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INTENSITY RULES FOR BETA AND GAMMA TRANSITIONS TO NUCLEAR ROTATIONAL STATES

BY

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I. Introduction.

The systematic occurrence of rotational spectra in strongly deformed nuclei implies many simple relations within the decay schemes of these nuclei. The great regularity in the properties of these states makes it possible to predict with considerable accuracy their energies and sequence of spins[1]. Moreover, the close relationship between the wave functions of the different states in a rotational sequence implies that the relative strengths of the transitions from a given nuclear state to the various members of a rotational family are governed by simple rules. In many cases, one obtains general quantitative relationships similar to the intensity rules for the fine structure and hyperfine structure of atomic spectra. These relationships may often be a valuable tool in the classification of decay schemes.

We shall here consider the intensity rules that apply to β- and γ-transitions in the deformed nuclei[2]. The available empirical evidence appears to be consistent with these rules and thus to lend further support to the assumed coupling scheme for these nuclei.

II. Spectra of Strongly Deformed Nuclei.

For nuclei whose equilibrium shape deviates strongly from spherical symmetry, one can distinguish approximately between two essentially different modes of excitation, rotational and intrinsic. The former is associated with a collective motion which

[1] Bohr and Mottelson, 1953a, 1953b, 1953c (the latter to be referred to in the following as BM); Ford, 1953; Asaro and Perlman, 1953. For a recent discussion of the theory and summary of the empirical data on nuclear rotational states, cf. A. Bohr, 1954; Bohr and Mottelson, 1954; Newton, 1954.

[2] Similar intensity rules governing transitions to the different members of a rotational sequence can also be given for the α-decay process (Bohr, Fröman, and Mottelson, 1955) and the stripping reaction (Satchler, 1955).

affects only the orientation in space while preserving the internal structure of the nucleus; the latter may be associated with the excitation of individual particles or with collective vibrations of the nuclear shape.

The rotational spectrum depends essentially on the nuclear equilibrium shape, and is especially simple for axially symmetric nuclei. The rotational motion can then be characterized by

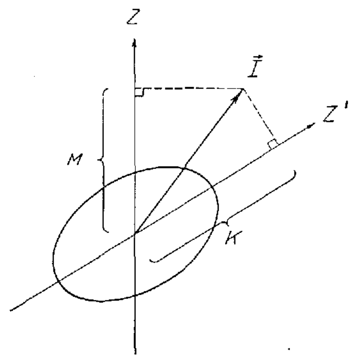

Fig. 1. *Angular momentum quantum numbers for a strongly deformed nucleus.* For strongly deformed nuclei possessing axial symmetry, the coupling scheme is characterized by the three constants of the motion: the total angular momentum, I, its projection, M, on an axis fixed in space (the z-axis in the figure), and its projection, K, on the nuclear symmetry axis, z'.

the quantum numbers I, K, M, representing the total angular momentum, its projection on the nuclear symmetry axis, and its projection on the space fixed axis, respectively (cf. Fig. 1).

The separation of the nuclear motion into rotational and intrinsic modes corresponds to the existence of approximate solutions of the nuclear wave equation of the simple product type

$$\Psi = \sqrt{\frac{2I+1}{8\pi^2}}\,\varphi_{\tau K}\,\mathfrak{D}^I_{MK}(\theta_i), \tag{1}$$

where φ represents the intrinsic structure characterized by K, and the additional set of quantum numbers, τ. For some purposes, one may attempt to describe the intrinsic wave function in greater detail in terms of the binding states of the individual nucleons in the deformed field and the collective vibrations of the nuclear shape (cf. the quantum numbers Ω_p, Ω, n_β, and n_γ employed in BM; more detailed calculations of single particle states in deformed potentials have been performed by NILSSON (1955) and by GOTTFRIED (1955)). Since, however, the intensity rules discussed below are independent of the intrinsic nuclear structure, it suffices in the present context to characterize this structure by the unspecified set of quantum numbers, τ.

The rotational wave functions $\mathfrak{D}(\theta_i)$, depending on the Eulerian angles θ_i of the nuclear coordinate system, are the proper functions for the symmetric top normalized so as to give a unitary transformation from the space fixed to the nuclear coordinate system. Due to the reflection symmetry of the nuclear shape, the complete wave function must be symmetrized by adding the term obtained from (1) by a rotation of 180° about an axis perpendicular to the nuclear symmetry axis (cf. BM, eq. II. 15).

The states in a rotational band are characterized by the same intrinsic wave function $\varphi_{\tau K}$ and are labeled by different values of I. In an odd-A nucleus, where K is a positive half integer number, I may take on the values

$$I = K, K+1, K+2, \ldots\ldots \quad \left.\begin{matrix}\text{all same parity as the}\\ \text{intrinsic structure.}\end{matrix}\right\} \tag{2}$$

In an even-even nucleus, the ground state has $K = 0$ and the symmetrization of the wave function limits the rotational band to

$$I = 0,\ 2,\ 4,\ 6 \ldots\ldots \quad \text{even parity.} \tag{3}$$

In an odd-odd nucleus or in excited intrinsic states of even-even nuclei with $K \neq 0$, the rotational sequence is again given by (2).

The energies of the states in a rotational band are given, apart from a constant, by the expression

$$E_I = \frac{\hbar^2}{2\mathfrak{J}} \left\{ I(I+1) + a(-)^{I+1/2}(I+1/2)\,\delta_{K,1/2} \right\}, \tag{4}$$

where the moment of inertia $\mathfrak{J}$ depends on the nuclear deformation and is thus expected to vary fairly smoothly with A, increasing as one moves away from closed shell configurations. The second term in (4), which occurs only for odd-A nuclei with $K = 1/2$, is associated with the symmetrization of (1). The parameter, a, can be expressed in terms of the properties of the intrinsic wave function (cf., e. g., BOHR and MOTTELSON, 1954, eq. 5a).

