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ON THE COUPLING CONSTANTS IN β-DECAY. EVIDENCE FROM ALLOWED TRANSITIONS

BY

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1. Introduction.

It is the aim of the present paper to discuss the information on the coupling in β-decay which can be gained from the allowed transitions. At the present stage of experimental information, it seems that the best determination of the coupling is achieved by comparing the experimental ft-values with the calculated nuclear matrix elements. In the following, we consider the mirror transitions and a few other favoured transitions, since methods for estimating matrix elements for unfavoured transitions are more uncertain. The precise investigation of the shape of β-spectra and the angular correlation in β-decay (recoil experiments) is also valuable for the determination of the coupling and will be discussed in relation to the present considerations. Information may also be obtained from polarization experiments which have already been discussed in detail by de Groot and Tolhoek[1].

In the following, we shall consider an arbitrary mixture of the five linearly independent invariants in β-theory (Table 1), which, for allowed transitions $\Delta I = \begin{cases} 0 \\ 1 \end{cases}$ no, leads to the following β-spectrum[1]*):

$$
\left.\begin{aligned}
P_{\pm}(E) &= \frac{m^5 c^4}{2\pi^3\hbar^7} F(Z,E)\, pE\,(E_{\max}-E)^2 \\
&\quad \left[(g_1^2+g_2^2)\left|\int 1\right|^2+(g_3^2+g_4^2)\left|\int\vec{\sigma}\right|^2 \mp \frac{2\gamma}{E}\left(g_1 g_2\left|\int 1\right|^2+g_3 g_4\left|\int\vec{\sigma}\right|^2\right)\right] \\
&= \frac{m^5 c^4}{2\pi^3\hbar^7} F(Z,E)\, pE\,(E_{\max}-E)^2 \\
&\quad \left[g_F^2(1\mp b_F/E)\left|\int 1\right|^2+g_{GT}^2(1\mp b_{GT}/E)\left|\int\vec{\sigma}\right|^2\right] \\
&= \frac{m^5 c^4}{2\pi^3\hbar^7} F(Z,E)\, pE\,(E_{\max}-E)^2(1+b/E) \\
&\quad \left[g_F^2\left|\int 1\right|^2+g_{GT}\left|\vec{\sigma}\right|^2\right],
\end{aligned}\right\} \quad (1)
$$

*) In formula (1), we have omitted the pseudoscalar term which, according to its selection rules, contributes to allowed transitions only through higher order terms[22]).

where we have used the notation from ref. 1, with

$$\left.\begin{aligned}
&g_1^2+g_2^2 = g_F^2 \quad \text{and} \quad g_3^2+g_4^2 = g_{GT}^2 \\
&b_F = \frac{2\gamma g_1 g_2}{g_1^2+g_2^2} \quad b_{GT} = \frac{2\gamma g_3 g_4}{g_3^2+g_4^2} \\
&b = \mp \frac{2\gamma\left[g_1 g_2 \left|\int 1\right|^2 + g_3 g_4 \left|\int \vec{\sigma}\right|^2\right]}{g_F^2 \left|\int 1\right|^2 + g_{GT} \left|\int \vec{\sigma}\right|^2} \\
&\gamma = \sqrt{1-(\alpha Z)^2}.
\end{aligned}\right\} \quad (1\,\text{a})$$

The cross terms $g_1 g_2$ and $g_3 g_4$ are the so-called Fierz terms. They are in general assumed to vanish. In section 2, where we shall discuss the information on the coupling, which can be gained from experimental ft-values in combination with calculated matrix elements, these terms are neglected. In section 3,

TABLE 1.

