$, where n is the number of sections in the track. Where mean masses are quoted later in the paper, a further error of $2\,\%$ is added, to take into account the possible errors in the calibration. This figure for the calibration error is deduced from the consistency of the apparent masses of all calibration particles. The weighted mass distribution of the K-particles, including the τ-meson, and the calibration tracks whose residual range is > 4 mm, is shown in Fig. 2. The mean mass of the five measured K-particles is $(986 \pm 25)\,m_e$.

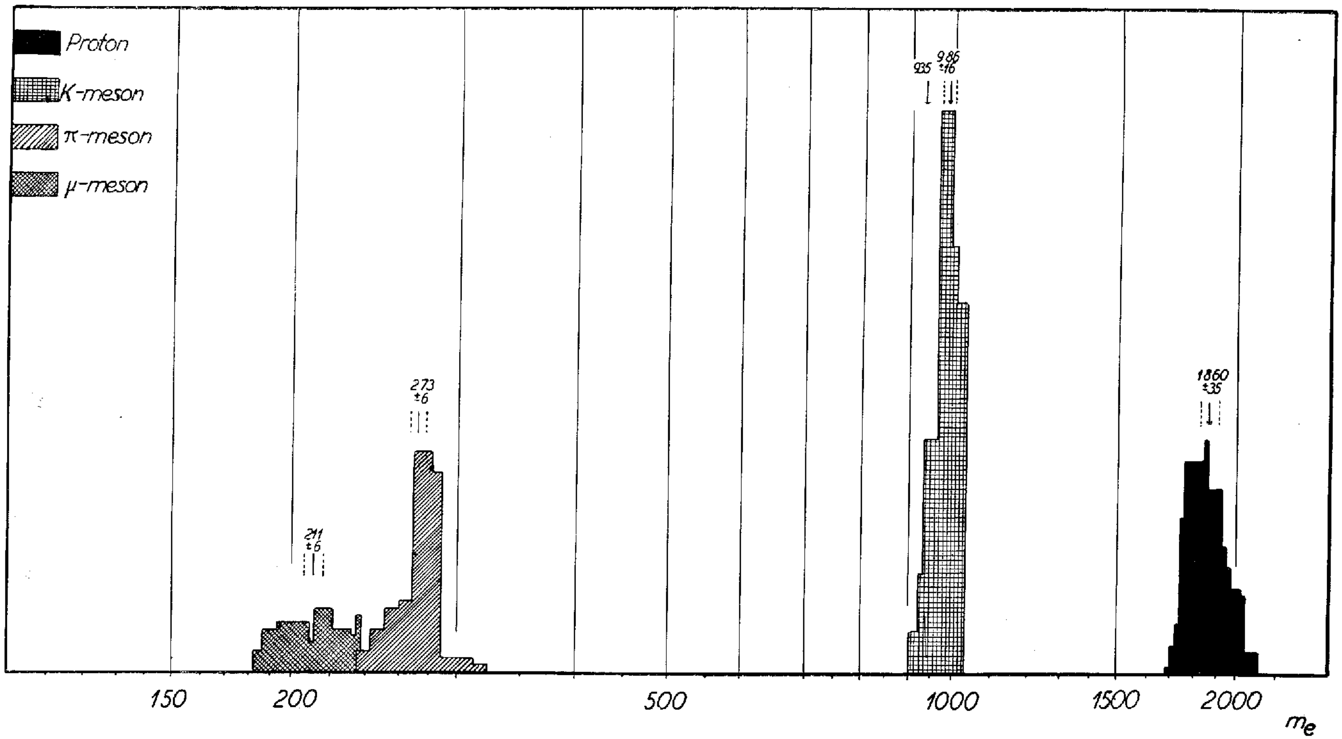

Fig. 2. A weighted distribution (constructed in the same way as Fig. 1) of the masses of the six K-mesons (K 5, K 6, K 7, K 12, K 20, and τ 2) and the calibration particles, obtained from mean gap-length vs. range measurements.

III. Measurements on the Secondary Particle Tracks.

Six of the secondary particles were flat enough to enable reasonably accurate measurements of the grain-density and scattering to be made. Of these six, five lead to values of $p\beta c$ close to 220 MeV., and a grain-density close to plateau. The sixth secondary, that associated with $K5$, was emitted with $p\beta c = (170 \pm 8)$ MeV. and a "blob" density, $b^* = (1.08 \pm 0.02)$ times the plateau value.

The ionization was estimated by comparing the "blob" count with that of nearby electron tracks. It was found to be very important that the electron tracks selected should be near the track under consideration, laterally as well as in depth. It was usually possible to find sufficient electron track which lay within one millimetre of the meson secondary at the same depth in the emulsion. Calibration electrons were chosen by rough scattering measurements which ensured that their energy lay between 30 and 80 MeV. When available, higher energy electrons were used which originated in high energy pairs or other readily identifiable electromagnetic processes. In each plate at least twice as many electron track grains as grains of the secondary track were counted.

The values of the "blob" density, quoted in Table I, are given for the same point on the track as that to which the values of $p\beta c$ given in column 9 refer, while the errors quoted have been calculated from the internal consistency along each track, and not from the number of "blobs" actually counted.