Rotational spectra of the type (4), characteristic of axially symmetric nuclei, are found to occur systematically in certain regions of elements (especially for $155 < A < 185$ and $A > 225$; for an example, cf. Fig. 2). The intensity rules discussed below thus apply particularly to these nuclei[1].

The energies associated with the excitation of individual particles depend on the level spacings for particle motion in the deformed field which are on the average a few hundred keV for a heavy nucleus. Intrinsic excitations of a collective vibrational character are in general expected to have somewhat higher energies, of the order of one or a few MeV.

Although intrinsic particle excitations and rotational excitations may have comparable energies, the two types of excitations can be rather easily distinguished, partly on the basis of the regularities in spins and energies in a rotational band, and partly on the basis of their essentially different transition probabilities.

The γ-transitions within a rotational family are of $E2$ and $M1$ type (cf. (2) and (3)). The $E2$ transition probabilities are strongly enhanced as compared with those corresponding to the transitions of a single proton; the enhancement which results from the collective nature of the nuclear quadrupole field increases with the nuclear deformation and is observed in some cases to exceed a factor of one hundred (BOHR and MOTTELSON, 1953a, 1954; BM, Chapter VII). The $M1$ rotational transition probability is related to the static magnetic moment of the nucleus and is of the order of magnitude of estimates for single-particle transitions, since the gyromagnetic ratio for the collective motion is of the same order of magnitude as that for single-particle motion (BM, Chapter VII).

In contrast, the transition probabilities for β- and γ-transitions between different particle configurations are usually smaller than those corresponding to single-particle transitions in a fixed spherical potential. Thus, the observed $M4$ γ-transitions and the allowed β-transitions are found to be retarded

[1] Outside these regions of elements, other regularities in the nuclear spectra have been observed, also suggestive of collective excitations (SCHARFF-GOLDHABER and WENESER, 1955; cf. also HEYDENBURG and TEMMER, 1954a). The spectra observed, however, differ essentially from (3) and (4), and exhibit features characteristic of collective vibrations about a spherical equilibrium shape.

by a factor (the unfavoured factor) in the range 10^{-1} to 10^{-2}. Such a retardation may result partly from the modification of the individual-particle wave functions implied by the non-spherical potential, and partly from a collective adjustment of the nuclear shape (BM, Chapters VII and VIII). A further retarda-

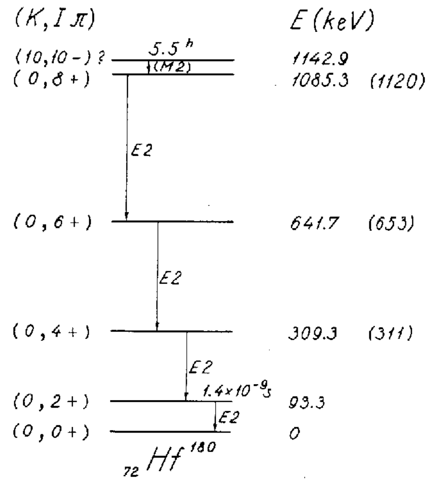

Fig. 2. *Decay scheme for 5.5 h isomeric state in* Hf^{180}. The empirical data are taken from Mihelich, Scharff-Goldhaber, and McKeown (1954). The internal conversion and angular correlation data are consistent with the spins and multipolarities shown in the figure. The states are labeled by the quantum numbers $(K,\ I\ \pi)$.

The observed excitation energies are listed in keV, and the values obtained from (3) and (4) with the moment of inertia adjusted to the first excited state are given in parenthesis. The small deviations from these calculated values are negative and have just the $I^2\,(I+1)^2$ dependence expected from the rotation-vibration interaction (Bohr and Mottelson, 1954).

The experimental evidence regarding the multipolarity of the 5.5 h isomeric transition is not conclusive. The very high degree of K-forbiddenness of this transition (cf. p. 17 below) implies that it should be many orders of magnitude slower than a single-particle transition of the same multipole order. While the observed lifetime may suggest an $M3$ or $E4$ classification on the basis of single-particle estimates, the expected retardation therefore suggests a lower multipole order. Thus, if the transition were $M2$, the retardation would amount to a factor of about 10^{-9}, which seems not excessive.

tion may result if the transition involves configurations of more than one particle (Moszkowski, 1953).

The simple separation between intrinsic excitations and collective rotations is realized in the limit of large deformations, where the rotational motion is so slow that it does not disturb the nucleonic configuration or distort the nuclear shape. With decreasing deformation, and increasing rotational frequency, the intrinsic nuclear structure is excited by the rotational motion, and the quantum numbers K, τ are no longer exact constants of the motion. This implies a modification in the rotational spectrum (4), which may often be described by a term proportional to $I^2 (I+1)^2$, as is characteristic of the rotation-vibration interaction in molecules. The magnitude of this correction term provides a measure of the adequacy of the rotational description. In the regions of the especially well developed rotational spectra ($155 < A < 185$ and $A > 225$), these correction terms in the energy amount to one per cent or less for the lowest rotational excitations (cf. Fig. 2, and also Bohr and Mottelson, 1954, especially Fig. 6 and Table II). More specific perturbations, associated with the particle-rotation coupling, may occur if the particle structure possesses a low lying state which is strongly excited by the rotational motion (cf. footnote on p. 16 below).

III. Classification of β- and γ-Transitions in Strongly Deformed Nuclei.

It is convenient to characterize β- and γ-transitions by their multipolarity, L, and their parity, π, representing the angular momentum and parity of the emitted radiation (cf., e. g., Rose, 1954).

