

the bands of the cupric ammines are transition group spectra of the first kind, while all the bands of Mn^{++} are transition group spectra of the second kind. It is hoped that further investigations will throw more light on these questions.

I wish to express my sincere thanks to Professor J. BJERRUM for suggesting this problem, and for his great interest and constant encouragement during the work. My thanks are further due to Mr. AAGE WINTHER for many helpful discussions.

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THE ENERGY PRODUCTION IN CONVECTIVE CORES IN STARS

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1. The following tables give the functions

$$I_m(\xi) = \int_0^\xi [\theta(\xi')]^{m+3} \xi'^2 d\xi' \quad (1)$$

and

$$W_m(\xi) = \xi^3 [\theta(\xi)]^{m+3} / I_m(\xi) \quad (2)$$

where θ is the Lane-Emden function of index $n = 3/2$. The functions have been calculated for all integral values of the parameter m from 1 to 23 inclusive.

2. The tables of $I_m(\xi)$ will facilitate the calculation of the energy production in convective cores in stars. The structure of the convective core is described by:

Distance from center, $r = \alpha\xi$

Temperature, $T = T_c\theta$

Density, $\rho = \rho_c\theta^{3/2}$

where α is a scale factor, and T_c and ρ_c the central temperature and density. The energy production inside a sphere of radius r , L_r , will be

$$L_r = \int_0^r 4\pi r^2 \rho \varepsilon dr. \quad (3)$$

If the energy production is expressed by the power law

$$\varepsilon = \varepsilon_0 \rho^\delta T^\nu, \quad (4)$$

we get

$$L_{r(=\alpha\xi)} = 4\pi\alpha^3 \varepsilon_0 \rho_c^{1+\delta} T_c^\nu I_m(\xi) \quad (5)$$

with $m = \nu + 3(\delta - 1)/2$.

3. The function W is the homology invariant quantity corresponding to I . The tables are primarily calculated as an aid to starting calculations of the interior structure of stars where homology invariant quantities are used as the primary variables. Such a choice will be convenient whenever the energy production and the opacity can be expressed with sufficient accuracy by power expressions, such as (4) above for ϵ and

$$\kappa = \kappa_0 \rho^{1-\alpha} T^{-3-s} \quad (6)$$

for the opacity, and the radiation pressure can be neglected. In this case we can choose the variables

$$\left. \begin{aligned} V &= -\frac{d \log P}{d \log r} \\ U &= \frac{d \log M_r}{d \log r} \\ W &= \frac{d \log L_r}{d \log r} \\ H &\equiv \frac{V}{n+1} = -\frac{d \log T}{d \log r} \end{aligned} \right\} \quad (7)$$

where we have used the usual notation: P is the pressure, M_r is the mass within the sphere of radius r , and n is the polytropic index.

In terms of these variables the usual four differential equations governing the internal structure of a star in radiative equilibrium reduce themselves to the following three differential equations

$$\left. \begin{aligned} \frac{dU}{dV} &= \frac{U(3-V+H-U)}{V(U+H-1)} \\ \frac{dW}{dV} &= \frac{W(3-(1+\delta)V-(\nu-1-\delta)H-W)}{V(U+H-1)} \\ \frac{dH}{dV} &= \frac{H((9+s-\alpha)H-(2-\alpha)V+W-1)}{V(U+H-1)} \end{aligned} \right\} \quad (8)$$

and the quadrature

$$\log r/r_0 = \int \frac{dV}{V(U+H-1)} \quad (9)$$

Inside the convective core the functions U , V , and H are independent of the opacity and energy production. U and V are tabulated in British Association Tables, Vol. II. The variable H follows immediately as $2V/5$. The remaining variable, W , is the function tabulated in the present publication. Contrary to what is the case of V , U and H , W depends on the energy production law. The present tables cover the ground where a law of the form (4), with $1 \leq \nu + 3(\delta - 1)/2 \leq 23$, is concerned.

The tables of W have been calculated only with the argument ξ . The applications to integrations of stellar structure require V to be the independent variable. Therefore an auxiliary table, which gives U and ξ as functions of V , has been provided.

