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ON THE ORIGINAL ORBIT OF COMET 1899 I

BY

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Comet 1899 I belongs to the group of comets for which a definitive calculation gives a hyperbolic orbit and therefore is fit to be used as basis of an investigation of the original orbit before the comet entered the region of the Sun and the major planets. The following elements have been computed by Merfield from 580 observations from March 4 to August 10, 1899 (159 days), all perturbations being taken into account (*Astronomische Nachrichten* 3748).

Osculation 1899, March 12

$$
\begin{aligned}
T &= 1899,\ \text{April } 12.978010\ \text{G.M.T.} \\
\omega &= 8^\circ 41' 46''.48 \\
\Omega &= 24\ 59\ 59.93 \quad \Big\}\ 1900.0 \\
i &= 146\ 15\ 30.29 \\
q &= 0.32657237 \pm 0.00000162 \\
e &= 1.00035029 \pm 0.00000404 \quad \Big\}\ \text{Probable errors.}
\end{aligned}
$$

From the values of q and e we compute the reciprocal value of the semi-major axis and its mean error:—

$$\frac{1}{a} = -0.0010726 \pm 0.0000184 .$$

The orbit does not quite fulfil Elis Strömgren's requirement, not having a period of observations of at least 6 months. As the number of observations, however, is large and as moreover another computation by Wedemeyer has given practically the same elements, I have all the same carried out a computation of the perturbed orbit in the years before the time of perihelion.

As the distance of perihelion is small, we have to start the computation by Encke's method. If we take all perturbations into account we get the following rectangular, ecliptical perturbations in units of the 8th decimal referred to the equinox of 1900.0:—

G. M. T.			ξ	η	ζ
1899	Mar.	30....	+ 32	0	+ 18
		20....	7	0	4
		10....	1	0	1
	Feb.	28....	15	— 5	15
		18....	54	31	58
		8....	123	87	135
	Jan.	29....	228	182	246
		19....	367	322	389
		9....	537	518	563
1898	Dec.	30....	730	778	769
		20....	941	1111	1012
	Nov.	30....	1377	2022	1613
		10....	1848	3299	2400
	Oct.	21....	2413	4971	3414
		1....	3112	7061	4707
	Sep.	11....	3968	9596	6340
	Aug.	22....	4983	12616	8388
		2....	6142	16181	10940
	July	13....	7431	20374	14106
	June	23....	+ 8832	— 25296	+ 18018

On July 13, 1898, the comet had a distance from the Sun sufficient for the direct integration of the co-ordinates. From the elements we compute the unperturbed co-ordinates and velocities:—