For γ-transitions, this classification just corresponds to the usual classification in terms of electric and magnetic multipole orders (EL and ML). The parity is

$$\pi = \left\{ \begin{array}{ll} (-)^L & \text{for } EL \\ (-)^{L+1} & \text{for } ML. \end{array} \right\} \qquad (5)$$

For β-transitions, the classification of the various transition operators in multipole orders is given in Table I. The number in parenthesis is the order of forbiddenness, which is often used in the analysis of ft-values, but which may differ from the multipole order.

The selection rules on L and π for a transition from a state with quantum numbers $(K_i, I_i\,\pi_i)$ to a state with $(K_f, I_f\,\pi_f)$ are

$$|I_i - I_f| \equiv \Delta I \leqslant L \leqslant I_i + I_f \qquad \pi = \pi_i\,\pi_f \tag{6}$$

$$|K_i - K_f| \equiv \Delta K \leqslant L. \tag{6a}$$

While the selection rules involving I and π are rigorous, that involving K depends on the adequacy of the wave function (1). Small deviations from the rotational wave functions, of the type discussed at the end of Section II, will relax this latter selection rule, which then acts to retard, rather than completely forbid the corresponding multipole transition. We shall refer to such transitions as K-forbidden and characterize the degree of this forbiddenness by the number ν, where

$$\nu = \Delta K - L. \tag{7}$$

The consequences of K-forbiddenness are discussed further in Section V.

IV. Branching Ratios.

The strength of a nuclear β- or γ-transition of multipole order, L, between an initial state, i, and a final state, f, may be characterized by the reduced transition probability

$$B(L, I_i \rightarrow I_f) = \sum_{\mu M_f} |\langle I_i M_i | \mathfrak{M}(L, \mu) | I_f M_f \rangle|^2, \tag{8}$$

where $\mathfrak{M}(L, \mu)$ is the μ-component of the transition operator of multipole order L. For γ-emission of frequency ω, the transition probability per second is

$$T = \frac{8\pi(L+1)}{L[(2L+1)!!]^2}\,\frac{1}{\hbar}\left(\frac{\omega}{c}\right)^{2L+1} B(L). \tag{9}$$

Similarly, the cross section for Coulomb excitation is simply proportional to $B(L)$. For β-transitions of a given multipole order, the ft-value is inversely proportional to the reduced transition probability for the corresponding multipole operator (cf. Table I).

TABLE I.

		$\pi = +$		$\pi = -$	
L	0	1	(0)	$\vec{\sigma} \cdot \vec{r}$, $\beta\gamma_5$	(1)
	1	$\vec{\sigma}$	(0)	$\vec{r}$, $\vec{r} \times \vec{\sigma}$, $\vec{\alpha}$	(1)
	2	R_{ij}, A_{ij}, T_{ij}	(2)	B_{ij}	(1)
	3	B_{ijk}	(2)	R_{ijk}, A_{ijk}, T_{ijk}	(3)

Beta decay transition operators, classified according to multipole order, L, and parity, π.
The transition operators are written in the notation of KONOPINSKI and UHLENBECK (1941). The number in parenthesis gives the degree of forbiddenness.

For transitions between two states which can be represented by pure wave functions of the type (1), it is convenient to express the multipole operators in the coordinate system fixed in the nucleus

$$\mathfrak{M}(L, \mu) = \sum_{\nu} \mathfrak{D}^L_{\mu\nu}(\theta_i)\, \mathfrak{M}'(L, \nu), \tag{10}$$

where $\mathfrak{M}'(L, \nu)$ has the same form as $\mathfrak{M}(L, \nu)$ but is a function of the nucleon coordinates in the primed coordinate system (cf. Fig. 1).

The reduced transition probability then takes the form

$$B(L, I_i \to I_f) = \sum_{\mu, M_f} \left| \langle \tau_i K_i I_i M_i \left| \sum_{\nu} \mathfrak{D}^L_{\mu\nu} \mathfrak{M}'(L, \nu) \right| \tau_f K_f I_f M_f \rangle \right|^2. \tag{11}$$

The integration over the Eulerian angles, θ_i, appearing in the matrix element (11) can be performed explicitly and only the term with $\nu = K_i - K_f$ gives a non-vanishing contribution. The reduced transition probability can thus be written as a product of a geometrical factor, depending only on the angular momenta I, K, and L, and a factor involving integrations over the intrinsic wave function of the initial and final states and thus depending only on τ, K, and L.

For transitions between two members of a rotational family, the intrinsic wave functions of initial and final states are the same, and the intrinsic part of the matrix element reduces to an expectation value. The absolute transition probabilities can then be directly expressed in terms of the intrinsic nuclear moments (cf. BM, § VIIc. ii).

In transitions involving a change of the intrinsic nuclear state, the absolute value of the matrix elements depends on more specific features of the intrinsic nuclear structure. However, when one compares the reduced transition probability for the emission of a given multipole radiation from a state, i, to different members $f, f', \ldots\ldots$ of a rotational family, the factor involving the intrinsic wave functions is the same. One thus obtains a ratio which depends only on the geometrical factors and can be written

$$\frac{B(L, I_i \to I_f)}{B(L, I_i \to I_{f'})} = \frac{\langle I_i L K_i K_f - K_i \,|\, I_i L I_f K_f \rangle^2}{\langle I_i L K_i K_f - K_i \,|\, I_i L I_{f'} K_f \rangle^2}, \tag{12}$$

where $\langle I_i L K_i \nu \,|\, I_i L I_f K_f \rangle$ is the vector addition coefficient for the addition of the angular momenta I_i and L to form the resultant I_f (cf., e.g., Condon and Shortley, 1935). The relation (12), of course, also holds where the states i and f, f' belong to the same rotational family.