4. In a recent paper by OSTERBROCK and the present author¹ it has been shown how an upper limit to the extent of the convective core in a star where (4) and (6) are valid can be derived. This limit has the following form: A convective core, extending to the point in the star where $V = V_0$, is only possible if

$$U_0 - W_0 - (1.2 + 0.4s + 0.6\alpha)V_0 \geq 0 \quad (10)$$

or

$$s + \frac{3}{2}\alpha \leq \frac{5(U_0 - W_0)}{2V_0} - 3. \quad (11)$$

If ϵ is of the form (4), the quantity on the right hand side of (11) depends only on V_0 and $\nu + 3(\delta - 1)/2$. It has been plotted in figure 1.

It is of considerable interest to know the behaviour of the convective core in a star where two energy sources, each with an output of the form (4), are active. There would be no difficulties in evaluating the W functions for such mixtures and then plotting the corresponding curves in the diagram. Since, however, the most important feature of the curve is its intersection with the $s + 3\alpha/2$ axis, we shall confine our attention to the evaluation of the position of this point. For this purpose, let us write the total energy production rate as

$$\epsilon = \epsilon_c (\gamma \theta^{m_1} + (1 - \gamma) \theta^{m_2}) \theta^{3/2}. \quad (12)$$

¹ Ap. J., 117, 306, 1953.

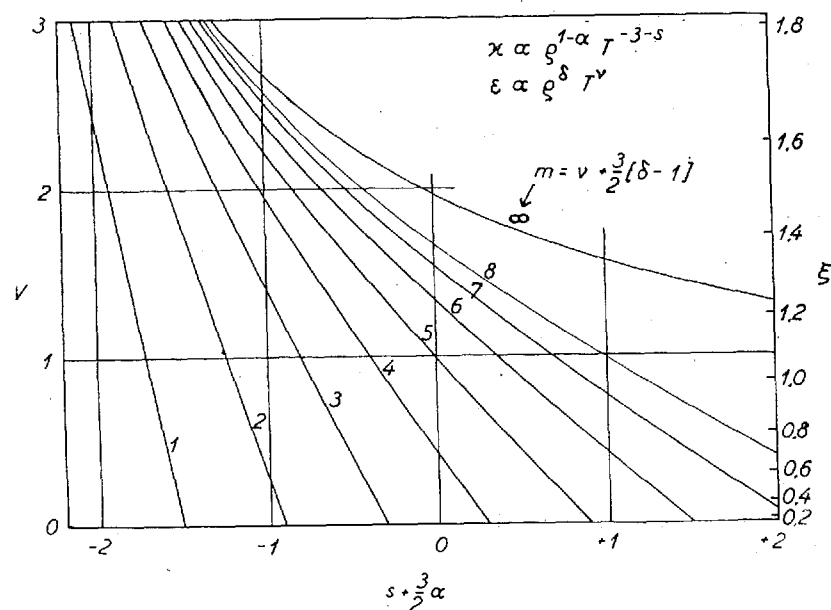


Figure 1. The curves give the upper limit to the extent of the convective core as a function of the opacity and energy production laws.

Thus ϵ_c is the total energy production rate at the center while γ measures the relative contribution to this total from the reaction characterized by the subscript 1. Using the power expansions of the Lane-Emden function it is easy to find the value of the relevant function at $V = 0$:

$$\left[\frac{5(U-W)}{2V} \right]_{V \rightarrow 0} - 3 = 0.6 \gamma m_1 + 0.6 (1 - \gamma) m_2 - 2.1. \quad (13)$$

Thus the abscissa of the required intersection on the horizontal axis is a mean of the values of m for the two reactions, the weighing factor being the contribution from the particular process to the total energy production at the center.

The tables.

The tables giving ξ and U as functions of V have been obtained by interpolation of the tables given in British Association Tables, Vol. II. The second difference, Δ'' , or the modified second difference, $M'' = \Delta'' - 0.184 \Delta''''$, has been given. Thus interpolation to the fraction n follows from Everett's formula:

$$f(x_i + n\Delta x) = (1 - n)f(x_i) + nf(x_{i+1}) \\ + E_0'' \Delta''(x_i) + E_1'' \Delta''(x_{i+1})$$

or the same expression with M'' replacing Δ'' . The coefficients E_0'' and E_1'' have been tabulated with argument n in Interpolation and Allied Tables, H. M. Stationary Office (reprinted from the Nautical Almanac for 1937). For small values of V the table for ξ becomes unmanageable. This difficulty is avoided if one works with the function $\sqrt{6V/5} - \xi$.

