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A SIX GAP
β-RAY SPECTROMETER

BY

O. B. NIELSEN AND O. KOFOED-HANSEN

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1. Introduction.

The theory of a new type of β-ray spectrometer has been developed previously[1)] for the purpose of more efficiently utilizing a radioactive source than has been possible so far. In principle, this can be accomplished by letting several double focusing spectrometers of a conventional type operate on the same source, and preferably having the same focus. However, a more suitable solution was found in a design where the individual spectrometers consist of a number of air gaps in the same electromagnet. Source and focus are situated in field-free space, and the gaps are arranged symmetrically around the line connecting them.

A preliminary investigation of a spectrometer consisting of a single air gap has also been described.

Since then we have constructed a spectrometer with six air gaps. The instrument is characterized by a large transmission and is especially well suited for coincidence experiments with scintillation counters, as the operation of the multiplier tubes is undisturbed by magnetic fields.

The present paper gives a description of some experience with this spectrometer which has now been operating for more than a year. In addition, a few possible new applications of the focusing principle will be mentioned.

2. The electron-optical system.

Let us first repeat the most essential features of the principle of focusing.

The instrument is shown in Figs. 1a and 1b. The electromagnet consists of six segments placed as slices of an orange, each occupying 40° of the entire circle around the symmetry

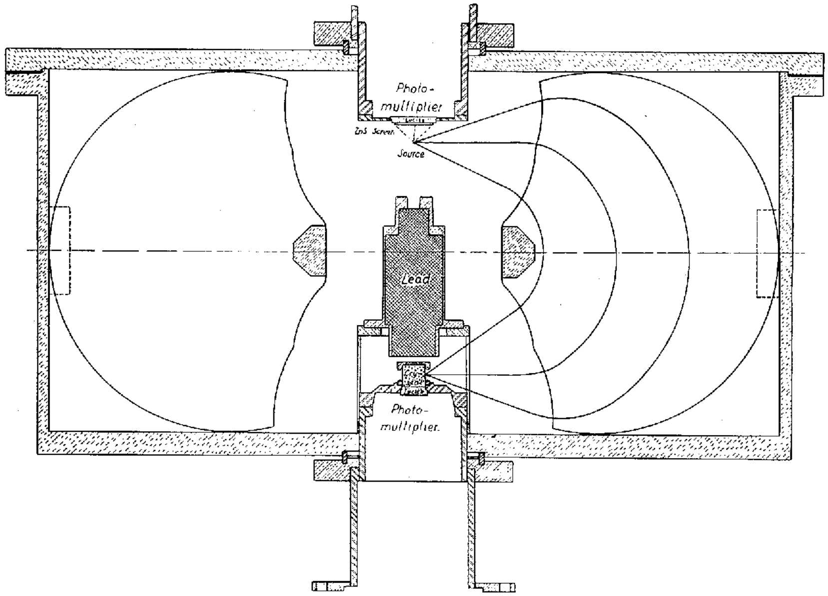

Fig. 1a. Vertical section through the spectrometer.

axis. The magnetic field is localized within the six wedge-shaped air gaps, the angle between the pole faces thus being 20°. If fringing field effects are neglected, all the lines of force will be circular.

It is evident that such a magnetic field is focusing along its own direction. Any particle starting from the source will stay in a plane containing the symmetry axis and, after sufficient deflection in the field, will return to the axis.

In the vertical plane (Fig. 1a), focusing can be obtained by choosing a suitable limiting curve for the pole pieces. It was shown theoretically (ref. 1) that a double infinity of such curves exists corresponding to a fixed position of the source and the counter. If symmetry around a plane perpendicular to the axis is demanded, only a single infinity remains.

The possibility of obtaining the ideal focusing is limited only by effects from fringing fields which cause deviations from the theoretically assumed field shape.

The existence of a fringing field, which the particles have to pass before entering the gap, has no serious disadvantages for