Invariant	Coupling constant	Nuclear matrix element. Allowed transitions	Selection rules
Scalar	g_1 } g_F	$\int 1$	$\Delta I = 0$ no
Vector............	g_2 } g_F	$\int 1$	$\Delta I = 0$ no
Tensor............	g_3 } g_{GT}	$\int \vec{\sigma}$	$\Delta I = \begin{cases}0\\1\end{cases}$ no no $0 \to 0$
Pseudovector......	g_4 } g_{GT}	$\int \vec{\sigma}$	$\Delta I = \begin{cases}0\\1\end{cases}$ no no $0 \to 0$
Pseudoscalar	g_5	$\int \gamma_5$	$\Delta I = 0$ yes

however, it will be shown how uncertain is the usual argument against the existence of cross terms, and a discussion of the influence of cross terms on the results obtained in section 2 will be given, together with a few remarks on the interpretation of angular correlation experiments in view of the possible existence of cross terms.

2. Information from *ft*-values and matrix elements.

If we neglect cross terms the total disintegration probability is given by

$$\lambda = \frac{m^5 c^4}{2\pi^3 \hbar^7}\left[g_F^2 \left|\int 1\right|^2 + g_{GT}^2 \left|\int \vec{\sigma}\right|^2\right] f$$

or

$$B = ft\left[(1-x)\left|\int 1\right|^2 + x\left|\int \vec{\sigma}\right|^2\right], \tag{2}$$

where

$$B = \frac{2\pi^3 \hbar^7 ln\, 2}{(g_F^2 + g_{GT}^2)\, m^5 c^4} \quad \text{and } x = \frac{g_{GT}^2}{g_F^2 + g_{GT}^2}. \tag{2 a}$$

It is seen that the experimental *ft*-value combined with calculated matrix elements determines a straight line in a B-x diagram for each allowed β-transition. The common intersection point for such lines determines g_F^2 and g_{GT}^2.

For mirror transitions, we can neglect the change in the radial wave functions and thus compute $\left|\int 1\right|^2$ and $\left|\int \vec{\sigma}\right|^2$ from the angular wave functions alone. In this case, the Fermi matrix element is given by

$$\left.\begin{aligned} \left|\int 1\right|^2 &= \sum_{M'} \left|\langle JM' \left| \sum_i Q_i \right| JM\rangle\right|^2 \\ &= \left|\langle JM \left| \sum_i Q_i \right| JM\rangle\right|^2 \end{aligned}\right\} \tag{3}$$

where Q_i changes the i'th neutron into a proton. We have specified the total angular momentum J and its z component M only.

The Gamow-Teller matrix element is, correspondingly,

$$\left.\begin{aligned} \left|\int \vec{\sigma}\right|^2 &= \sum_{M'} \sum_k \left|\langle JM' \left| \sum_i \sigma_{ki} Q_i \right| JM\rangle\right|^2 \\ &= \frac{J(J+1)}{M^2} \left|\langle JM \left| \sum_i \sigma_{zi} Q_i \right| JM\rangle\right|^2 \end{aligned}\right\} \tag{4}$$

according to the general rules for matrix elements of vector components[2)].

If the state of the mirror nuclei in question can be described in terms of a single particle outside a core, coupled to an angular momentum zero, one obtains

$$
\left.\begin{aligned}
\left|\int 1\right|^2 &= \left|\langle 1\rangle\right|^2 = 1\\
\left|\int \vec{\sigma}\right|^2 &= \frac{J(J+1)}{M^2}\left|\langle\sigma_z\rangle_{J_z=M}\right|^2\\
&= \frac{J(J+1)}{M^2}\left|\left\langle\frac{\vec{\sigma}\cdot\vec{J}}{J^2}\right\rangle\langle J_z\rangle\right|^2\\
&= \begin{cases}\dfrac{J}{J+1} & \text{for}\quad J = l-{}^1/_2\\[2mm] \dfrac{J+1}{J} & \text{for}\quad J = l+{}^1/_2\end{cases}
\end{aligned}\right\}\qquad (5)
$$

where l is the orbital angular momentum of the odd particle.