The tables of I and W are based on the values of θ given in British Association Tables, Vol. II. The calculations were carried out to seven decimals and only in the copy prepared for printing the functions were rounded to five and four decimals. For interpolation to arbitrary values of ξ the modified second differences have been given. Most of the work of calculating these functions was done by means of the IBM 602-A calculators at the IBM Watson Scientific Computing Laboratory, New York City.

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V	\$	M	U	Z
0.0	0.00000	—	3.00000	+ 30
0.1	0.34606	—	2.96414	30
0.2	0.48888	—	2.92858	30
0.3	0.59811	—1567	2.89332	29
0.4	0.68986	1065	2.85835	30
0.5	0.77040	776	2.82368	31
0.6	0.84294	601	2.78932	31
0.7	0.90986	477	2.75527	31
0.8	0.97093	396	2.72153	31
0.9	1.02850	333	2.68810	32
1.0	1.08271	291	2.65499	33
1.1	1.13401	251	2.62221	32
1.2	1.18280	224	2.58975	33
1.3	1.22935	200	2.55762	32
1.4	1.27390	180	2.52581	34
1.5	1.31665	163	2.49434	33
1.6	1.35777	149	2.46320	33
1.7	1.39740	139	2.43239	33
1.8	1.43564	128	2.40191	34
1.9	1.47260	118	2.37177	34
2.0	1.50838	111	2.34197	34
2.1	1.54305	105	2.31251	35
2.2	1.57667	97	2.28340	33
2.3	1.60932	93	2.25462	34
2.4	1.64104	87	2.22618	34
2.5	1.67189	83	2.19808	35
2.6	1.70191	80	2.17033	34
2.7	1.73113	75	2.14292	35
2.8	1.75960	71	2.11586	33
2.9	1.78736	70	2.08913	34
3.0	1.81442	66	2.06274	35
3.1	1.84082	63	2.03670	33
3.2	1.86659	61	2.01099	34
3.3	1.89175	58	1.98562	34
3.4	1.91633	57	1.96059	33
3.5	1.94034	55	1.93589	34
3.6	1.96380	53	1.91153	+ 33