Scattering measurements were made on a Koristka microscope (noise level 0.03 μ independent of cell-length (Bøggild and Scharff, 1954)) by the coordinate method due to Fowler (1950). Each track was measured in 50 μ cells. $\bar{D}$ was calculated, using a $4 \times \bar{D}$ cut-off, and the noise was eliminated between 50 μ and 100 μ cells on the assumption that it was independent of the cell-length. We were able to use such a small cell-size because the total noise on the readings was 0.1 μ, and this was an advantage not only because it increased the statistics, but also because it reduced the effect of distortion. The distortion in these emulsions was low, but corrections were made for some of the steeper tracks, the distortion being measured directly on neighbouring parallel tracks, and the appropriate correction being added to each second difference.

The values of the scattering from sections of track in different emulsions were plotted as a function of distance from the decay point of the K-mesons, and a straight line, of slope corresponding to the appropriate rate of energy loss, was then fitted to the experimental points by the method of least squares, and was extrapolated back to the decay point. From the value of $\bar{\alpha}$ so found, the value of $p\beta c$ with which the secondary was emitted was calculated.

The value of the scattering constant was checked, on the assumption that the theoretical dependence on velocity and cell size (WILLIAMS, 1940; GOTTSTEIN et al., 1951) was correct, from measurements on a number of stopping particles in the emulsion. For each section of track, the value of $p\beta c$ was calculated from the range-energy relation published by BARONI et al. (1954), which was first tested at low energy on μ-mesons from the decay of π^+-mesons stopping in the stack. The true μ-meson range was determined independently of the shrinking by the regression method of FRY and WHITE (1954), which, with a slight modification, was adapted to thick emulsions. The mean range of 20 μ-mesons was $(0.0 \pm 1.4)\ ^0/_0$ from that given by the range-energy relation for 4.11 MeV. μ-mesons. It was found that the scattering constant in these plates should be increased by $(3 \pm 3)\ ^0/_0$ over the value determined by VOYVODIC and PICKUP (1952). The errors on the values of $p\beta c$ at emission including those arising from this source are set out separately in column 12 of Table I.

IV. Identification of the Secondary Particles.

The secondaries of $K3$, $K7$, $K12$, $K14$, and $K20$ were all emitted with a value of $p\beta c$ between 215 MeV. and 233 MeV. The deviations of the individual values from the mean, (224 ± 7) MeV., were consistent with the assumption that all five particles were ejected with the same energy. In the following discussion we shall therefore assume that all these particles arose from the same two-body decay process. The ionization of four of them was the same within experimental limits, but that of $K7$ was considerably higher.

The grain density and scattering results are presented in Fig. 3. The variables chosen for this figure and their theoretical relationship are discussed in the Appendix. The measured "blob" densities, b^*, were converted to grain densities, g^*, on the assumption of an exponential gap-length distribution, using the approximate formula

$$g^* = b^* \left\{ 1 + \frac{\sigma}{G} (g^* - 1) \right\}.$$

Here, G is the mean gap-length and σ is the minimum distance between the centres of resolved grains. We used the value $\frac{\sigma}{G} = 0.1$, which is sufficiently accurate for our purpose, as the quantity $0.1\,(g^* - 1)$ never exceeded 0.02.

The K-meson secondaries are each represented by two points, one calculated on the assumption that the particle is a μ-meson, and the other on the assumption that it is a π-meson. The secondary of $K5$ is well identified as a π-meson, fitting well the points obtained for the calibration tracks. With the exception of $K7$, the other secondaries are best fitted on the assumption that they are μ-mesons. The fast calibration tracks seem to lie below the theoretically calculated limit, but it may be that one of them was a μ-meson.

Independent evidence in favour of the group of secondaries being really μ-mesons is obtained when the mass measurements on the primary particles are taken into account. If we assume that they are π-mesons, and that the events do represent the decay of heavy mesons, then the primary mass, from the observed value of $p\beta c$ at emission of the secondaries, would have to be $(1070 \pm 20)\, m_e$, or greater. This value is well outside the standard error of the combined direct measurements on the tracks of particles $K7$, $K12$, and $K20$, which give a value $(981 \pm 23)\, m_e$. This argument rests on the assumption that we are studying a decay process, but it can be shown that any form of interaction involving a nucleus which could lead to the emission of a π-meson in the required energy region (Rossi, 1954) would require that the spectrum of the secondary emission energies have a width