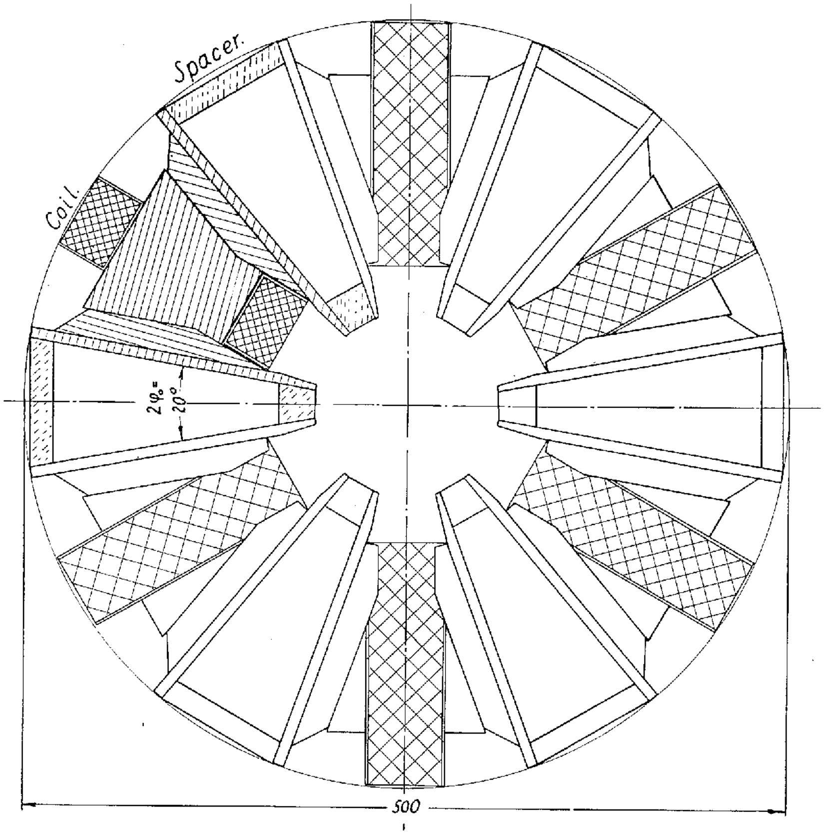

Fig. 1b. Horizontal section through the spectrometer.

the focusing in the plane of Fig. 1a, because it can be corrected for by shaping the pole pieces. In Section 6, a practical method for this correction is described.

Outside the central plane in each wedge-shaped gap the rays will be influenced by the fringing field as by a cylindrical lens[2)3)] with the focal distance

$$f \sim \frac{\varrho}{\operatorname{tg} \psi}, \tag{1}$$

where ψ is the angle between the ray and the edge of the pole piece (Fig. 2) and ϱ the radius of curvature for the electron immediately inside the edge.

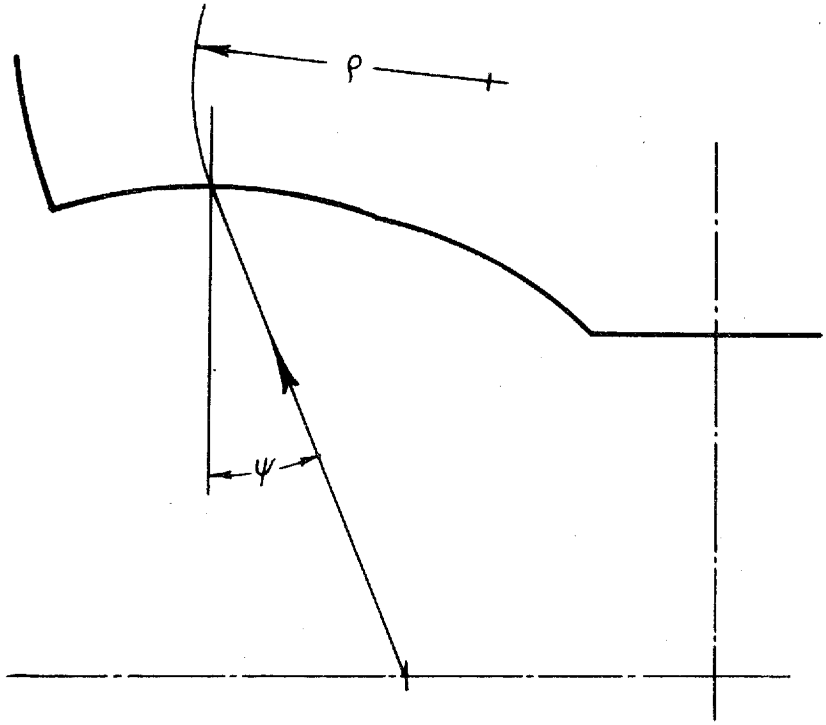

Fig. 2. Entrance of β-ray into the magnetic field.

We have made an attempt to reduce this effect by choosing the boundary of the pole plates approximately as circles with centers near the source and the focus. However, ψ, and hence f, cannot be entirely constant along the limiting curve. Thus, the image of the source has a certain extension along the direction of the magnetic field. This has no direct bearing on the resolving power, but the counter must of course be constructed so as to accept a bundle of rays with finite width.

In the present instrument, practically the entire gap is used with an entrance diaphragm to the counter of 10 mm height.

Another deviation from the ideal field shape is due to the finite radial extension of the pole plates, which causes the field to be stronger at the surface of the iron than in the central plane of the gap. This constitutes the essential limit for the resolution in the present construction. In principle, the effect can be reduced by employing more extended pole pieces or by shimming.