\$	I ₁	I ₂	I ₃	I ₄	I ₅	M
0.0	0.00000	0	0.00000	0	0.00000	0
0.1	0.00033	+ 199	0.00033	+ 199	0.00033	+ 198
0.2	0.00262	382	0.00261	377	0.00260	373
0.3	0.00868	535	0.00861	520	0.00853	514
0.4	0.02002	647	0.01970	612	0.01940	578
0.5	0.03773	710	0.03682	647	0.03593	589
0.6	0.06245	720	0.06030	625	0.05825	538
0.7	0.09428	681	0.08995	550	0.08585	436
0.8	0.13283	596	0.12502	434	0.11776	298
0.9	0.17727	474	0.16437	288	0.15261	+ 141
1.0	0.22641	328	0.20658	+ 130	0.18887	+ 16
1.1	0.27881	169	0.25009	— 28	0.22500	158
1.2	0.33290	9	0.29335	170	0.25961	273
1.3	0.38709	— 141	0.33495	289	0.29154	354
1.4	0.43990	272	0.37371	377	0.32000	401
1.5	0.49004	378	0.40876	432	0.34451	414
1.6	0.53645	455	0.43955	456	0.36493	399
1.7	0.57837	502	0.46582	452	0.38141	364
1.8	0.61531	521	0.48761	426	0.39426	316
1.9	0.64710	514	0.50517	384	0.40397	262
2.0	0.67377	487	0.51892	332	0.41105	209
2.1	0.69561	445	0.52935	276	0.41604	159
2.2	0.71301	392	0.53703	221	0.41943	116
2.3	0.72651	334	0.54250	170	0.42165	81
2.4	0.73668	275	0.54625	126	0.42303	54
2.5	0.74409	219	0.54872	90	0.42386	35
2.6	0.74930	168	0.55029	61	0.42433	21
2.7	0.75283	124	0.55123	40	0.42458	12
2.8	0.75510	87	0.55176	24	0.42471	6
2.9	0.75649	58	0.55205	14	0.42477	3
3.0	0.75728	37	0.55218	7	0.42479	1
3.1	0.75770	21	0.55225	3	0.42480	—
3.2	0.75789	11	0.55227	1	0.42480	0
3.3	0.75797	5	0.55228	0	0.42480	0
3.4	0.75800	2	0.55228	0	0.42480	0
3.5	0.75800	0	0.55228	0	0.42480	0
0.0	0.00000	0	0.00000	0	0.00000	0
0.1	0.00033	+ 199	0.00033	+ 198	0.00033	+ 198
0.2	0.00262	382	0.00261	377	0.00260	373
0.3	0.00868	535	0.00861	520	0.00853	514
0.4	0.02002	647	0.01970	612	0.01940	578
0.5	0.03773	710	0.03682	647	0.03593	589
0.6	0.06245	720	0.06030	625	0.05825	538
0.7	0.09428	681	0.08995	550	0.08585	436
0.8	0.13283	596	0.12502	434	0.11776	298
0.9	0.17727	474	0.16437	288	0.15261	+ 141
1.0	0.22641	328	0.20658	+ 130	0.18887	+ 16
1.1	0.27881	169	0.25009	— 28	0.22500	158
1.2	0.33290	9	0.29335	170	0.25961	273
1.3	0.38709	— 141	0.33495	289	0.29154	354
1.4	0.43990	272	0.37371	377	0.32000	401
1.5	0.49004	378	0.40876	432	0.34451	414
1.6	0.53645	455	0.43955	456	0.36493	399
1.7	0.57837	502	0.46582	452	0.38141	364
1.8	0.61531	521	0.48761	426	0.39426	316
1.9	0.64710	514	0.50517	384	0.40397	262
2.0	0.67377	487	0.51892	332	0.41105	209
2.1	0.69561	445	0.52935	276	0.41604	159
2.2	0.71301	392	0.53703	221	0.41943	116
2.3	0.72651	334	0.54250	170	0.42165	81
2.4	0.73668	275	0.54625	126	0.42303	54
2.5	0.74409	219	0.54872	90	0.42386	35
2.6	0.74930	168	0.55029	61	0.42433	21
2.7	0.75283	124	0.55123	40	0.42458	12
2.8	0.75510	87	0.55176	24	0.42471	6
2.9	0.75649	58	0.55205	14	0.42477	3
3.0	0.75728	37	0.55218	7	0.42479	1
3.1	0.75770	21	0.55225	3	0.42480	—
3.2	0.75789	11	0.55227	1	0.42480	0
3.3	0.75797	5	0.55228	0	0.42480	0
3.4	0.75800	2	0.55228	0	0.42480	0
3.5	0.75800	0	0.55228	0	0.42480	0
0.0	0.00000	0	0.00000	0	0.00000	0
0.1	0.00033	+ 198	0.00033	+ 198	0.00033	+ 198
0.2	0.00262	373	0.00261	373	0.00259	369
0.3	0.00868	514	0.00865	504	0.00845	490
0.4	0.02002	578	0.01940	578	0.01909	546
0.5	0.03773	589	0.03593	589	0.03507	534
0.6	0.06245	538	0.05825	538	0.05628	459
0.7	0.09428	436	0.08585	436	0.08199	337
0.8	0.13283	298	0.11776	298	0.11101	185
0.9	0.17727	+ 141	0.15261	+ 141	0.14188	+ 27
1.0	0.22641	+ 16	0.18887	+ 16	0.17303	+ 119
1.1	0.27881	158	0.22500	158	0.20304	239
1.2	0.33290	273	0.25961	273	0.23072	323
1.3	0.38709	354	0.29154	354	0.25524	369
1.4	0.43990	401	0.32000	401	0.27614	380
1.5	0.49004	414	0.34451	414	0.29328	362
1.6	0.53645	399	0.36493	399	0.30684	325
1.7	0.57837	364	0.38141	364	0.31717	275
1.8	0.61531	316	0.39426	316	0.32476	222
1.9	0.64710	262	0.40397	262	0.33012	170
2.0	0.67377	209	0.41105	209	0.33378	125
2.1	0.69561	159	0.41604	159	0.33616	88
2.2	0.71301	116	0.41943	116	0.33766	59
2.3	0.72651	81	0.42165	81	0.33856	37
2.4	0.73668	54	0.42303	54	0.33907	23
2.5	0.74409	35	0.42386	35	0.33935	13
2.6	0.74930	21	0.42433	21	0.33949	7
2.7	0.75283	12	0.42458	12	0.33955	3
2.8	0.75510	6	0.42471	6	0.33958	2
2.9	0.75649	3	0.42477	3	0.33960	—
3.0	0.75728	1	0.42479	1	0.33960	0
3.1	0.75770	—	0.42480	—	0.33960	0
3.2	0.75789	0	0.42480	0	0.33960	0
3.3	0.75797	0	0.42480	0	0.33960	0
3.4	0.75800	0	0.42480	0	0.33960	0
3.5	0.75800	0	0.42480	0	0.33960	0