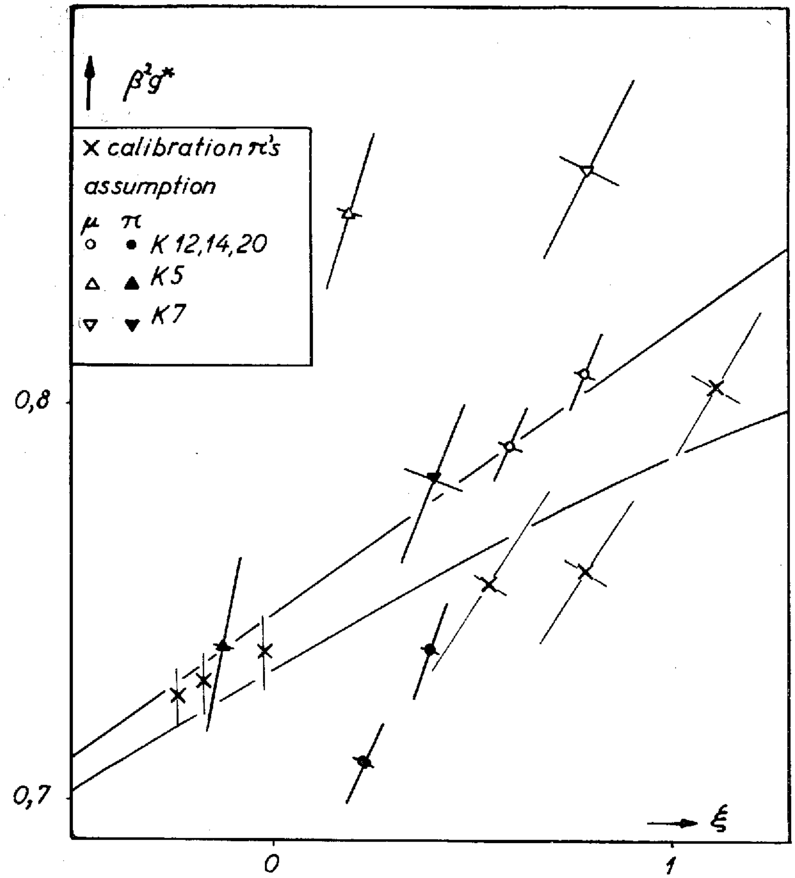

Fig. 3. $\beta^2 g^*$ vs. $\xi = ln \frac{\beta^2}{1-\beta^2} - \beta^2$ for the K-meson secondaries and π-meson calibration tracks. Each secondary particle is represented by two points; one calculated on the assumption that the secondary is a π-meson and the other on the assumption that it is a μ-meson. The curves represent what are believed to be limiting forms of the theoretical relationship (cf. Appendix). They are fitted at the low energy calibration points.

of about 50 MeV., due to the motion of the nucleons in the nucleus. This would be incompatible with the observed homogeneity of the group. It should be remarked, however, that the mean $p\beta c$ of π-mesons, arising from a reaction of the type observed by Rossi, $K^- + n \rightarrow \pi^- + \Lambda^\circ$, would be very close to that of the μ-mesons arising from K_μ decay. In view of the grain-density vs. scattering evidence that $K7$ is a π-meson (Fig. 3), it might well be possible to interpret this one event as being due to the above reaction. There is good evidence that most of the 18 K-mesons observed are, in fact, positive, since none has been found to be associated with an Auger electron at the end of its range. None of the secondaries was observed to interact in flight, but this evidence is poor, for the total path length in the emulsion was only just over one mean free path, assuming geometrical cross section.

The assumption that the secondary particles are μ-mesons, on the other hand, leads to a mass of the primary particle, calculated from the observed energy of emission of the secondaries, and assuming a decay scheme $K_{\mu} \rightarrow \mu + \nu$, of $(998 \pm 25)\, m_e$, in good agreement with the measured primary mass. The above decay scheme was proposed by GREGORY et al. (1954) for the so-called K_{μ} events which they had observed in their double cloud-chamber arrangement at the Pic-du-Midi. It must be remarked, however, that while their evidence suggests that the mass of the K_{μ}-meson is less than that of the τ-meson, our work suggests that it is the same or greater.

There remains the possibility that some or all of the secondary particles are electrons. This cannot be ruled out on the basis of grain density and scattering measurements for any of the secondaries except that of $K5$. There is, however, no indication of energy loss by bremsstrahlung, although the total track length measured corresponds to more than 8 radiation lengths. This may be seen from Fig. 4, in which the values of $p\beta c$ are plotted as a function of distance from the decay point. The shortest measured length was 2.2 cm., and the total length was 24 cm. The probability that any one secondary is an electron is less than 15 $^0/_0$ (EKSPONG, 1955), and if they are indeed a group of particles from events of the same type, the probability that they are all electrons is negligible.

The decay of $K5$, in which a π-meson of initial $p\beta c = (170 \pm 8)$ MeV. is emitted, is clearly an example of the χ-meson, which is believed to decay according to the scheme $\chi^+ \rightarrow \pi^+ + \pi^\circ$. Assuming this decay scheme and using the observed emission energy and the directly measured mass of the primary particle, the mass of the neutral secondary is found to be $(316 \pm 68)\, m_e$. The interpretation of a large number of events similar to $K5$ has recently

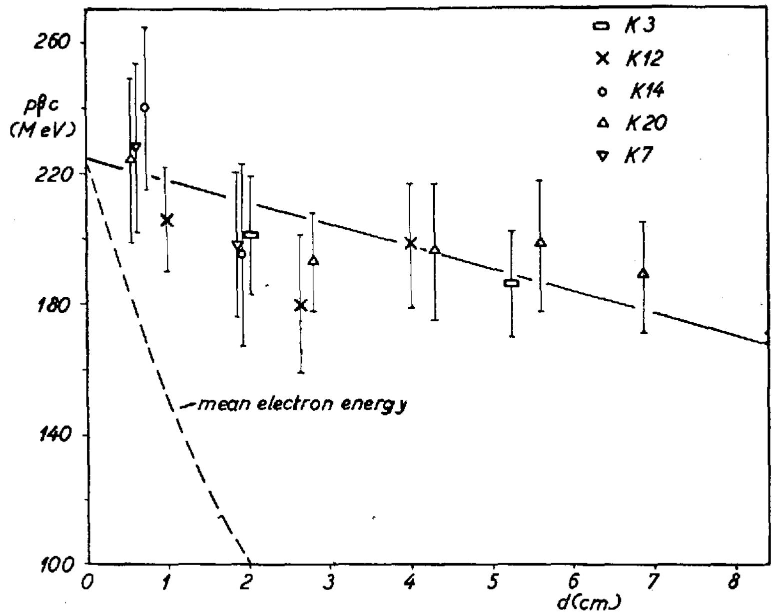

Fig. 4. $p\beta c$ as a function of the distance from the decay point, for the secondary particles. The full line, corresponding to ordinary ionization loss alone, fits the observations well. The dotted curve, based on the assumption that the secondaries are electrons, and taking energy loss by "bremsstrahlung" into account, is much too steep.