3. The magnet.

From the theory of focusing it followed that three parameters may vary within certain limits, as far as the construction of the pole pieces is concerned. The three parameters are: the quantity b defined in eq. (2) of ref. 1, the relative distance between source and focus, and the function ζ (a) defined by eq. (9) of ref. 1. Furthermore, when constructing the spectrometer, one has to decide on the number of gaps and the angle covered by each of them. The final design must represent a compromise between the partly opposing demands of transmission, resolving power, maximum β-momentum measurable, and economy.

The magnet with the corrected pole shape is shown in Fig. 1. As mentioned before, the individual segments have some similarity in shape with the slices of an orange. Each consists of five parts of iron screwed together. The spacers fixing their relative positions are of aluminium. For ease of machining, the limiting curves for the pole pieces are built up of circles.

The iron is of Swedish origin and has a low coercive force. The magnetic circuit is designed so as to avoid local saturation, which might influence the shape of the field. Because of the large air gaps the effect of hysteresis is small, and the field can be reproduced to within .1 per cent by adjusting the current only, provided that a suitable magnetization procedure is followed.

Each of the six coils consists of about 900 turns of 1.2 mm dia. copper wire with glass insulation, the resistance being approximately 6 ohms. The coil forms are water-cooled, but the thermal conductivity through the coils is poor since they are placed in vacuum. They cannot continuously carry more than 3 amps. corresponding to focusing of 1.5 MeV β-particles, although saturation of the iron first takes place at 5 amps. Hence, the present construction is not quite satisfactory*.

The current is furnished by a servo-controlled generator. At 1.5 MeV the power consumption amounts to about 500 Watts.

The total weight of the magnet and the coils is only about 110 kg.

* A spectrometer of the same type is now under construction by C. A. Mallmann (Buenos Aires) who places the coils in closed containers filled with oil.

4. The vacuum system.

The cylindrical vacuum container is cast in aluminium alloy. No troubles with vacuum leaks have been encountered. The pumping system consists of one 2″ oil diffusion pump and a mechanical forepump.

5. Counter equipment.

Originally we used a GM counter with six mica windows of 12 mm dia. It has now been replaced by a scintillation counter arranged as shown in Fig. 1a. The diameter of the anthracene crystal is 15 mm, and the entrance slit can be varied from 1 to 9 mm. Due to the absence of a magnetic field, the multiplier tube can be placed immediately below the crystal.

It is evident that the fringing fields from the different gaps tend to eliminate each other, so that they are relatively less extended than in a one-gap spectrometer. From the symmetry it follows that the axis is field-free, and the multipliers can be shielded by iron without any effect on the focusing.

The only disadvantage of the scintillation counter is its higher background when low energy β-particles are to be counted. We hope to reduce this effect considerably by replacing the present multiplier tube type EMI 6260 by an EMI 6094, which has a photo-cathode of only 10 mm dia.

6. Investigation of a single gap.

a. Empirical determination of the shape of the pole pieces.

The necessary corrections to the shape of the pole pieces were found by measurements similar to those carried out on the single-gap model of ref. 1.

The aperture was subdivided into a series of regions which were investigated individually. The corresponding openings in the entrance diphragms are shown in Fig. 3. For each diaphragm the magnetization current required to focus the F line from ThB was measured. Source and counter could be moved along the

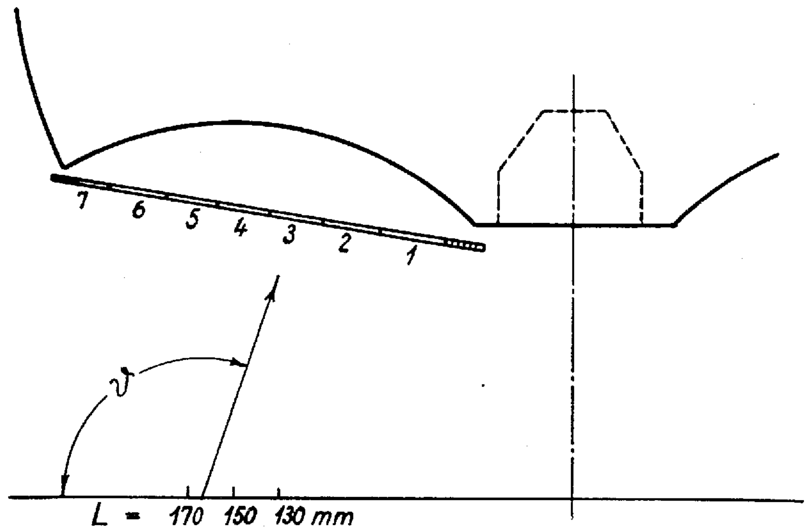

Fig. 3. Entrance diaphragms used in the investigation of the pole shape corrections.

axis from outside the vacuum, and the measurements were repeated for a set of different positions, all being symmetrical about the central plane.