ξ	I_6	M''	I_7	M''	I_8	M''	I_9	M''	I_{10}	M''
0.0	.00000	0	.00000	0	.00000	0	.00000	0	.00000	0
0.1	.00033	+197	.00033	+197	.00033	+197	.00033	+196	.00033	+196
0.2	.00257	360	.00256	356	.00255	351	.00254	347	.00253	343
0.3	.00830	461	.00823	446	.00816	433	.00809	419	.00802	406
0.4	.01851	485	.01822	456	.01794	428	.01767	402	.01740	376
0.5	.03342	434	.03263	389	.03187	346	.03112	306	.03040	269
0.6	.05257	322	.05083	263	.04916	210	.04756	162	.04602	+119
0.7	.07488	174	.07161	+109	.06851	+52	.06558	+4	.06281	-37
0.8	.09890	+16	.09347	-46	.08841	-95	.08369	-134	.07929	164
0.9	.12311	-127	.11492	175	.10741	209	.10052	232	.09420	245
1.0	.14612	237	.13468	264	.12439	278	.11512	280	.10675	276
1.1	.16682	306	.15189	309	.13869	301	.12700	285	.11662	264
1.2	.18455	333	.16607	313	.15004	286	.13609	256	.12390	226
1.3	.19901	324	.17718	287	.15858	248	.14265	210	.12894	176
1.4	.21028	290	.18546	242	.16466	198	.14712	159	.13223	126
1.5	.21867	243	.19134	190	.16877	146	.15000	111	.13425	83
1.6	.22465	190	.19530	140	.17141	102	.15175	73	.13541	51
1.7	.22872	141	.19786	97	.17301	66	.15276	44	.13604	29
1.8	.23136	99	.19942	64	.17394	40	.15330	25	.13636	16
1.9	.23300	65	.20033	39	.17444	23	.15358	14	.13652	8
2.0	.23398	41	.20083	23	.17470	12	.15372	7	.13659	4
2.1	.23452	24	.20109	12	.17483	6	.15378	3	.13662	2
2.2	.23482	14	.20122	6	.17488	3	.15380	1	.13663	-1
2.3	.23496	7	.20128	3	.17491	1	.15381	-1	.13663	0
2.4	.23503	4	.20131	1	.17492	-1	.15382	0		
2.5	.23507	2	.20132	-1	.17492	0	.15382	0		
2.6	.23508	-1	.20132	0						
2.7	.23508	0								