In Table 2, we have listed the experimental ft-values for all mirror transitions together with spins and magnetic moments for the daughter nuclei. In the table we have also listed the shell model configuration assignments for the particles outside closed configurations and the corresponding magnetic moments.

a) Closed shell $\pm$ 1 nuclei[4].

It is generally believed that single-particle states are most purely realized for those mirror nuclei which have closed shells of 0, 2, 8, 20 protons and neutrons $\pm$ one nucleon. This assumption is supported by the fact that the experimental magnetic moments generally agree rather well with those calculated from single-particle wave functions. In Table 2, under the heading closed shell $\pm$ 1, we have listed the matrix elements for these transitions found from formula (5). Using these together with the ft-values one obtains the $B(x)$-lines in Fig. 1. These lines are inside the experimental errors consistent with a common intersection point of $(B_0, x_0) = (2650 \pm 85,\ .50 \pm .05)$, where the errors are mean square deviations found from internal consistency of the data. However, these errors should not be taken too literally and some remarks in this connection will be given later*).

*) In a paper by Langer and Moffat[20], which came to our knowledge after the conclusion of this paper, a redetermination of the H^3 ft-value is given. The result is $ft = 1014 \pm 20$, which is in clearcut disagreement with a common intersection point in Fig. 1. Langer's value for $E_{max} = 17.95 \pm .10$ disagrees also with another recent value given by Hamilton, i.e. $E_{max} = 19.4 \pm .4$ keV[19]).

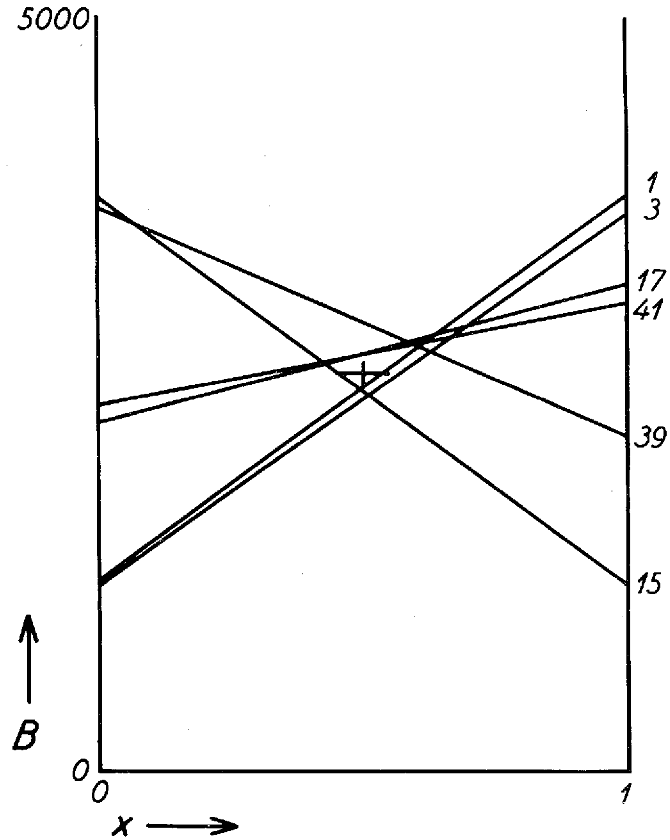

Fig. 1. $B(x)$ lines for closed shell $\pm$ one nucleon transitions. Mass numbers are indicated.

b) Other mirror transitions.

Also in other cases than those discussed under a) the shell model predicts closed configurations $\pm$ one single nucleon (i.e. for mass numbers 11, 13, 19, 27, 29, 31, 33). In these cases, the matrix elements may also be calculated from formula (5).

For several particles outside closed configurations the *ft*-value depends sensitively on the coupling scheme.