The results of a series of such measurements are shown in Fig. 4. The magnitization current I_m is given as a function of the diaphragm no., i. e. of the angle ϑ. Each curve represents a value of the distance L between source and counter.

Changes in L influence the focusing current less in the outer part of the gap than in the inner part. This is due to the variation of the dispersion with ϑ, as found theoretically in ref. 1. As a consequence, the curves show maxima or minima when the distance L is within certain limits.

Around such maxima the aberration is at most of second order. The condition for first order focusing is thus fulfilled. Even considerable deviations from the theoretical field shape will not prevent the occurrence of such extrema.

After the choice of a certain L the aberration can be reduced by working the pole pieces in the regions where the deflection is too great. We have chosen $L = 165$ mm, and after the final correction the focusing current for a line is constant to within one per cent. A further reduction of the aberration was not con-

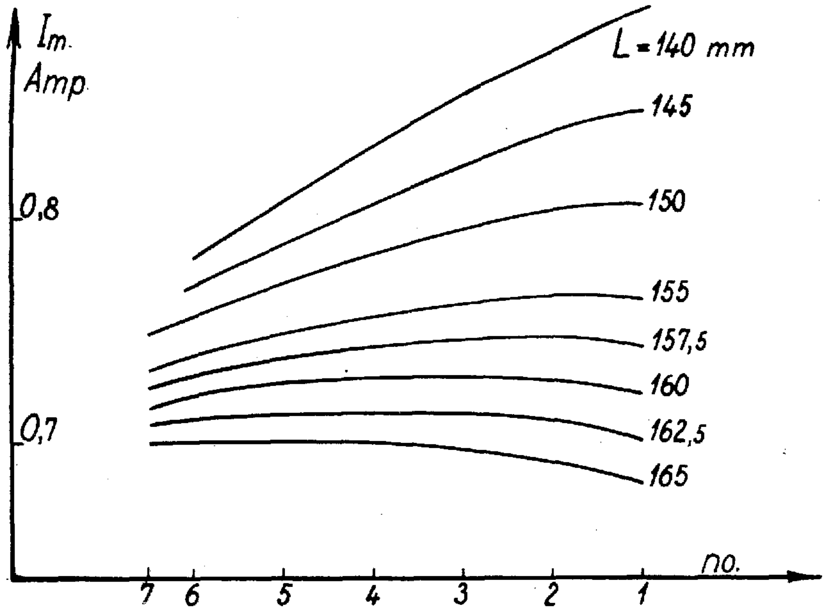

Fig. 4. The focusing current for the F line as a function of the diaphragm no. Each curve corresponds to a certain distance between source and counter.

sidered necessary because other contributions to the line width are of the same order of magnitude.

b. Some practically important properties of the electron orbits.

Curves like those drawn in Fig. 4 can also be represented in a diagram where I_m is plotted as a function of L for each diaphragm. Ideal focusing means that all curves have to pass through a common point. For the corrected pole pieces the case is illustrated in Fig. 5.

In a symmetric spectrometer (ζ (a) = 0) an image is formed with magnification of unity, as shown theoretically in ref. 1. This implies that a small shift in the position of the source introduces the same shift in the focus. This effect was verified by observing that a simultaneous displacement of source and focus by 10 mm in the same direction caused only small changes of the curves in Fig. 5.

The slope of the lines in Fig. 5 determines the dispersion D for the corresponding part of the gap, since