In special cases, where $L \geqslant K_i + K_f$, the symmetrization of (1) may imply that the transition matrix elements become a sum of two products of geometrical and intrinsic factors. The ratio of reduced transition probabilities can then be written in the form

$$\left.\begin{array}{c} \dfrac{B(L, I_i \to I_f)}{B(L, I_i \to I_{f'})} = \\ \left[\dfrac{\langle I_i L K_i K_f - K_i \,|\, I_i L I_f K_f \rangle + b(-)^{I_f + K_f} \langle I_i L K_i, -K_f - K_i \,|\, I_i L I_f - K_f \rangle}{\langle I_i L K_i K_f - K_i \,|\, I_i L I_{f'} K_f \rangle + b(-)^{I_{f'} + K_f} \langle I_i L K_i, -K_f - K_i \,|\, I_i L I_{f'} - K_f \rangle}\right]^2, \end{array}\right\} \tag{13}$$

where b is a parameter depending on the intrinsic wave function (cf. the similar parameter a in (4)). When either K_i or K_f is zero, (13) reduces to (12) regardless of L. A case of interest in which the parameter b enters significantly in the transition probabilities is that of $M1$ γ-radiation within a rotational sequence with $K = 1/2$[1].

The recent extensive use of the Coulomb excitation reaction for the study of nuclear rotational states has provided a number

[1] For collective $E2$ transitions within such a rotational sequence with $K = 1/2$, the quantity b vanishes.

TABLE II.

Nucleus	I_0	$E_{I_0+1}-E_{I_0}$	$E_{I_0+2}-E_{I_0}$	$\frac{B(E2;\, I_0\to I_0+2)}{B(E2;\, I_0\to I_0+1)}$	$\frac{B(M1;\, I_0+2\to I_0+1)}{B(M1;\, I_0+1\to I_0)}$	References
$_{45}Rh^{103}$	1/2	295	357	1.8 (1.5)	—	a)
$_{47}Ag^{107}$	1/2	320	415	1.7 (1.5)	—	a)
$_{47}Ag^{109}$	1/2	306	405	1.8 (1.5)	—	a)
$_{63}Eu^{153}$	5/2	84	195 (192)	0.4 (0.35)	~ 2 (1.45)	b) c)
$_{65}Tb^{159}$	3/2	58	138 (139)	0.3 (0.56)	—	b) c)
$_{67}Ho^{165}$	7/2	94	209 (209)	0.26 (0.26)	~ 2 (1.53)	b) c)
$_{71}Lu^{175}$	7/2	114	248 (252)	—	~ 1 (1.53)	b) c)
$_{72}Hf^{177}$	(7/2)[1]	113	250 (251)	0.20 (0.26)	—	b) c) d)
$_{73}Ta^{181}$	7/2	136	303 (304)	0.21 (0.26)	—	b) e) f)
$_{74}W^{183}$	1/2	46	99	~ 1 (1.5)	—	c) f) g)
$_{79}Au^{197}$	3/2	277	555 (665)	2 (0.56)	—	b) h) i)

[1] The spin of Hf^{177} is given as 7/2, as indicated by the observed rotational level spacings and the observed relative intensities. Earlier spectroscopic evidence (RASMUSSEN 1935) had tentatively indicated a spin value 1/2 or 3/2.

a) HEYDENBURG and TEMMER (1954).
b) HEYDENBURG and TEMMER (1954a).
c) HUUS et. al. (1955).
d) MARMIER and BOEHM (1955).
e) HUUS and ZUPANČIČ (1953).
f) MCCLELLAND, MARK, and GOODMAN (1955).
g) MURRAY et al. (1955).
h) COOK, CLASS, and EISINGER (1954).
i) GOLDBURG and WILLIAMSON (1954).

Relative transition intensities from Coulomb excitation data.

In the Coulomb excitation of odd-A nuclei, one strongly excites the two lowest members of the ground state rotational band, having spins I_0+1 and I_0+2, where I_0 is the ground state spin. The energies of the states so populated are given in columns three and four, and the value of $E_{I_0+2}-E_{I_0}$ calculated by means of (4) from the observed $E_{I_0+1}-E_{I_0}$ is listed in parenthesis in column four. The nuclei with $I_0=K=1/2$ exhibit the effect of the second term in (4). From the observed cross sections for the excitation of the two states in question, one may obtain the ratio of the $E2$ reduced transition probabilities listed in column five; the theoretical ratio (12) is listed in parenthesis. The study of the radiative de-excitation has permitted in some cases an estimate of reduced $M1$ transition probabilities; the observed ratios are listed in column six, together with the theoretical value (12) (in parenthesis). For a discussion of the derivation of the reduced transition probabilities from the Coulomb excitation cross section, cf. the forthcoming review article by ALDER et al. (1955).

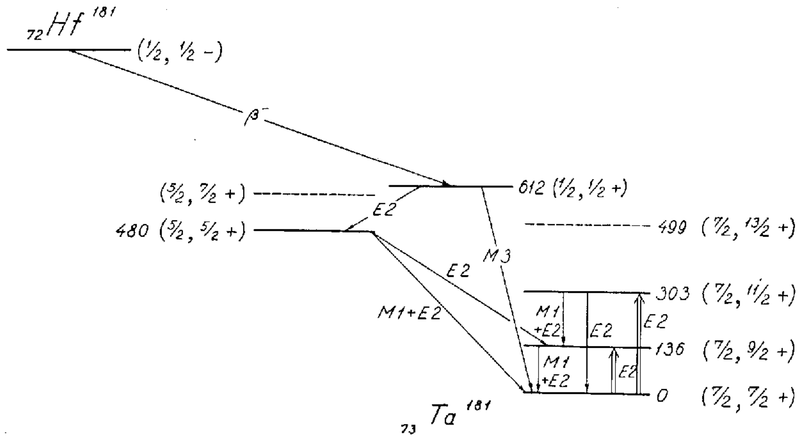

Fig. 3. *Level scheme for Ta*^{181}. The figure illustrates the information on the level structure of Ta^{181} obtained from the analysis of the β-decay of Hf^{181} (McGowan, 1954) and of the Coulomb excitation (double arrows) (Huus and Zupančič, 1953; Huus and Bjerregård, 1953; Eisinger, Cook, and Class, 1954). These reactions appear to populate states belonging to three different rotational bands, and this is indicated by drawing members of the same bands (having same K and π) above each other, while the different bands are displaced sideways. The excitation energies are listed in keV, and the states are labeled by the quantum numbers (K, $I\ \pi$). The absolute parity is uncertain, and the value relative to that of the ground state is given. The levels drawn dotted are not populated in these reactions, but their position is inferred from formulae (2) and (4). The spin assignments for the 480 keV and 612 keV levels tentatively suggested here differ from those of McGowan (1954), but are consistent with the angular correlation and conversion data given in this reference and, in addition, seem more compatible with the long lifetime for the 612 keV γ-transition which is here assigned the multipolarity $M3$, rather than $M1$.