If one assumes that the even structures couple to zero angular momentum, as is often done in shell model calculations of mag-

TABLE 2. The E_{max} quoted are often taken from reaction data (ref. 1
where the formulas of FEENBER

	Transition	E_{max} (MeV)	t	ft-value	spin	μ_{exp} daughter
1	n → p	0.782 ± 0.001[15])	12.8^m ± 2.5	1280 ± 250	$^1/_2$	2.7
3	H → He	0.0191 ± 0.0005[14]) [18])	12.45^y ± 0.2	1240 ± 120	$^1/_2$	—2.1
7	Be → Li	0.863 ± 0.002[15])	52.9^d ± 0.2	2240 ± 40	$^3/_2$	3.2
11	C → B	0.958 ± 0.003[15])	20.39^m ± 0.06	3840 ± 70	$^3/_2$	2.6
13	N → C	1.200 ± 0.005[15])	10.1^m ± 0.1	4560 ± 100	$^1/_2$	0.7
15	O → N	1.683 ± 0.005[14])	2.1^m ± 0.1	3800 ± 200	$^1/_2$	—0.2
17	F → O	1.745 ± 0.006[16])	65^s ± 2	2320 ± 100	$^5/_2$	—1.8
19	Ne → F	2.234 ± 0.005[16])	19.5^s ± 1.0	1970 ± 100	$^1/_2$	2.6
21	Na → Ne	2.50 ± 0.03[14])	22.8^s ± 0.5	3700 ± 200	$(^3/_2)$	<0
23	Mg → Na	3.073 ± 0.010[16])	12.0^s ± 0.2	4780 ± 150	$^3/_2$	2.2
25	Al → Mg	..	7.3^s	..	$(^5/_2)$	—0.8
27	Si → Al	3.48 ± 0.10[17])	5.0^s ± 0.4	3350 ± 600	$^5/_2$	3.6
29	P → Si	3.60 ± 0.15[14])	4.6^s ± 0.2	3510 ± 700	$^1/_2$	—0.5
31	S → P	4.06 ± 0.12[17])	3.1^s ± 0.2	4020 ± 600	$^1/_2$	1.1
33	Cl → S	4.43 ± 0.13[17])	2.0^s ± 0.2	3800 ± 650	$^3/_2$	0.6
35	A → Cl	4.4 ± 0.2[14])	1.90^s ± 0.05	3420 ± 800	$^3/_2$	0.8
37	K → A	4.57 ± 0.13[17])	1.2^s ± 0.2	2520 ± 600	..	..
39	Ca → K	5.13 ± 0.15[17])	1.06^s ± 0.03	3740 ± 500	$^3/_2$	0.3
41	Sc → Ca	4.9 ± 0.2[14])	0.87^s ± 0.03	2430 ± 800	..	..

*) probably in strong admixture with $d^2_{5/2}\, s_{1/2}$ which would make $\left|\int 1\right|^2$ and

netic moments, etc., the j—j coupling scheme leads to unique wave functions. The matrix elements are easily calculated using the formulae (3) and (4) together with the rules for matrix elements of sums of single-particle operators[3]). The result is shown nd 16). The ft-values are calculated numerically except for $Z < 7$, nd Trigg[18)] have been used.