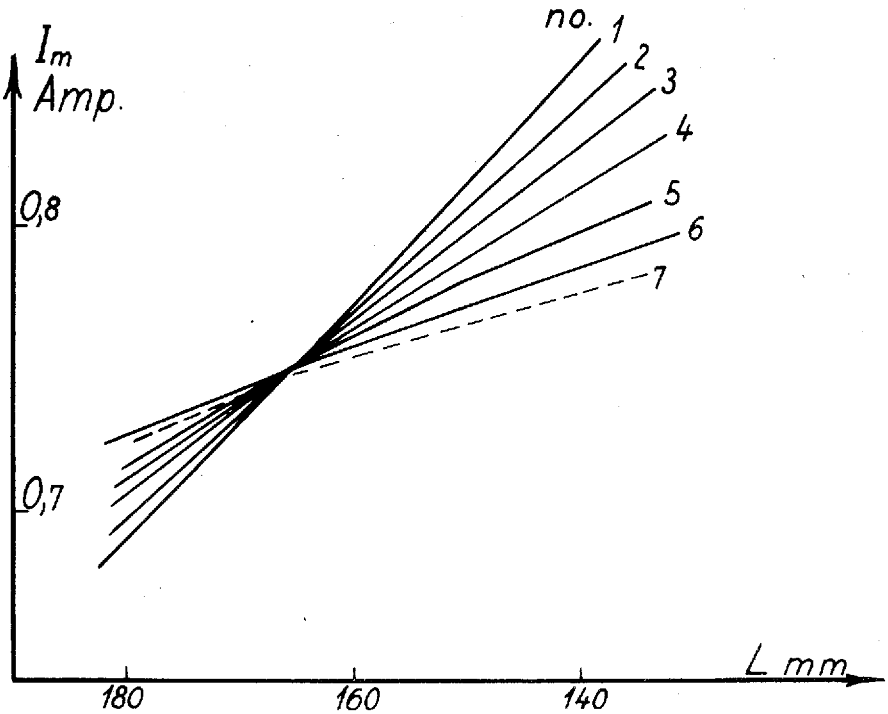

Fig. 5. The focusing current as a function of the distance between source and counter. Each curve corresponds to a diaphragm as numbered in Fig. 3. Results obtained with corrected pole pieces.

$$D = \Delta L \frac{p}{\Delta p} \approx \frac{\Delta L}{\Delta I_m} I_m.$$

The mean value of D is 310 mm or 1.9 times the distance L.

The variation of D causes a smearing out of the image when the magnetization current deviates from the focusing value (Fig. 6). However, at the same time a contraction in the pencil of rays appears outside the axis of symmetry. In fact, a focus line exists which represents a surface on which a portion of the spectrum is focused with good resolution. Each point on this line is characterized by a curve like those in Fig. 4 with a broad maximum in the central part.

By moving the entrance slit to the counter along the focus line, a resolution of $I_m/I_m \lesssim$ two per cent was found over an

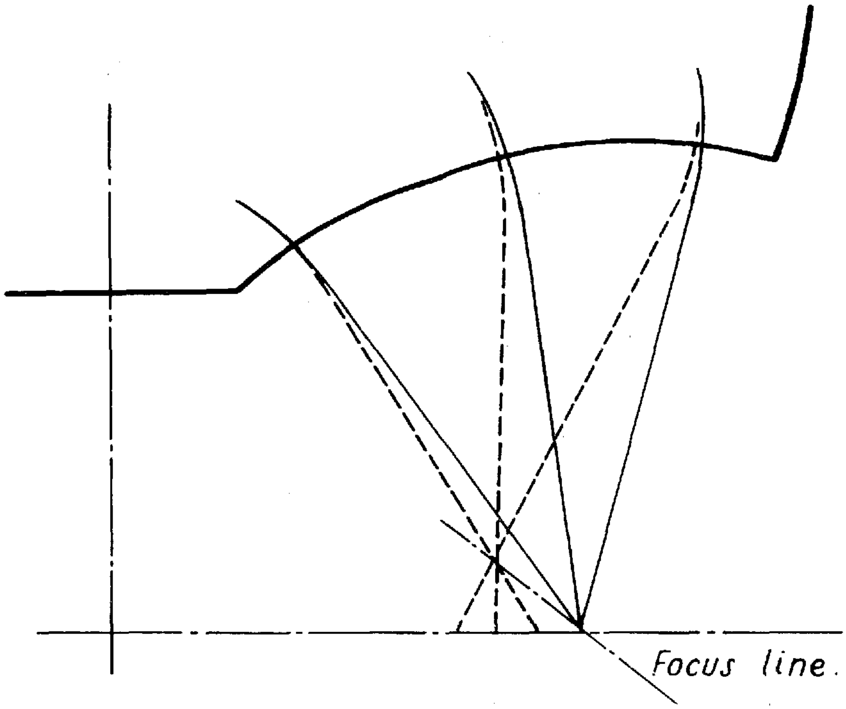

Fig. 6. Position of the focus line.

interval covering 15 per cent change in electron momentum. The existence of the line of focus leaves some freedom in the choice of the counter diameter.

Image formation outside the axis of symmetry is accompanied by a moderate increase in the height of the image, corresponding to a divergence of 20° for the bunch of rays in the direction of the lines of force.

c. The aberration across the gap.