In this level scheme, one may compare in a number of cases relative transition probabilities to two different members of the ground state rotational band.

a) The relative cross sections for the population of the (7/2, 9/2+) and (7/2, 11/2+) levels in the Coulomb excitation process depend on the ratio of the two $E2$ reduced transition probabilities, $B(E2;\ 7/2,\ 7/2 \rightarrow 7/2,\ 11/2)$ and $B(E2;\ 7/2,\ 7/2 \rightarrow 7/2,\ 9/2)$, which according to (12) should be 0.26. The observed value for this ratio is about 0.2.

b) The $M1$ transition probability in the (7/2, 11/2+) → (7/2, 9/2+) transition can be determined from the data discussed in a), together with the observed branching ratio for this transition in competition with the (7/2, 11/2+) → (7/2, 7/2+) $E2$ cross-over transition. By means of (12) one can then calculate the $M1$ transition probability for the (7/2, 9/2+) → (7/2, 7/2+) transition, and one finds that this transition should have an $E2$ intensity of 10–20 %. From the angular correlation data of McGowan (1954), interpreted in terms of the spin assignments given in the figure, one obtains an $E2$ admixture in this transition of about 7 %; attenuation effects due to quadrupole coupling may somewhat increase this value. The K-shell internal conversion coefficient is also consistent with an $E2$ admixture of the order of 10 %.

A knowledge of the amount of $E2$ radiation in the (7/2, 11/2+) → (7/2, 9/2+) transition would make possible a further test of the intensity rules, by comparison with the transitions discussed under a).

c) The relative strength of the $E2$ transitions from the 480 keV level with $K = 5/2$ to the two first members of the ground state rotational band give the ratio $B\,(E2;\,5/2,\,5/2 \rightarrow 7/2,\,7/2)\!:\!B\,(E2;\,5/2,\,5/2 \rightarrow 7/2,\,9/2)$. The value calculated from (12) is 1.25 while the observed ratio is about 1.1, assuming the 480 keV transition to be 85 $^0/_0$ $E2$ and 15 $^0/_0$ $M1$, as indicated by the angular correlation data.

The large $E2\!:\!M1$ ratio in the 480 keV transitions may be ascribed to a coupling between the two rotational bands, which results in a small admixture of (5/2, 7/2+) to the Ta ground state, with a consequent great enhancement of the $E2$ transitions. While in general such rotational admixtures may affect the intensity rules, in the present case of $\Delta K = 1$ and $E2$ radiation, the rules are not affected, provided one assumes that the intrinsic quadrupole moment is about the same in the two rotational bands (cf. pp. 19 ff.). From the observed lifetime of the 480 keV level one can estimate the squared amplitude of admixture to be about 10^{-3}.

of tests of the intensity rules (12) as applied to the transitions within a rotational band. Some of the general features of the decay schemes studied in this manner are illustrated in Figs. 3 and 4, and a summary of the available data from this source is given in Table II. The intensity rule has also been tested by the evidence on the relative lifetimes of the 2+ and 4+ rotational states in $_{88}\mathrm{Ra}^{226}$ as deduced from the relative attenuation of the α—γ correlation associated with the two states (Falk-Vairant, Teillac, Valadas, and Benoist, 1954).

The intensity rules (12) applied to transitions involving a change in the intrinsic structure are illustrated by the β-decay branching ratios of the odd-odd nuclei $_{69}\mathrm{Tm}^{170}$ (cf. Fig. 5), $_{45}\mathrm{Rh}^{106}$, and $_{47}\mathrm{Ag}^{106}$ (cf. Fig. 6), and by the γ-transition branching ratios in the even-even nucleus $_{74}\mathrm{W}^{182}$ (cf. Fig. 7).

V. Effect of Perturbations.

Although the evidence on rotational spectra suggests that in the regions of the strongly deformed nuclei the wave functions (1) give a good representation of the nuclear states, the expected small deviations from this description may sometimes be significant.

Such deviations may be described by a wave function of the form

$$\Psi = \sqrt{\frac{2I+1}{8\pi^2}}\left\{\varphi_{\tau K}\,\mathfrak{D}^I_{MK} + \sum_{\tau' K'} \epsilon_{\tau' K'}\,\varphi_{\tau' K'}\,\mathfrak{D}^I_{MK'}\right\}, \tag{14}$$

where the expansion amplitudes $\epsilon_{\tau' K'}$ can be estimated from the effect of the perturbation terms, representing the partial decoupling of the rotational from the intrinsic motion (cf. A. Bohr, 1952, eq. 96). Thus, the decoupling of the last odd particle gives rise, in first order ($K' = K \pm 1$), to terms in (14) with

$$\left.\begin{aligned} e_{\tau' K \pm 1} &= \frac{1}{E_{\tau K I} - E_{\tau' K' I}} \langle \tau K I \,|\, -\frac{\hbar^2}{\Im}(I_{x'} j_{x'} + I_{y'} j_{y'}) \,|\, \tau' K \pm 1\, I \rangle \\ &= -\frac{1}{\Delta E}\frac{\hbar^2}{2\Im}\sqrt{(I \mp K)\,(I \pm K + 1)}\, \langle \tau K \,|\, j_{x'} \mp i j_{y'} \,|\, \tau' K \pm 1 \rangle, \end{aligned}\right\} \quad (15)$$

where $\Delta E = E_{\tau K I} - E_{\tau' K' I}$ is the energy difference between the unperturbed states. The x' and y' axes are perpendicular to the intrinsic nuclear symmetry axis (cf. Fig. 1) and $\vec{j}$ is the angular momentum of the particle.