onfiguration	$\mu_{\text{theor.}}$	closed shell $\pm$ 1		j-j coupling even structure 0		T-multiplet		semi-empirical	
		$\lvert\int 1\rvert^2$	$\lvert\int \vec{\sigma}\rvert^2$	$\lvert\int 1\rvert^2$	$\lvert\int \vec{\sigma}\rvert^2$	$\lvert\int 1\rvert^2$	$\lvert\int \vec{\sigma}\rvert^2$	$\lvert\int 1\rvert^2$	$\lvert\int \vec{\sigma}\rvert^2$
$s_{1/2}$	2.79	1	3	..	..	..	..	1	
$\overset{3}{s}_{1/2}$	—1.91	1	3	..	..	..	..	1	
$\overset{3}{p}_{3/2}$	3.79	..	..	$^1/_4$	$^5/_{12}$	..	..	1	0.98
$\left(\overset{3}{p}_{3/2}\right)^{1/2,\,1/2}_{3/2}$	3.03	..	..	..	..	1	$^{121}/_{135}$		
$\overset{5}{p}_{3/2}$	3.79	..	..	..	..	1	$^5/_3$	1	0.45
$p_{1/2}$	0.64	..	..	..	..	1	$^1/_3$	1	0.40
$\overset{3}{p}_{1/2}$	—0.26	1	$^1/_3$	..	..	..	..	1	0.35
$d_{5/2}$	—1.91	1	$^7/_5$	..	..	..	..	1	1.37
$\overset{3}{s}_{1/2}$*)	2.79	..	..	..	..	1	3	1	2.59
$\left(\overset{5}{d}_{5/2}\right)_{3/2}$	—1.14	..	..	0	0	1	not unique	1	—
$\left(\overset{7}{d}_{5/2}\right)_{3/2}$	2.89	..	..	0	0	1		1	0.16
$\overset{9}{d}_{5/2}$	—1.91	..	..	$^1/_9$	$^7/_{45}$	..	..	1	0.28
$\left(\overset{9}{d}_{5/2}\right)^{1/2,\,1/2}_{5/2,\,5/2}$	—1.04	..	..	..	..	1	0.784		
$\overset{11}{d}_{5/2}$	4.79	..	..	..	..	1	$^7/_5$	1	0.35
$s_{1/2}$	—1.91	..	..	..	..	1	3	1	0.25
$\overset{3}{s}_{1/2}$	2.79	..	..	..	..	1	3	1	0.23
$d_{3/2}$	1.15	..	..	..	..	1	$^3/_5$	1	0.19
$\overset{3}{d}_{3/2}$	0.13	..	..	$^1/_4$	$^3/_{20}$	..	..	1	0.15
$\left(\overset{3}{d}_{3/2}\right)^{1/2,\,1/2}_{3/2}$	0.27	..	..	..	..	1	$^{121}/_{375}$		
$\overset{5}{d}_{3/2}$	1.15	..	..	$^1/_4$	$^3/_{20}$	..	..	1	—
$\left(\overset{5}{d}_{3/2}\right)^{1/2,\,1/2}_{3/2}$	1.01	..	..	..	..	1	$^{121}/_{375}$		
$\overset{7}{d}_{3/2}$	0.13	1	$^3/_5$	..	..	..	..	1	0.39
$f_{7/2}$	—1.91	1	$^9/_7$	..	..	..	..	1	—

$\lvert\int \vec{\sigma}\rvert^2$ smaller without altering μ.

in Table 2 under the heading "j—j coupling, even structure 0". The ft-values calculated from these numbers, using the $B_0\, x_0$ value given in a), are in very poor agreement with the experimental ft-values. This is not surprising since the wave functions used are not consistent with the charge independence of nuclear forces.

In general, the wave function will depend on the nuclear forces. However, in some cases (i.e. mass number 7, 25, 35, 37), the wave functions may be uniquely constructed from the assumption that the ground state has the lowest possible value of the total isotopic spin T[5].

The wave function for Li^7, where $J = M = {}^3/_2$ and $T = T_\zeta = {}^1/_2$ is thus given by[23]

$$\left(p^3_{3/2}\right)^{1/2,\,1/2}_{3/2,\,3/2} = \sqrt{\frac{4}{15}}\, \overset{N}{p}_{3/2,\,1/2}\, \overset{N}{p}_{3/2,\,-1/2}\, \overset{P}{p}_{3/2,\,3/2} - \sqrt{\frac{9}{15}}\, \overset{N}{p}_{3/2,\,3/2}\, \overset{N}{p}_{3/2,\,-3/2}\, \overset{P}{p}_{3/2,\,3/2}$$
$$+ \sqrt{\frac{1}{15}}\, \overset{N}{p}_{3/2,\,3/2}\, \overset{N}{p}_{3/2,\,-1/2}\, \overset{P}{p}_{3/2,\,1/2} - \sqrt{\frac{1}{15}}\, \overset{N}{p}_{3/2,\,3/2}\, \overset{N}{p}_{3/2,\,1/2}\, \overset{P}{p}_{3/2,\,-1/2}$$

where we have left out the antisymmetrization operator

$$\frac{1}{\sqrt{3!}} \sum_P (-1)^P P,$$

where P means the permutation of all three particle coordinates. For mass numbers 21 and 23, where $j = {}^5/_2$, the wave functions are not uniquely determined from the assumption $T = {}^1/_2$.