The shape of the F line was measured through diaphragm no. 4 with apertures corresponding to three different values of the angle φ. The results are shown in Fig. 7. The contributions to the line width from the extension of the source and the entrance slit of the counter amount to $\sim$.5 per cent. It is seen that the particles passing close to the surface of the iron reach the counter for a slightly lower current than those moving in the central plane

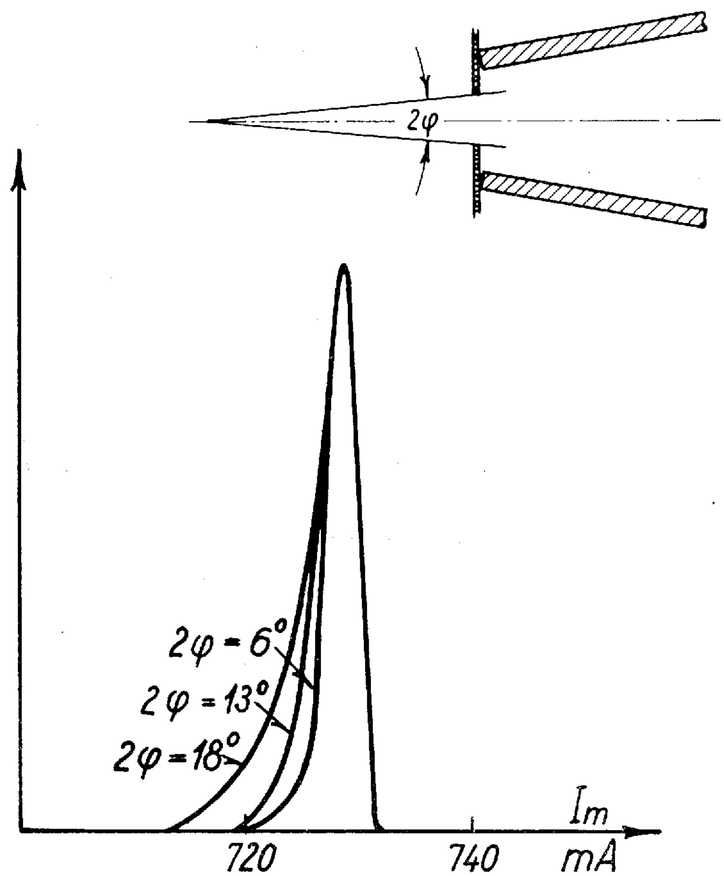

Fig. 7. Line shape measured with different opening angles φ in diaphragm no. 4. The curves are normalized at the maximum point.

of the gap. This contribution is of the order $\sim .3 \times \varphi^2$ in the part of the gap considered. As mentioned previously, this aberration depends upon the construction of the magnet and can be reduced in principle.

d. The transmission from a point source.

As in other spectrometers, the resolving power depends on the utilized solid angle. We have carried out an experimental determination of the relationship between the resolution $R = \frac{p}{\Delta p}$

and the transmission T, defined as the fraction of particles emitted from a mono-energetic source, which are counted at the peak of the line.

The total emission was found by a measurement of the line (F line from ThB) through a small hole in a diaphragm. The

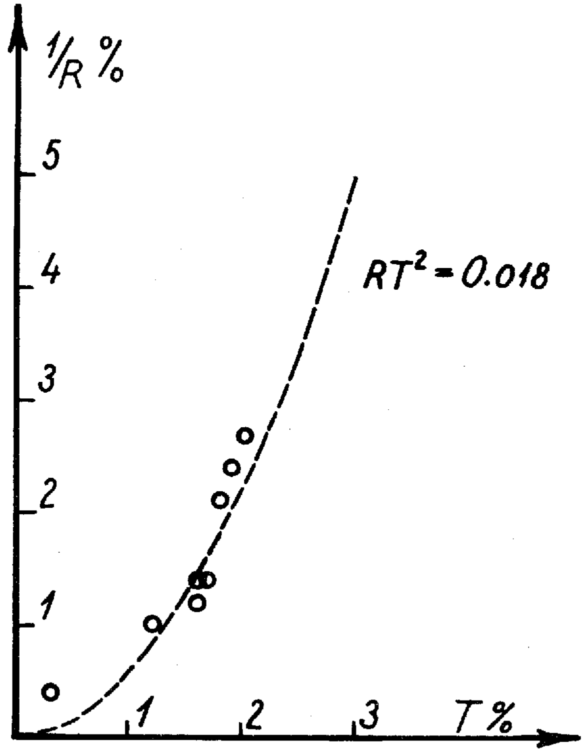

Fig. 8. Transmission versus resolution.

counter slit was so wide that the height of the line represented the solid angle defined by the diaphragm.

The transmission could then easily be determined for a number of different combinations of entrance diaphragms and counter slits. In each case, the latter were adjusted to approximate the same magnitude as the image. During these experiments the source dimensions were so small that their influence on the shape could be neglected.

Some of the results are seen in Fig. 8. The curve $RT^2 = .018$ gives a reasonable fit to the measurements. The maximum transmission of 2.0 per cent for a single gap may be compared with the total opening for each gap of 2.3 per cent of 4π.

e. Extended source.

The mean dispersion 310 mm can be used for an evaluation of the influence on the line shape of the dimensions of source and counter slit. In practice, a source area of 5×10 mm can be used at a resolution of 2.5 per cent.

7. The use of several gaps.

The use of the same source with several gaps demands of course a more detailed analysis.