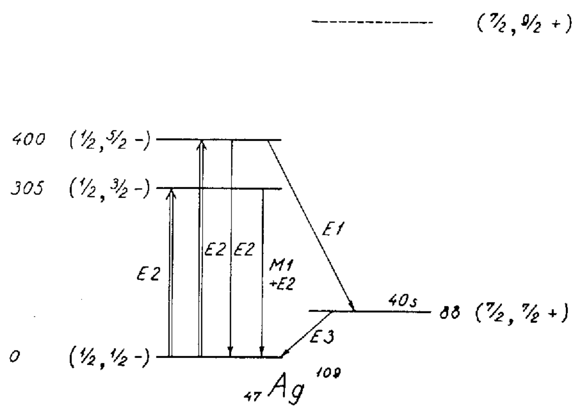

Fig. 4. *Level scheme for* Ag^{109}. The figure illustrates the information on the level structure of Ag^{109} obtained from Coulomb excitation (Huus and Lundén, 1954; Heydenburg and Temmer, 1954). The notation is similar to that of Fig. 3.

The negative parity levels provide an example of a rotational series with $K = 1/2$, in which the second term in (4) gives rise to anomalous spacings. The decoupling parameter a deduced from the observed levels has the value 0.67.

The relative cross sections for the population of the (1/2, 3/2—) and (1/2, 5/2—) levels in the Coulomb excitation yield the ratio $B\,(E2;\, 1/2,\, 1/2 \rightarrow 1/2,\, 5/2)$: $B\,(E2;\, 1/2,\, 1/2 \rightarrow 1/2,\, 3/2) = 1.8$ to be compared with the value 1.50 obtained from (12).

The 88 keV level belongs to the well-known class of 7/2+ isomeric states found in this region of elements. It is populated in the Coulomb excitations by a weak $E1$ branch from the 400 keV level. The observed branching ratio from this level to the ground state and the isomeric state can be combined with the cross section for Coulomb excitation to yield a value for the reduced transition probability for the $E1$ γ-ray of $B\,(E1;\ 5/2 - \rightarrow 7/2+) = 2 \times 10^{-32} e^2\,\mathrm{cm}^2$. This value is smaller by a factor of about 10^6 than the value corresponding to a single-particle $E1$ transition (cf., e.g., Blatt and Weisskopf, 1952, p. 627). The reduction may be, at least partly, ascribed to the K-forbiddenness of the transition.

Similar level structures have been observed for Ag^{107} and Rh^{103} (Heydenburg and Temmer, 1954); cf. Table II.

The intrinsic matrix element appearing in the final expression (15) is of similar type as those which enter in the probabilities for nuclear transitions involving a change in the particle configuration. While a quantitative evaluation is difficult, such matrix elements are known to involve unfavoured factors (cf. p. 7

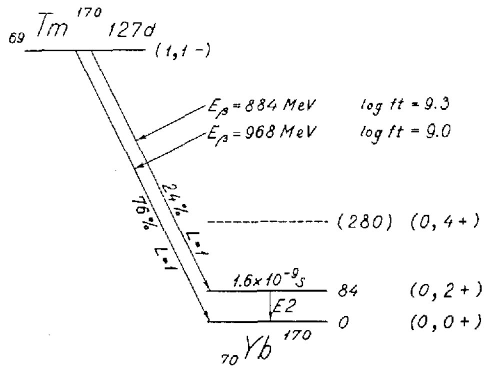

Fig. 5. *Beta decay of* Tm^{170}. The experimental data for the Tm^{170} decay are taken from GRAHAM, WOLFSON, and BELL (1952). The notation in the figure is the same as in Fig. 3. The 84 keV level in Yb^{170} can be identified as the first rotational excitation on the basis of its short lifetime.

The experimental ratio of 1.9 ± 0.2 for the ft-values of the two branches of the β-decay is in good agreement with the value 2.0 obtained from (12), assuming Tm^{170} to have $I = 1$ and the β-transitions to be of multipole order $L = 1$ (cf. Table I).

In the region of the well developed rotational spectra, similar branching ratios have been observed in the decays of Ho^{164} (BROWN and BECKER, 1954), Re^{186} (METZGER and HILL, 1951; STEFFEN, 1951; KOERTS, 1954) and Np^{236} (PASSELL, 1954). In two additional cases (Lu^{176} (GOLDHABER and HILL, 1952) and Ta^{180} (BROWN, BENDEL, SHORE, and BECKER, 1951)), there is some evidence for an odd-odd nucleus of spin 1 which β-decays to the ground state and first excited state of the neighbouring even-even nucleus, but with an ft-ratio appreciably smaller than the expected value of 2; in these cases, however, the assignments seem not to have been definitely established.

above) which may imply a rather small value for ϵ ($\epsilon^2 \sim 10^{-1}$ to 10^{-2}) even in cases where ΔE is of the order of rotational energies[1].