been strikingly confirmed by the observation by the Padua group of two examples in a very large stack of stripped emulsions in which the secondaries are brought to rest (Rostagni, private communication). Both in respect of the energy with which the secondary is emitted, and the directly measured mass of the decaying particle, $(996 \pm 34)\, m_e$, $K5$ is in good agreement with the observations hitherto reported (Dilworth et al., 1954).

V. The Production Stars.

All the K-mesons found in our stack of plates could be traced back to their parent stars, and the classification of each is included in Table I. The mean numbers of heavily and lightly ionizing particles emerging from these stars are very similar to those found by other workers, and no charged hyperons have been found to emerge from any of the six stars which were examined in detail.

The origin of $K15$ deserves special mention, for it was found to be produced without any other visible associated tracks. Similar origins have been found for K-mesons in Bristol and Bombay, and for a Y^+-particle at Rome. In view of the evidence for associated production obtained at Brookhaven by the Cosmotron group (FOWLER et al., 1954) it seems possible that these events may be of the type

$$n + p \rightarrow Y^\circ + K^+ + n \quad \text{or} \quad n + p \rightarrow Y^+ + K^\circ + n\,.$$

VI. Discussion and Comparison with other Results.

Evidence for a decay scheme $K_\mu \rightarrow \mu + \nu$ has previously been put forward by GREGORY et al. (1954) on the basis of certain S-events found in the lower chamber of a double cloud-chamber arrangement. Further evidence is provided by some of the events observed in the large multi-plate chamber at M.I.T. (ROSSI, 1954). In both instances two groups of particles were reported; one producing secondaries of range about 100 gm. cm.$^{-2}$ Pb, and another secondaries of range about 60 gm. cm.$^{-2}$ Pb. These particles can be shown to be L-mesons and, on the basis of the directly measured primary masses, the first group cannot be π-mesons, and are therefore assumed to be μ-mesons. The second group of secondary particles is identified with the π-mesons from the decay of the χ-meson, a process previously observed in the nuclear emulsion. The secondary momentum spectrum of slow, charged V-events observed in cloud chambers shows a definite peak, which probably includes both K_μ and χ secondaries. There is no evidence for μ-secondaries with a continuous energy spectrum. The cloud-chamber has provided no strong evidence for a negative counterpart to either the K_μ or the χ-meson.

On the other hand, emulsion work has provided evidence in favour of a particle, named by the Bristol workers the $\varkappa$-meson, which decays with the emission of a μ-meson whose energy varies within wide limits. There has hitherto been no very good

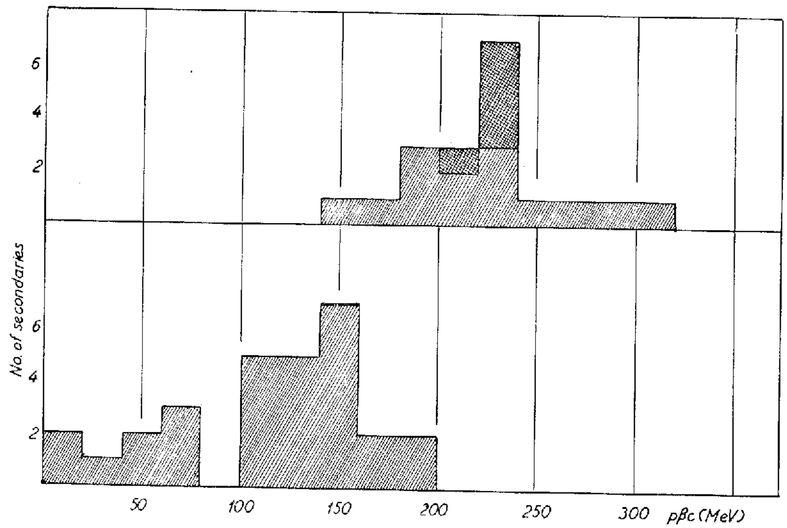

Fig. 5. The momentum spectrum of the secondaries of K-mesons, excluding those which are identified as π-mesons or electrons. The upper diagram includes only events which are likely to be examples of K_μ decay. The lower includes the remainder, and presumably represents the spectrum of secondaries from $\varkappa$ decay. The cross-hatched portion represents our present experimental results.

evidence for a decay of the K_μ type. This is not particularly surprising, for, at least in glass backed plates, it is very difficult to distinguish the peak due to the K_μ decays from the nearby, and possibly overlapping, spectrum of the secondaries of $\varkappa$-decay.

Fig. 5 shows the momentum spectrum of K-meson secondaries which have not been identified as π-mesons or electrons. We have used the collected results listed by Dilworth et al. (1954) together with our own. Most of these secondaries are probably μ-mesons, for the π-mesons from χ-decay have a grain density appreciably above minimum when they are emitted, and are therefore comparatively easily identified. The upper histogram includes secondaries whose $p\beta c$ at emission was within two standard deviations of 214 MeV., the value expected from K_μ decay if the primary mass is equal to that of the τ-meson. The lower histogram includes the remainder of the examples. To obtain an upper limit to the proportion of K_μ-mesons in the sample, one may assume that the upper diagram includes only K_μ events, and the lower diagram only $\varkappa$ events. Thus, the evidence obtained up to the time of the Padua conference would

TABLE I a. Primary Particles.