It is mainly the same magnetic flux which has to pass through the entire magnetic circuit. Therefore, mechanical and electrical differences in the construction of the parts of the magnet have only a minor influence on the magnetic properties of the individual gaps. The maximum difference in field strength was reduced to less than .5 per cent by small corrections to the number of turns in some of the coils.

At present the entire spectrometer is used with a resolution of all six gaps of $1/R \sim 1.6$ per cent without any appreciable effect on the line shape from the differences in field strength in the gaps. The corresponding transmission amounts to 9 per cent of 4π.

In a measurement of the maximum energy of the continuous β-spectrum of P^{33}[4], the resolution was reduced to 2.5 per cent in order to obtain higher intensity. The spectrum could then be measured with a total activity of 5×10^4 source disintegrations per minute. The background counting rate in the GM tube was 10 counts per minute.

When high resolution is desired, only one gap is employed. Only three gaps can be used when the energy of the spectrum is so low that the passage of particles through the supporting foil must be avoided.

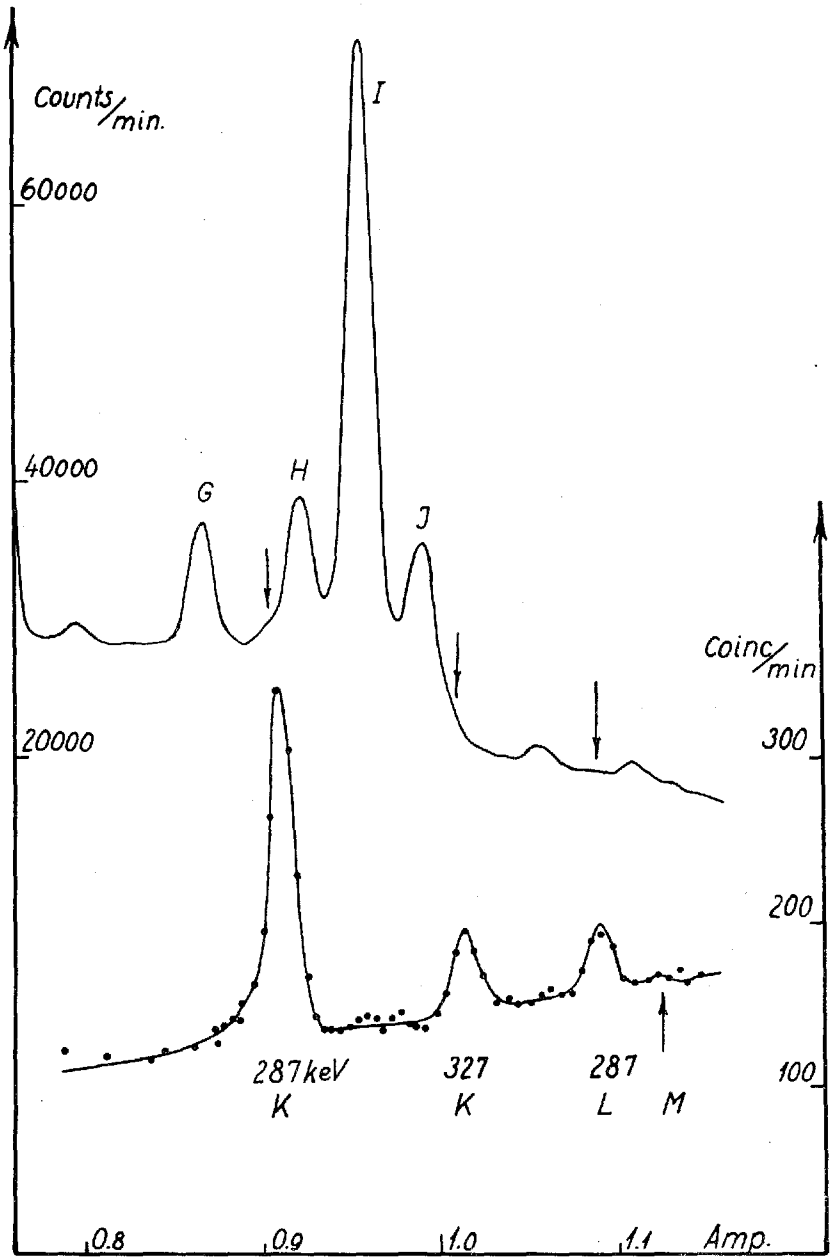

Fig. 9. Above: A part of the complex β-spectrum of $Th(B + C + C')$. The lines are given with the notation of Ellis. — Below: The corresponding α—β coincidence spectrum. Random coincidences subtracted.