[1] *Note added in proof:*

The recent high precision measurements on the W^{183} level structure (MURRAY, SNELGROVE, MARMIER, and DUMOND, 1954; MURRAY, BOEHM, MARMIER, and DUMOND, 1955) have revealed deviations from the rotational spectrum (4) of the magnitude of a few per cent. The observed deviations cannot be accounted for in terms of a rotation-vibration interaction and indicate a relatively large coupling to an excited particle configuration. The inclusion of a particle-rotation coupling of the type (15), acting between the ground state band and the first excited band, has made possible an interpretation of these effects (KERMAN, 1955). The squared amplitudes of the admixed states are found to be of the order of ten per cent and their unusually large magnitude in this case affects in an important way the transition intensities.

Higher order terms involving $|K-K'| > 1$ will be appreciably smaller, decreasing rather rapidly with the change in K.

The admixed states in (14) have an especially important effect on the intensity rules if either the unperturbed matrix element is very small (as is, for instance, often the case with low energy $E\,1$ radiation) or even vanishes (as for the K-forbidden transitions; cf. below), or if the admixed amplitude is associated with an especially large matrix element (cf. the rotational admixtures discussed below).

a) *K-forbiddenness.*

The above estimate of the deviations from the wave function (1) indicates that K-forbidden transitions (cf. p. 9) may be appreciably retarded, but that this retardation is usually not large enough to alter the predominant multipole order of a nuclear transition (cf., however, the γ-transitions in W^{182} between the $K = 2-$ and $K = 0+$ rotational bands (Fig. 7)). It will, however,

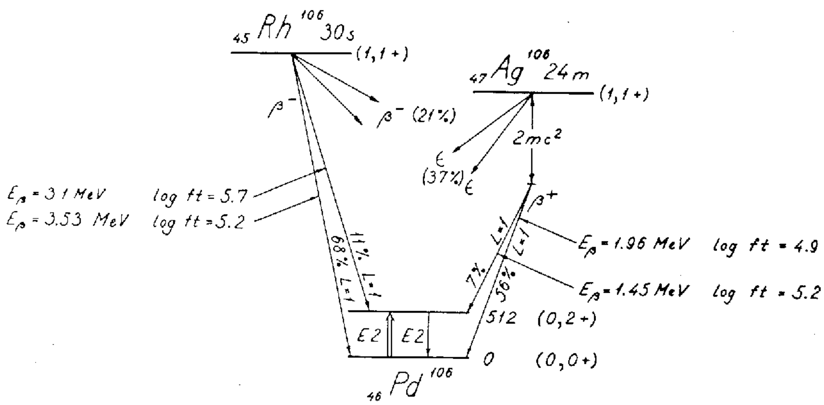

Fig. 6. *Rotational branchings in the β-decay of Rh^{106} and of Ag^{106}.* The notation in the figure is the same as used in Fig. 3. The experimental data for the Rh^{106} decay are taken from Alburger (1952) and for the Ag^{106} decay from Bendel, Shore, Brown, and Becker (1953), who besides the transitions shown have also found evidence for β-transitions from Rh^{106} and electron capture transitions from Ag^{106} to higher states in Pd^{106}. We here only consider the branching between transitions to the ground state and the 512 keV first excited 2 + state. Assuming the quantum numbers indicated in the figure, the ratio of ft-values for the transitions to the first excited state and the ground state should be 2.0 (cf. (12)), which is in fairly good agreement with the measured ratios of about 2 for the Ag^{106} decay and about 3 for the Rh^{106} decay. There is, however, evidence from the higher excited states in Pd^{106} that the coupling scheme may be rather different from that of Fig. 1 (cf. footnote on p. 6).

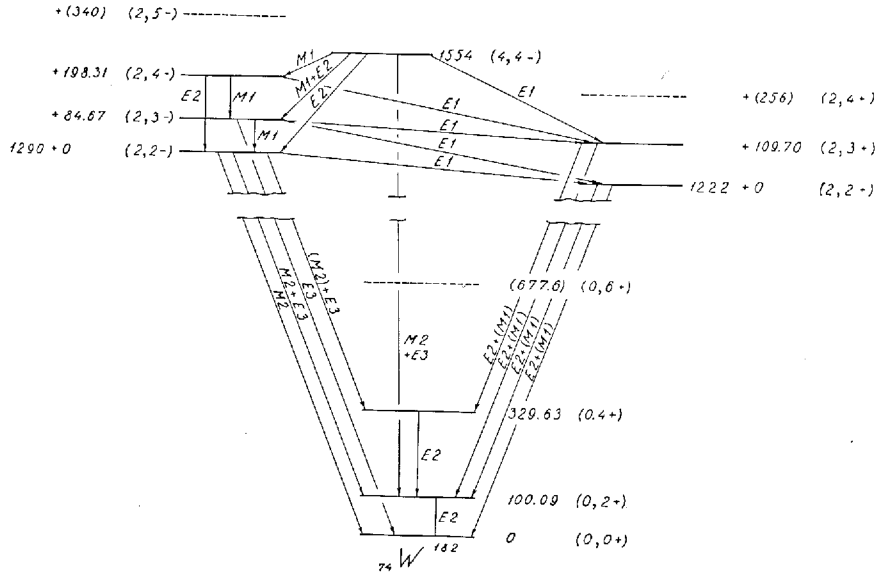

Fig. 7. *Level scheme for* W^{182}. The notation is the same as that used in Fig. 3. The experimental data, which are obtained from a study of the radiation following the β-decay of Ta^{182}, is taken from BOEHM, MARMIER, and DUMOND (1954) (cf. also MIHELICH, 1954; FOWLER, KRUSE, KESHISHIAN, KLOTZ, and MELLOR, 1954). Apart from the indicated levels, there is evidence for two very weakly populated states at energies of about 1255 keV and 1437 keV. Otherwise, the level scheme drawn above differs from that given by BOEHM et. al. only in the spin assignments of the 1331 keV and 1554 keV levels; the assignments suggested here are, however, also in agreement with the data in this reference.