1 Event	2 Star	3 Total Length (mm.)	4 Length used for G. R. measurement (mm.)	5 Mass from $\bar{\alpha}$-R (m_e)	6 Mass from G-R (m_e)	Observer*
3 +	12 + 2p	3.0		1000 ± 300	1110 ± 160 ÷	A. K.
4	11 + 4p	21		1070 ± 180		M. W. H.
5	11 + 12p	30	15.7	850 ± 140	996 ± 29	»
6	6 + 1p	5.3	3.5	620 ± 110	971 ± 59	»
7	16 + 2n	44	14.0	1290 ± 210	998 ± 30	»
8	23 + 9p	15		920 ± 170		»
9	17 + 5p	1.7		610 ± 200		»
10	8 + 4p	8.9		1030 ± 200		»
11	13 + 1p	6.1		1000 ± 190		R. M. P.
12	17 + 2p	36	14.0	740 ± 130	969 ± 29	J. K. B.
13	24 + 3p	19		1100 ± 200		K. N.
14	6 + 2p	22		1200 ± 200		M. W. H.
15	1 + 0n	22		1090 ± 180		»
16	18 + 2n	25		800 ± 120		»
17	19 + 2p	11		1210 ± 300		R. M. P.
18	13 + 2n	11		940 ± 160		M. W. H.
19	5 + 2n	27		1050 ± 180		E. T.
20	3 + 3p	8.1	3.5	890 ± 140	968 ± 59	E. B.

\+ Found in an earlier stack.
÷ Measured photoelectrically by VON FRIESEN and his coworkers.
* Observers: Miss E. BACH, Dr. J. K. BØGGILD, Mr. M. WOLF HANSEN, Miss A. KOLDING, Miss K. NIELSEN, Miss R. MØLLER PEDERSEN, Miss E. TROLLE.

seem to suggest that $\varkappa$-mesons were rather more frequently stopped in the emulsion than K_μ-mesons.

While the statistical weight of our result is small, the fact that we have found five probable K_μ-decays and no $\varkappa$-mesons emitting a lower energy μ-meson among six identified events, would seem to require that the K_μ-meson be considerably more common than the $\varkappa$-meson. This is in accord with the results of the cloud-chamber workers, but is very different indeed from that obtained from the earlier emulsion work, discussed above. It is, of course, possible that the discrepancy is a result of different scanning efficiencies. If not, it would seem that a greater proportion of K_μ-mesons is stopped in the larger blocks of emulsion.

TABLE I b. Secondary Particles.

Event	7 Total Length (mm.)	8 Length used for $p\beta c$ measurement (mm.)	9 $p\beta c$ (MeV.)	10 b^*	11 $p\beta c$ at decay (MeV.)	12 $\oplus$	13 Identity
3+	63	52	199 ± 13	1.00 ± 0.01	220 ± 12	± 13	$(\pi, e)\ \mu$
4△	Nuclear Absorption						
5	58	55	141 ± 8	1.17 ± 0.01	170 ± 6	± 8	π
6	Steep						
7	32	26	218 ± 16	1.02 ± 0.02	221 ± 15	± 16	$(\mu, e)\ \pi$
8	Steep						
9	Steep						
10	Steep						
11	Steep						
12	46	44	199 ± 13	0.97 ± 0.01	215 ± 11	± 13	$(\pi, e)\ \mu$
13	Steep						
14	52	19	228 ± 15	0.98 ± 0.01	233 ± 14	± 15	$(\pi, e)\ \mu$
15	Steep						
16	Steep						
17	Steep						
18	23						
19	Steep						
20	78	69	197 ± 10	0.98 ± 0.01	227 ± 7	± 10	$(\pi, e)\ \mu$

$\oplus$ Error in $p\beta c$ at decay, including the uncertainty in the scattering constant.
△ BØGGILD et al., 1954.

This would imply that the K_μ-mesons are emitted with a higher mean kinetic energy than the $\varkappa$-mesons.

One major difference between our results and those of the cloud-chamber work is the mass of the K_μ-meson. The Pic-du-Midi group quote a best value of $(935 \pm 15)\ m_e$, obtained as a weighted mean between that measured directly, and that obtained indirectly from the assumed decay scheme and the momentum of emission of the secondary. Further, their value of $p\beta c$ at emission, (206 ± 1.5) MeV., is considerably lower than that which we have found. In both experiments there is approximate agreement between the directly and indirectly obtained masses on the basis of the assumed K_μ decay scheme.

In our work, the direct and indirect masses are completely independent. The $p\beta c$ of the secondary particles is obtained by measurements of the multiple scattering, and depends on the range-energy relation which was used in the determination of the scattering constant. While the range-energy relation in this region could be one or two per cent in error, it does not seem possible on this basis to bring our results down as low as those of the Paris group. Any distortion or noise which was not completely eliminated would increase the apparent scattering and produce too low a value of $p\beta c$, so that correction for such errors would increase the divergence between the two sets of experimental results. Alternatively, if the scattering constant is changed sufficiently to produce agreement, the value of $p\beta c$ with which the secondary of $K5$ is emitted would be reduced from (170 ± 6) MeV. to (153 ± 5) MeV. This would destroy completely the excellent identification of this good event as a χ-meson, and it could not then be identified with any well-established decay scheme.