8. Application as coincidence spectrometer.

The field-free space around the axis implies that the spectrometer is especially well suited for coincidence investigations. In Fig. 1a is shown the position of a ZnS-screen and a multiplier tube immediately behind the source. Since the ZnS-screen covers 30 per cent of the total solid angle, the transmission for β-lines is $\sim$ 3 per cent in the coincident spectrum.

The upper part of Fig. 9 shows a portion of the complex $Th(B + C + C')$ β-spectrum; below we see the corresponding spectrum for coincidences between electrons and α-particles. The weak lines following the α-decay of ThC[5] can hereby be studied without any disturbance from the extremely strong neighbouring lines. The continuous background is due to β-particles from ThC in coincidence with the α-group from ThC″, the half life of which, 3×10^{-7} sec., is of the order of magnitude of the resolving time of the coincidence circuit.

Instead of the ZnS-screen we can of course use an anthracene crystal for β—β-coincidences or a NaI crystal for β—γ-coincidences. It is our intention in the near future to insert one further multiplier with a crystal where the lead screen is now situated (Fig. 1a). This will imply, on the one hand, a doubling of the efficiency and, on the other hand, will make possible triple coincidence investigations of decays where three particles are emitted in cascade.

9. Further applications of the focusing principle.

C. A. Mallmann[6][7] has shown the possibility of constructing a double spectrometer for β—β-coincidences by placing two magnets above each other. The β-rays can be focused simultaneously in two spectrometers when the gaps are limited to accept particles at angles $\vartheta > 90°$.

A spectrometer with pole pieces shaped as sketched in Fig. 10 can be used as a pair spectrometer, since positive and negative particles can be focused simultaneously. Focusing is achieved by shaping of the two outer boundaries, while the central limiting curve can be kept symmetrical and close to a circle.

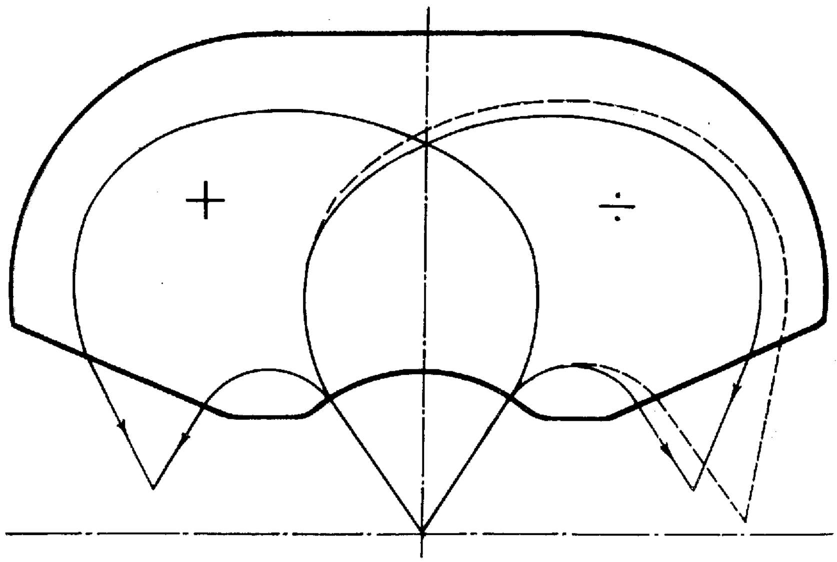

Fig. 10. Pole shape for proposed pair spectrometer.

A better relation between R and T will be obtained in this wedge-shaped field than in a homogeneous field. Furthermore, the efficiency is multiplied by the number of gaps. Also some of the advantages gained in the spectrometer developed by McDaniel and Walker[8] are obtained. Thus, the existence of the focal line makes it possible to work with, say, up to 2×4 counters placed close to each other. The extension of the converter is of relatively minor importance, since the sum of the momenta of electron and positron to the first approximation depends only on the relative distance between the two struck counters.

A magnet with a wedge-shaped field in a single air gap will be advantageous as double focusing spectrometer for heavy particles. In such instruments, the utilization of the theoretical solid angle demands very large magnets. In this respect, the proposed design is cheaper than the type used at present[9]. This results from the fact that the magnetic field should exist only in the space occupied by the beam of particles, whereas the conventional types utilize only the central part of the field in full height.

We wish to express our gratitude to Professor NIELS BOHR for his interest in our work and for the excellent working conditions at his institute. We are also indebted to Mr. J. LINDHARD for theoretical discussions.

Summary.

A β-spectrometer is described. It is constructed according to a new principle developed previously[1]. A transmission of 9 per cent is obtained at a resolution of 1.6 per cent. The instrument is especially well suited for coincidence investigations with scintillation counters.

Some of the properties of the spectrometer are discussed from a practical point of view. A few new applications for the focusing principle are suggested.

Institute for Theoretical Physics,
University of Copenhagen, Denmark.

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