The level scheme is interpreted as involving primarily states associated with four rotational series. The first series, comprising the three lowest levels, is the systematically occurring $K = 0+$ ground state rotational band of even-even nuclei. (For lifetime and Coulomb excitation of the 100 keV level, cf. SUNYAR, 1954; HUUS and BJERREGÅRD, 1953; MCCLELLAND, MARK, and GOODMAN, 1954). The calculated energy of the $6+$ member of this band includes a small correction for the rotation-vibration interaction, as deduced from the observed energies of the $2+$ and $4+$ states.

The second series, beginning with the 1222 keV level, has $K = 2+$ and a moment of inertia rather close to that of the ground state band, although a little smaller. The third series, beginning with the 1290 keV level, has $K = 2-$ and a moment of inertia again of the same order of magnitude as that of the ground state, but in this case somewhat larger. This larger moment is in agreement with a general tendency observed for configurations, as in odd-A nuclei, having particles in addition to those filled in pairs. (Cf. BOHR and MOTTELSON, 1954; BOHR, FRÖMAN, and MOTTELSON, 1955). In this view, the somewhat smaller moment of inertia for the $2+$ series might indicate an intrinsic excitation of collective vibrational type leaving the particles in a paired configuration (cf. further below).

The β-decay of Ta^{182} further populates a level at 1554 keV, which seems to be the lowest member of a fourth rotational band.

The measured γ-intensities provide a number of tests of the intensity rules (12):

a) The branching ratio of the 1222 keV level to the ground state and 100 keV levels yields $B(E2; 2{,}2 \rightarrow 0{,}0) : B(E2; 2{,}2 \rightarrow 0{,}2) = 0.62$, assuming pure $E2$ radiations for both transitions. The ratio calculated from (12) is 0.70. The amount of $M1$ radiation observed in these high energy transitions appears small, but is difficult to determine quantitatively. According to (12), the intensity of the 893 keV $E2$ transition from the 1222 keV level to the 329 keV level, which has so far not been observed, should be 1.5 % of the ground state transition.

b) The branching ratio of the transitions from the (2,3+) level to the 100 keV and 329 keV levels gives $B(E2;\ 2{,}3 \rightarrow 0{,}2) : B(E2;\ 2{,}3 \rightarrow 0{,}4) = 2.1$, to be compared with the theoretical value 2.5.

The observation of rather pure $E2$ radiation in the $\Delta I = 1$ transitions between the $K = 2+$ and $K = 0+$ rotational bands may be understood in terms of the K-forbiddenness for $M1$ radiation. The $K = 2+$ band may possibly represent vibrational excitations of the ground state; such an interpretation would account for the strength of these transitions, as reflected in the fact that the $(2{,}3+) \rightarrow (2{,}2+)$ rotational transition is too weak to be observed. A crucial test of the vibrational character of these levels would be provided by a determination of the cross section for Coulomb excitation of the (2,2+) level.

c) The ratio of the intensities of the $E1$ transitions from the (2,3—) level to the (2,3+) and (2,2+) levels is calculated from (12) and (9) to be 0.031:1. While the low energy transition has been detected, its weak intensity has so far prevented a quantitative determination of this branching ratio.

d) The assignment of quantum numbers in the figure also provides an interpretation of the observation that the $\Delta I = 0$ and 1 transitions between the $K = 2-$ and $K = 0+$ rotational bands appear to be mainly of $M2$ or $E3$ type, rather than $E1$, which is K-forbidden.

Such a K-forbiddenness may also account for the fact that no β-transitions are observed directly to the members of the $K = 0+$ ground state rotational band.

lead to increased admixtures of higher multipole components, which may be especially significant in the so-called parity unfavoured transitions (γ-transitions with $\pi = (-)^{\Delta I + 1}$ and β-transitions with $\pi = (-)^{\Delta I}$ ($\Delta I = 0$ excepted)), where mixed multipole transitions are most likely to occur.

An example of a highly K-forbidden transition ($\nu = 8$) where a large retardation factor is expected is provided by the 5.5 h isomeric transition in Hf^{180} (cf. Fig. 2). Another such example is the β-decay of Lu^{176}, where the high order of K-forbiddenness ($\nu = 5$ or 6) suggests the interpretation of the observed $\log ft = 18.9$ as a second forbidden transition with an unfavoured factor of the order of 10^{-7}.

b) *Rotational admixtures.*

In the calculation of nuclear transition probabilities, small admixtures in the wave functions (1) may be significant, even where there is no K-forbiddenness, if the associated transition matrix element is large. Thus, the large transition matrix element for $M1$ and $E2$ radiation within a rotational family (cf. p. 6) implies that, in a transition involving a change in the particle structure, special significance attaches to the admixtures to one of the combining states of a wave function belonging to the rotational family of the other. Especially for $E2$ radiation, these rotational admixtures are expected to contribute a significant, and in many cases a dominant part of the transition matrix element (especially for $\Delta K = 1$, where the admixed amplitudes are largest (cf. pp. 14 ff.)).

Since the γ-transitions within a rotational family are generally of mixed $E2 + M1$ character for $\Delta I = 1$, the rotational admixtures may lead to mixed multipole radiation also for transitions associated with a change in the intrinsic nuclear structure and having $\Delta I = 0$ or 1 and $\pi = +$.

Another consequence of the rotational admixtures may be to modify the relative intensity rules (12) and (13) for $M1$ and $E2$ γ-radiation. In certain cases, the modified rules may be derived by considering the I-dependence of the admixed amplitudes.

For $\Delta K = 1$, however, it is found that the intensity rules for the $E2$ radiation are not affected if the nuclear deformations for the two states are approximately the same. An interesting example is provided by the decay of the 480 keV state in Ta^{181} (cf. Fig. 3). The squared amplitude of the rotational admixture is here of the order of 10^{-3}, but is nevertheless mainly responsible for the $E2$ radiation as evidenced by the strong admixture of $E2$ in the 480 keV $\Delta I = 1$ transition.

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