The directly measured masses in our experiment, obtained from the measurements of mean gap-length, are entirely independent of the range-energy relation and, of course, of the scattering constant. The calibration tracks used for comparison included π- and μ-mesons on the one side, and protons on the other, and there is no evidence for any systematic divergence of the mean gap-length as a function of mass from the expected relation. In view of the complete independence of the direct and indirect measurements, the mean value, $(990 \pm 16)\, m_e$, is statistically significant. As it lies well above the accepted mass of the τ-meson, it is very improbable that the K_μ-meson is the lighter of the two.

In the cloud-chamber work, both the directly and the indirectly measured masses depend to some extent on the range-energy relation chosen. If the relation used has the correct shape, then any change in its absolute value will shift the directly and indirectly measured masses in opposite directions. However, an erroneous range-energy relation can produce an incorrect mass value if both its absolute value and its shape are wrong. Again, it must be emphasized that most cloud-chamber measurements on slow, charged V-particles lend support to the Pic-du-Midi results, rather than to the higher value which we have obtained.

It does not now seem possible to account for the discrepancy between the results of the two experiments, although the similarity in the mode of decay makes it seem extremely probable that the particles actually being studied are the same. A brief account of this work has already been published by Bøggild et al. (1955).*

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Mag. scient. N. Brene has assisted skilfully with the measurements. The contributions of the scanning group, recorded in Table I, are acknowledged with thanks.

* Note added in proof: – Since this paper was written the results presented at the recent International Conference at Pisa have confirmed that the K_μ-meson decays in the emulsion rather more frequently than the χ-meson, and that in large stacks the $\varkappa$-meson is much less frequently found. In addition, the evidence presented suggests that the mass of the K_μ-meson is witin a few electron masses of that of the τ-meson.

Appendix.

By M. Scharff.

It is a well-known fact that the identification of particles on the basis of grain-density and scattering measurements becomes increasingly difficult as the particle velocity, β, approaches unity. Near $\gamma = \frac{1}{\sqrt{1-\beta^2}} = 4$, where g^* as a function of energy has a broad minimum, π- and μ-meson tracks are indistinguishable. Still, at $\gamma = 2.5$, the region of interest in the present experiment, an identification is not quite hopeless, provided that the velocity dependence of g^* is accurately known. The best way to determine this relationship would be to measure the grain density of stopping π- and μ-meson tracks, for which the velocity is known from the range-energy relation. Due to the limited dimensions of our stack, however, the calibration could only be extended to $\gamma = 1.6$ by this method. For higher energy calibration tracks we were forced to use supposed π-mesons ejected from stars, relying upon the rather inaccurate values of β obtained from scattering measurements. There is necessarily some doubt about the identity of such particles, since a μ-meson may sometimes be ejected from a star, or may arise from an unobserved π-μ decay. It therefore seemed desirable to consider whether an extrapolation of the lower energy calibration might not be more reliable. The g^*-β curves obtained by various experimenters differ considerably in the minimum value of g^* and the value of γ for which saturation is reached, and previous experimental results do not provide a safe basis for the extrapolation from $\gamma = 1.6$ to $\gamma = 2.5$. Nevertheless, it turns out that the theoretical relation is, in this region, quite insensitive to rather widely different assumptions as to the mechanism of grain formation, and we feel it may be used with some confidence.

We shall first review the theory which has been developed by several authors. The grain density of a track produced by a particle of relativistic velocity is assumed to be proportional to the mean energy loss of the particle in emulsion (Fowler, 1950) or, more precisely, to the mean energy gained by those

AgBr crystals lying directly along the track (STILLER and SHAPIRO, 1953; MESSEL and RITSON, 1950). To calculate the latter quantity, one may then use the previous results of BETHE (1933) and MØLLER (1932) who calculated the energy loss, neglecting collisions with an energy transfer greater than a certain value, T', and obtained

$$\left(\frac{dE}{dR}\right)_{T'} = BNZ\left[ln\frac{2\,mc^2\cdot T'}{I^2} + ln\frac{\beta^2}{1-\beta^2} - \beta^2\right], \qquad (1)$$

where $B = \frac{2\,\pi e^4}{mc^2}\cdot 1/\beta^2$, Z is the atomic number, N the density of atoms and $I \approx (10\text{ eV.}) \times Z$ is the average excitation potential of the stopping medium. Formula 1 is only valid for γ of the order of unity. At higher energies the relativistic polarization, or "density" effect, of FERMI (1940) enters, causing $(dE/dR)_{T'}$ to saturate at a value which may be obtained from formula (1) by substituting $l = 2\,ln\frac{I}{\hbar\,\omega_0}$ for the velocity-dependent term $\xi(\beta^2) = ln\frac{\beta^2}{1-\beta^2} - \beta^2$. Here $\omega_0 = \left(\frac{4\,\pi e^2\,NZ}{m}\right)^{1/2}$ is the classical resonance frequency of the electrons in the medium. Taking for *AgBr* $I = 415$ eV. and $\hbar\omega_0 = 34$ eV., one finds $l = 5.0$.