$$x_0 = -3.9804483 \qquad 20\frac{dx_0}{dt} = +0.22685216$$

$$y_0 = +0.7341700 \qquad 20\frac{dy_0}{dt} = +0.01624977$$

$$z_0 = -1.5681124 \qquad 20\frac{dz_0}{dt} = +0.05420179$$

From the scheme of perturbations we get:—

$$\xi = +0.0000743 \qquad 20\frac{d\xi}{dt} = -0.00001342$$

$$\eta = -0.0002037 \qquad 20\frac{d\eta}{dt} = +0.00004540$$

$$\zeta = +0.0001411 \qquad 20\frac{d\zeta}{dt} = -0.00003515$$

The additions give the perturbed co-ordinates and velocities:—

$$x = -3.9803740 \qquad 20\frac{dx}{dt} = +0.22683874$$

$$y = +0.7339663 \qquad 20\frac{dy}{dt} = +0.01629517$$

$$z = -1.5679713 \qquad 20\frac{dz}{dt} = +0.05416664$$

These values lead to the following perturbed co-ordinates:—

G. M. T.	x	y	z
1898 July 13....	— 3.9803740	+ 0.7339663	— 1.5679713
June 23....	4.2044239	0.7171655	1.6210429
3....	4.4232304	0.6994595	1.6720819
May 14....	4.6372405	0.6809836	1.7212956
Apr. 24....	4.8468382	0.6618484	1.7688573
4....	5.0523545	0.6421448	1.8149146
Mar. 15....	5.2540889	0.6219487	1.8595938
Feb. 23....	5.4522899	0.6013243	1.9030045
3....	5.6471875	0.5803266	1.9452424
Jan. 14....	5.8389844	0.5590028	1.9863921
1897 Dec. 25....	6.0278616	0.5373940	2.0265296
5....	6.2139831	0.5155363	2.0657228
Oct. 26....	6.5785402	0.4711939	2.1415169
Sep. 16....	6.9336942	0.4261793	2.2142024
Aug. 7....	7.2803209	0.3806469	2.2841321
June 28....	7.6191655	0.3347153	2.3515993
May 19....	7.9508669	0.2884762	2.4168488
Apr. 9....	8.2759793	0.2420035	2.4800880
Feb. 28....	8.5949862	0.1953550	2.5414930
Jan. 19....	8.9083123	0.1485781	2.6012166
1896 Dec. 10....	— 9.2163332	+ 0.1017119	— 2.6593910

G. M. T.			x	y	z
1896	Oct.	31....	− 9.5193835	+ 0.0547878	− 2.7161315
	Sep.	21....	9.8177625	+ 0.0078325	2.7715408
	Aug.	12....	10.1117395	− 0.0391322	2.8257091
	July	3....	10.4015580	0.0860880	2.8787173
	May	24....	10.6874388	0.1330196	2.9306380
	Apr.	14....	10.9695831	0.1799144	2.9815361
	Mar.	5....	11.2481749	0.2267614	3.0314711
1895	Dec.	16....	11.7953637	0.3202778	3.1286603
	Sep.	27....	12.3302033	0.4135133	3.2225805
	July	9....	12.8537214	0.5064280	3.3135462
	Apr.	20....	13.3668048	0.5989955	3.4018256
	Jan.	30....	13.8702286	0.6911966	3.4876483
1894	Nov.	11....	14.3646750	0.7830189	3.5712130
	Aug.	23....	14.8507477	0.8744537	3.6526935
	June	4....	15.3289852	0.9654958	3.7322420
	Mar.	16....	15,7998699	1.0561418	3.8099939
1893	Dec.	26....	16.2638365	1.1463891	3.8860687
	Oct.	7....	16.7212784	1.2362363	3.9605741
	July	19....	17.1725529	1.3256820	4.0336066
	Apr.	30....	17.6179854	1.4148261	4.1052531
1892	Nov.	21....	18.492489	1.591596	4.244696
	June	14....	19.346884	1.766834	4.379456
	Jan.	6....	20.182939	1.940422	4.509993
1891	July	30....	21.002159	2.112340	4.636698
	Feb.	20....	21.805827	2.282577	4.759908
1890	Sep.	13....	22.595045	2.451126	4.879910
	Apr.	6....	23,370768	2.617996	4.996957
1889	Oct.	28....	24.133827	2.783206	5.111272
	May	21....	24.884957	2.946793	5.223048
1888	Dec.	12....	25.624805	3.108802	5.332460
	July	5....	26.353957	3.269292	5.439662
	Jan.	27....	27.072941	3.428330	5.544792
1887	Aug.	20....	27.782241	3.585990	5.647975
	Mar.	13....	28.482303	3.742348	5.749324
1886	Oct.	4....	− 29.173540	− 3.897486	− 5.848941

G. M. T.			x	y	z
1886	Apr.	27....	−29.856341	−4.051488	−5.946919
1885	Nov.	18....	30.531069	4.204437	6.043342
	June	11....	31.198073	4.356416	6.138289
	Jan.	2....	31.857683	4.507503	6.231829
1884	July	26....	32.510216	4.657781	6.324029
	Feb.	17....	33.155984	4.807318	6.414948
1883	Sep.	10....	33.795284	4.956179	6.504643
	Apr.	3....	34.428409	5.104419	6.593163
1882	Oct.	25....	35.055646	5.252079	6.680556
	May	18....	35.677272	5.399183	6.766867
1881	Dec.	9....	36.293551	5.545740	6.852136
	July	2....	36.904735	5.691738	6.936401
	Jan.	23....	37.511055	5.837145	7.019699
1880	Aug.	16....	38.112718	5.981919	7.102065
	Mar.	9....	38.709904	6.126002	7.183530
1879	Oct.	1....	39.302761	6.269335	7.264126
	Apr.	24....	39.891415	6.411861	7.343884
1878	Nov.	15....	40.475960	6.553532	7.422832
	June	8....	41.056474	6.694311	7.500997
1877	Dec.	30....	41.633020	6.834176	7.578406
	July	23....	42.205647	6.973122	7.655083
	Feb.	13....	42.774405	7.111148	7.731053
1876	Sep.	6....	43.339337	7.248303	7.806338
	Mar.	30....	43.900489	7.384592	7.880959
1875	Oct.	22....	44.457911	7.520064	7.954936
	May	15....	45.011650	7.654769	8.028288
1874	Dec.	6....	45.561767	7.788758	8.101034
	June	29....	46.108323	7.922090	8.173190
	Jan.	20....	46.651385	8.054823	8.244773
1873	Aug.	13....	47.191028	8.187019	8.315796
	Mar.	6....	47.727332	8.318737	8.386275
1872	Sep.	27....	48.260387	8.450037	8.456223
	Apr.	20....	48.790289	8.580972	8.525653
1871	Nov.	12....	49.317141	8.711593	8.594575
	June	5....	−49.841056	−8.841938	−8.663002