ξ	I_{11}	M''	I_{12}	M''	I_{13}	M''	I_{14}	M''	I_{15}	M''
0.0	.00000	0	.00000	0	.00000	0	.00000	0	.00000	0
0.1	.00033	+196	.00033	+195	.00033	+195	.00033	+195	.00033	+194
0.2	.00252	338	.00251	334	.00250	330	.00249	326	.00248	322
0.3	.00795	393	.00788	380	.00781	368	.00774	355	.00767	343
0.4	.01713	352	.01687	328	.01662	305	.01637	283	.01612	263
0.5	.02970	234	.02901	202	.02835	172	.02771	+144	.02708	+118
0.6	.04454	+80	.04312	+45	.04176	+14	.04045	-14	.03919	-38
0.7	.06019	-71	.05770	-100	.05534	-124	.05310	144	.05098	160
0.8	.07518	187	.07134	203	.06776	214	.06441	221	.06127	224
0.9	.08839	251	.08305	252	.07814	248	.07361	241	.06942	232
1.0	.09918	265	.09233	252	.08611	236	.08046	219	.07532	201
1.1	.10739	242	.09915	218	.09178	195	.08518	173	.07924	152
1.2	.11322	197	.10382	169	.09552	144	.08817	122	.08164	103
1.3	.11709	145	.10680	119	.09781	97	.08993	78	.08300	63
1.4	.11951	99	.10857	77	.09912	59	.09090	45	.08370	35
1.5	.12092	62	.10956	46	.09981	33	.09138	24	.08404	18
1.6	.12169	36	.11008	25	.10015	17	.09161	12	.08420	8
1.7	.12209	19	.11033	13	.10031	8	.09171	5	.08426	3
1.8	.12228	10	.11044	6	.10038	4	.09175	2	.08428	-1
1.9	.12237	4	.11049	3	.10040	1	.09176	-1	.08429	0
2.0	.12240	2	.11051	-1	.10041	-1	.09177	0	.08429	0
2.1	.12242	-1	.11051	0	.10042	0	.09177	0		
2.2	.12242	0	.11052	0	.10042	0				
2.3	.12242	0	.11052	0						

ξ	I_{16}	M''	I_{17}	M''	I_{18}	M''	I_{19}	M''	I_{20}	M''
0.0	.00000	0	.00000	0	.00000	0	.00000	0	.00000	0
0.1	.00033	+194	.00033	+193	.00033	+193	.00033	+193	.00033	+192
0.2	.00247	317	.00246	313	.00245	309	.00244	305	.00243	301
0.3	.00761	332	.00754	320	.00748	309	.00741	298	.00735	287
0.4	.01588	243	.01564	223	.01541	205	.01518	188	.01496	171
0.5	.02647	+93	.02588	+71	.02530	+50	.02474	+31	.02420	+13
0.6	.03798	-60	.03682	-79	.03571	-95	.03463	-110	.03360	-122
0.7	.04897	172	.04706	181	.04524	188	.04352	193	.04188	196
0.8	.05834	224	.05559	222	.05301	218	.05059	212	.04832	205
0.9	.06556	221	.06199	210	.05869	197	.05562	184	.05278	172
1.0	.07064	184	.06635	167	.06244	151	.05885	136	.05556	121
1.1	.07389	132	.06907	116	.06469	101	.06073	87	.05712	75
1.2	.07582	86	.07061	72	.06593	60	.06172	49	.05791	40
1.3	.07686	50	.07141	40	.06655	31	.06219	25	.05828	19
1.4	.07738	26	.07179	20	.06683	15	.06240	11	.05843	8
1.5	.07762	12	.07196	9	.06695	6	.06248	5	.05849	3
1.6	.07772	5	.07203	4	.06699	2	.06251	2	.05851	-1
1.7	.07776	2	.07205	-1	.06701	-1	.06252	-1	.05852	0
1.8	.07777	-1	.07206	0	.06701	0	.06252	0	.05852	0
1.9	.07778	0	.07206	0						
2.0	.07778	0								

ξ	I_{21}	M''	I_{22}	M''	I_{23}	M''
0.0	.00000	0	.00000	0	.00000	0
0.1	.00033	+192	.00033	+191	.00033	+191
0.2	.00242	297	.00242	293	.00241	289
0.3	.00728	277	.00722	266	.00716	256
0.4	.01473	+155	.01452	+139	.01431	+125
0.5	.02367	-4	.02315	-19	.02265	-33
0.6	.03261	133	.03165	142	.03073	149
0.7	.04033	197	.03885	197	.03744	195
0.8	.04619	198	.04418	190	.04230	181
0.9	.05014	159	.04769	147	.04541	135
1.0	.05253	108	.04975	96	.04718	85
1.1	.05383	64	.05083	55	.04808	47
1.2	.05447	33	.05134	27	.04849	22
1.3	.05475	15	.05155	12	.04865	9
1.4	.05486	6	.05164	5	.04871	3
1.5	.05490	2	.05167	-2	.04873	-1
1.6	.05492	-1	.05167	0	.04874	0
1.7	.05492	0			.04874	0