The form of equation (1) suggests that we plot the quantity $\beta^2 g^*$, which may be termed the "reduced grain density", as a function of ξ. If g^* and $(dE/dR)_{T'}$ are in fact proportional, such a plot will, at low energies, be a straight line given by

$$\beta^2 g^* = \text{const} \times (L + \xi), \qquad (2)$$

where $L = ln\frac{2\,mc^2\cdot T'}{I^2}$, while at high energies

$$\beta^2 g^* = \text{const} \times (L + l). \qquad (2')$$

L may now be determined empirically from the value $\beta^2 g^* = 0.735$ at $\xi = -0.1$ measured on the stopping meson tracks, taking a saturation value $\beta^2 g^* = 1.0$. The result is $L = 14.3$, from which follows $\beta^2 g^* = 0.79$ at $\xi = 1.0$. This analysis may be refined by introducing a slowly-varying density correction, δ, (A. BOHR, 1948), and assuming $\beta^2 g^* \propto (L + \xi - \delta)$. This correction is included in curve 1 of Fig. 6, which is normalized to unity, not at $\xi \to \infty$, but in the energy region ($\xi \approx 8$—9) of the electron tracks chosen for standardization of the grain-density measurements. This refinement makes little difference to the curve, the value of L still being ≈ 14. Such a value for L, however, leads to the result $T' \approx 200$ keV., which is at variance with the usual interpretation of T' as the upper limit to the energy of struck electrons which can contribute appreciably to the ionization within the crystal. From this point of view, T' might reasonably be expected to lie between 2 and 10 keV., corresponding to L between 9.5 and 11, which in our stack would lead to a saturation value of $\beta^2 g^* = 1.1$.

Still, this apparent inconsistency can be removed without violating the assumption that grain density is proportional to local energy loss. A major part of the relativistic increase of $\beta^2 g^*$ arises from the effect of the transverse component of the field of the fast particle, and part of the energy lost in this way may be emitted as Čerenkov radiation. The fraction, α, absorbed locally depends on the damping of the atomic oscillators. If we use the theoretical value $L = 10$ and adjust α to fit the calibration point, we obtain curve 2 of Fig. 6, for which $\alpha = 0.70$. Without damping, ($\alpha = 0$), $\beta^2 g^* = 0.78$ at $\xi \to \infty$, while for strong damping, ($\alpha = 1$), $\beta g \to 1.1$, as previously mentioned. Actually, Sternheimer (1953) found $1 - \alpha = 0.02$ by a direct approximate calculation, so that it remains doubtful whether our data can be fitted by a consistent theory of this type. For this reason we shall discuss briefly a more general approach.

Let us assume that, for a given development, the probability that a crystal is developed is a function $f(\Delta)$ only of the energy, Δ, transferred to it. The grain density must then behave as $\int_0^\infty f(\Delta)\, \sigma(\Delta)\, d\Delta$, $\sigma(\Delta)$ being the cross section for an energy transfer Δ. The theory discussed above is a consequence of the rather special assumption $f(\Delta) \propto \Delta$. The general case of arbitrary $f(\Delta)$ can be analyzed by means of the distinction, introduced by N. Bohr (1948), between free collisions and resonance collisions, the latter containing the variation with ξ. It may then easily be shown that the reduced energy loss is still represented by expressions of the form (2) and (2′), although the values of L and

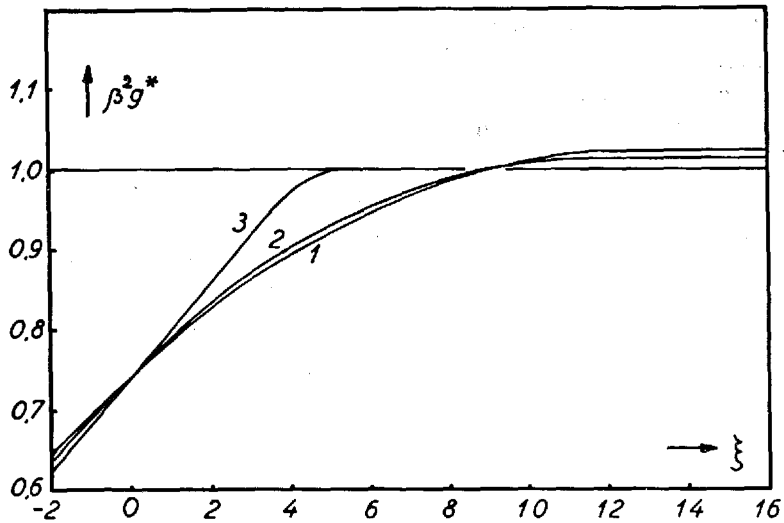

Fig. 6. The theoretical variation of $\beta^2 g^*$ as a function of ξ. The different curves result from the various possible initial assumptions about the mechanism of energy transfer to the silver bromide crystals and about grain formation, which are discussed in the text.

l will depend on $f(\Delta)$ as well as on the distribution of atomic oscillator strengths.

To test the sensitivity of the theoretical $\beta^2 g^*$ vs. ξ curve to the form of $f(\Delta)$, we choose a form which is extremely different from the previous one, viz.

$$f(\Delta) = \begin{cases} 0 \text{ for } \Delta < T_0 \\ 1 \text{ for } \Delta > T_0. \end{cases}$$

Then, g^* will vary as $\int_{T_0}^{\infty} \sigma(\Delta)\, d\Delta$, equivalent to the suggestion of Brown (1953) that a crystal is developed when the energy transferred to it exceeds a certain value T_0, which depends only on development. Curve 3 of Fig. 6 is the result of this assumption, calculated using the simple oscillator distribution of A. Bohr (1948) and assuming strong damping. T_0 was chosen to be 200 eV., to give an absolute plateau grain density of 10 per 50 μ. The curve is a straight line below $\xi = 3$, as in this region the polarization affects only atomic frequencies below $T_0/\hbar$. The slope of the line depends somewhat on the atomic model used, but an upper limit may be calculated from the extreme assumption that the rise is concentrated on oscillators of frequency $T_0/\hbar$.