On November 12, 1871, when the comet had a distance of 50.8 units from the Sun, it was so remote from the planets that the perturbations had only insignificant influence on the movement of the comet.

Now we compute the following velocity components $\frac{dx}{dt}$, $\frac{dy}{dt}$, $\frac{dz}{dt}$ and reductions to the centre of gravity of the Sun and the 8 major planets $\xi_\odot$, $\eta_\odot$, $\zeta_\odot$, $\frac{d\xi_\odot}{dt}$, $\frac{d\eta_\odot}{dt}$, $\frac{d\zeta_\odot}{dt}$. The addition gives the resulting centre co-ordinates and velocities $\bar{x}$, $\bar{y}$, $\bar{z}$, $\frac{d\bar{x}}{dt}$, $\frac{d\bar{y}}{dt}$, $\frac{d\bar{z}}{dt}$.

$x = -49.31714$	$y = -8.71159$	$z = -8.59458$
$\xi_\odot = +\quad 14$	$\eta_\odot = -\quad 320$	$\zeta_\odot = -\quad 1$
$\bar{x} = -49.31700$	$\bar{y} = -8.71479$	$\bar{z} = -8.59459$

$\frac{dx}{dt} = +0.00328353$	$\frac{dy}{dt} = +0.00081548$	$\frac{dz}{dt} = +0.00042916$
$\frac{d\xi_\odot}{dt} = +\quad 566$	$\frac{d\eta_\odot}{dt} = +\quad 173$	$\frac{d\zeta_\odot}{dt} = -\quad 10$
$\frac{d\bar{x}}{dt} = +0.00328919$	$\frac{d\bar{y}}{dt} = +0.00081721$	$\frac{d\bar{z}}{dt} = +0.00042906$

From these we find:—

$$\bar{r} = \sqrt{\bar{x}^2 + \bar{y}^2 + \bar{z}^2} = 50.81320$$

$$V^2 = \left(\frac{d\bar{x}}{dt}\right)^2 + \left(\frac{d\bar{y}}{dt}\right)^2 + \left(\frac{d\bar{z}}{dt}\right)^2 = 0.00001167070.$$

If these values are substituted in the equation of conservation of energy:—

$$\frac{V^2}{k^2(1+m)} = \frac{2}{\bar{r}} - \frac{1}{\bar{a}},$$

in which:—

$$k^2(1+m) = 0.0002963093,$$

we find:—

$$\frac{1}{\bar{a}} = -0.0000270 \pm 0.0000184.$$

Thus the computation has given the result that the original orbit was hyperbolic, though only in a slight degree.

The result of the investigations up to now is that in 22 out of 23 cases the orbits have changed in a hyperbolic direction during the time when the comets moved from far off to the region of perihelion. Only in 3 cases the computations have shown an original hyperbolic orbit, but two of the results must be considered inconclusive as all perturbations had not been taken into account. The third case is comet 1899 I, where the original $\frac{1}{a}$ is negative, and numerically larger than the mean error.

In fact we have here the first example of a comet for which all perturbations have been taken into account and which all the same shows an originally hyperbolic orbit. However, since a change in $\frac{1}{a}$ equal to 1.5 times the computed mean error would make the orbit elliptical, it cannot be said that an originally hyperbolic orbit has been established.

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