ξ	W_{16}	M''	W_{17}	M''	W_{18}	M''	W_{19}	M''	W_{20}	M''
0.0	3.0000	-764	3.0000	-804	3.0000	-845	3.0000	-885	3.0000	-926
0.1	2.9621	748	2.9601	786	2.9581	825	2.9561	864	2.9542	902
0.2	2.8501	698	2.8423	731	2.8345	764	2.8268	796	2.8190	828
0.3	2.6689	614	2.6521	638	2.6353	661	2.6187	683	2.6021	704
0.4	2.4269	495	2.3987	505	2.3708	514	2.3431	522	2.3156	528
0.5	2.1361	336	2.0956	329	2.0557	320	2.0162	308	1.9773	294
0.6	1.8122	-144	1.7602	-117	1.7092	-88	1.6593	-55	1.6104	-20
0.7	1.4743	+ 69	1.4133	+114	1.3542	+162	1.2970	+212	1.2416	+265
0.8	1.1432	279	1.0777	336	1.0151	394	0.9555	452	0.8986	511
0.9	0.8392	452	0.7747	509	0.7143	564	0.6578	616	0.6051	665
1.0	0.5793	557	0.5212	599	0.4681	636	0.4198	667	0.3758	692
1.1	0.3735	573	0.3259	591	0.2837	602	0.2465	605	0.2137	602
1.2	0.2236	509	0.1883	501	0.1581	488	0.1324	469	0.1106	445
1.3	0.1237	394	0.1000	370	0.0806	342	0.0648	313	0.0519	283
1.4	0.0630	268	0.0487	238	0.0375	209	0.0288	181	0.0220	155
1.5	0.0295	162	0.0217	135	0.0159	112	0.0116	91	0.0084	74
1.6	0.0126	86	0.0088	68	0.0061	53	0.0042	41	0.0029	31
1.7	0.0049	41	0.0032	30	0.0021	22	0.0014	16	0.0009	11
1.8	0.0018	17	0.0011	12	0.0007	8	0.0004	5	0.0002	4
1.9	0.0006	7	0.0003	4	0.0002	3	0.0001	+ 2	0.0001	+ 1
2.0	0.0002	2	0.0001	+ 1	0.0000	+ 1	0.0000	0	0.0000	0
2.1	0.0000	+ 1	0.0000	0	0.0000	0				
2.2	0.0000	0								

ξ	W_{21}	M''	W_{22}	M''	W_{23}	M''
0.0	3.0000	-966	3.0000	-1007	3.0000	-1048
0.1	2.9522	940	2.9502	978	2.9482	1017
0.2	2.8113	859	2.8036	891	2.7959	921
0.3	2.5855	724	2.5690	743	2.5526	761
0.4	2.2883	532	2.2612	535	2.2343	536
0.5	1.9389	-277	1.9010	-258	1.8636	-237
0.6	1.5625	+ 18	1.5157	+ 59	1.4699	+ 102
0.7	1.1880	319	1.1362	375	1.0861	432
0.8	0.8445	569	0.7930	626	0.7441	682
0.9	0.5559	710	0.5102	750	0.4677	786
1.0	0.3359	710	0.2998	723	0.2672	730
1.1	0.1849	593	0.1597	579	0.1377	561
1.2	0.0922	419	0.0767	391	0.0637	362
1.3	0.0415	253	0.0331	224	0.0263	197
1.4	0.0168	131	0.0128	110	0.0097	92
1.5	0.0061	59	0.0044	47	0.0032	37
1.6	0.0020	23	0.0014	17	0.0009	13
1.7	0.0006	8	0.0004	5	0.0002	4
1.8	0.0002	2	0.0001	+ 2	0.0001	+ 1
1.9	0.0000	+ 1	0.0000	0	0.0000	0
2.0	0.0000	0				

Det Kongelige Danske Videnskabernes Selskab

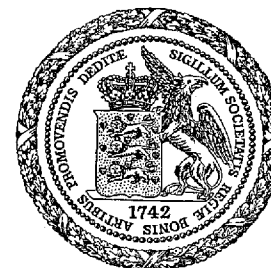
Matematisk-fysiske Meddelelser, bind 29, nr. 6

Dan. Mat. Fys. Medd. 29, no. 6 (1955)

A SIX GAP β -RAY SPECTROMETER

BY

O. B. NIELSEN AND O. KOFOED-HANSEN



København 1955

i kommission hos Ejnar Munksgaard