Previous experimental results fall mainly into two groups. Stiller and Shapiro (1953), and Fleming and Lord (1953), fit their data with curves of the form (2) (or (1)), whereas the results of Pickup and Voyvodic (1950), Morrish (1952), Daniel et al. (1952), and Michaelis and Violet (1953) correspond more closely to type (3). From the general treatment of the formation of grains given above, one would not expect such large differences between different batches of plates, but it may well be that, allowing for small differences in experimental techniques, all the data could be fitted by one curve intermediate between (2) and (3). In any case, however, all the empirical curves are consistent with the general scheme, in the sense that each one can be accounted for by some particular choice of $f(\Delta)$, intermediate to the two rather extreme assumptions mentioned above. We are therefore fairly confident that this is also true for the extrapolated curve required in the present experiment.

In Fig. 3, the two limiting curves are fitted so as to deviate about one standard deviation on each side of the calibration points. The upper curve is of type (3) with maximum slope; the lower one is of type (2) with $T' = 1$ keV.

A fuller account of the Appendix will be published shortly.

References.

Amaldi, E., E. Fabri, T. F. Hoang, W. O. Lock, L. Scarsi, B. Touschek, and B. Vitale (1954). Nuovo Cimento **12**, suppl. no. 2, 419.

Baroni, G., C. Castagnoli, G. Cortini, C. Franzinetti, and A. Manfredini (1954). Bureau of Standards, C.E.R.N., Bull. No. 9.

Bethe, H. A. (1933). Handbuch der Physik, 2nd ed., $^{24}/_{1}$, p. 273.

Bohr, A. (1948). Dan. Mat. Fys. Medd. **24**, no. 19.

Bohr, N. (1948). Dan. Mat. Fys. Medd. **18**, no. 8.

Brown, L. M. (1953). Phys. Rev. **90**, 95.

Bøggild, J. K., J. E. Hooper, and M. Scharff (1954), Nuovo Cimento **12**, suppl. no. 2, p. 223.

Bøggild, J. K., J. E. Hooper, W. C. G. Ortel, and M. Scharff (1955). Nuovo Cimento **1**, 1267.

Bøggild, J. K. and M. Scharff (1954). Nuovo Cimento **12**, suppl. no. 2, 374.

Daniel, R. R., J. H. Davies, J. H. Mulvey, and D. H. Perkins (1952). Phil. Mag. **43**, 753.

Davies, J. and C. Franzinetti (1954). Nuovo Cimento **12**, suppl. no. 2, 481.

Della Corte, M. (1953). Nuovo Cimento **10**, 958.

Della Corte, M., M. Ramat, and L. Ronchi (1953). Nuovo Cimento **10**, 509.

Della Corte, M. (1954). Nuovo Cimento **12**, 28.

Dilworth, C. C., A. Manfredini, G. D. Rochester, J. Waddington, and G. T. Zorn (1954). Nuovo Cimento **12**, suppl. no. 2, 433.

Ekspong, A. G. (1955). Arkiv f. Fysik **9**, no. 53.

Fay, H., K. Gottstein, and K. Hain (1954). Nuovo Cimento **11**, suppl. no. 2, 234.

Fermi E., (1940). Phys. Rev. **57**, 485.

Fleming, J. R. and J. J. Lord (1953). Phys. Rev. **92**, 511.

Fowler, P. H. (1950). Phil. Mag. **41**, 169.

Fowler, W. B., R. P. Shutt, A. M. Thorndike, and W. L. Whittemore (1954). Phys. Rev. **93**, 861.

Fry, W. F. and G. R. White (1954). Phys. Rev. **93**, 1427.

Gottstein, K., M. G. K. Menon, J. H. Mulvey, C. O'Ceallaigh, and O. Rochat (1951). Phil. Mag. **42**, 708.

GREGORY, B., A. LAGARRIGUE, L. LEPRINCE-RINGUET, F. MULLER, and C. PEYROU (1954). Nuovo Cimento **11**, 292.
MESSEL, H. and D. M. RITSON (1950). Phil. Mag. **41**, 1129.
MICHAELIS, P. P. and C. E. VIOLET (1953). Phys. Rev. **90**, 723.
MORRISH, A. H. (1952). Phil. Mag. **43**, 533.
MØLLER, C. (1932). Ann. d. Phys. (5), **14**, 531.
O'CEALLAIGH, C. (1954). Nuovo Cimento **12**, suppl. no. 21, 412.
PICKUP, E. and L. VOYVODIC (1950). Phys. Rev. **80**, 89.
ROSTAGNI, A. Private Communication.
ROSSI, B. (1954). Varenna School lecture notes, unpublished.
STERNHEIMER, R. M. (1953). Phys. Rev. **91**, 256.
STILLER, B. and M. M. SHAPIRO (1953). Phys. Rev. **92**, 735.
VOYVODIC, L. and E. PICKUP (1952). Phys. Rev. **85**, 91.
WILLIAMS, E. J. (1940). Phys. Rev. **58**, 292.

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