If the total isotopic spin T is a good quantum number, one can easily show that the Fermi matrix element for a transition between a state T, T_ζ and a state $T', T_\zeta - 1$ is given by[6]

$$\left|\int 1\right|^2 = (T + T_\zeta)\,(T - T_\zeta + 1)\,\delta_{TT'}. \tag{8}$$

For mirror transitions, where $T' = T = {}^1/_2$ and $T_\zeta = {}^1/_2$, we get $\left|\int 1\right|^2 = 1$.

The Gamow-Teller matrix element may be calculated using explicit wave functions inserted in Eq. (4). The results are listed in Table 2, under the heading "T-multiplet", together with the matrix elements for the above mentioned cases, where we have to do with closed configurations $\pm$ one particle. In Fig. 2, the corresponding $B\,(x)$ lines are plotted. The $B_0\, x_0$ value from section a) is indicated by a cross. It is seen that the agreement with this value of B, x (or any other value) is very poor. However, it should be noted that the lines which deviate most from B_0, x_0 correspond to transitions between nuclei for which the theoretical

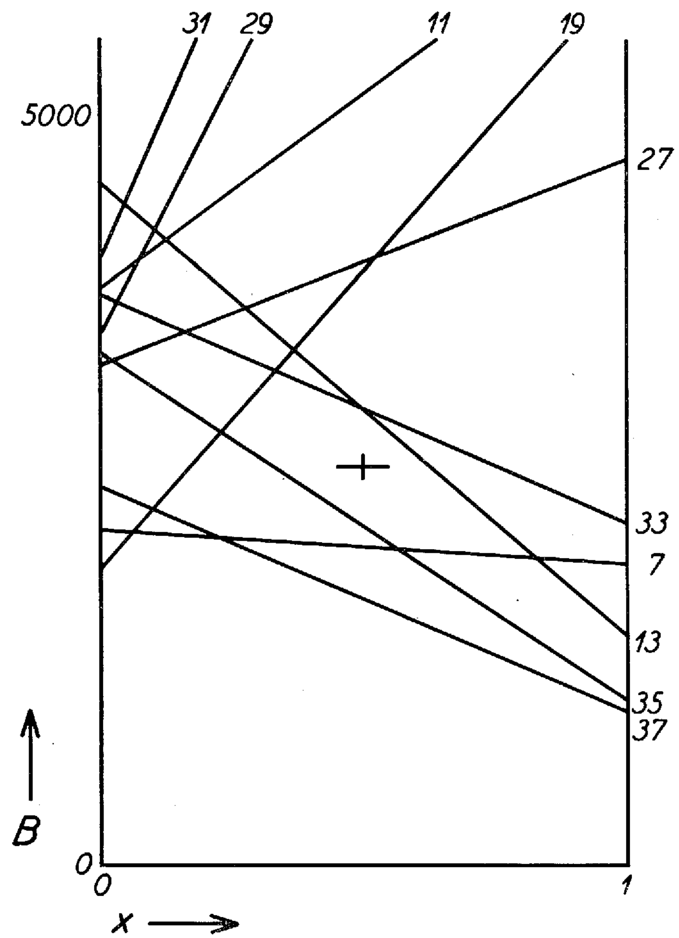

Fig. 2. $B(x)$ lines for mirror transitions with unadjusted matrix elements. Mass numbers are indicated.

magnetic moments deviate essentially from the empirical ones. It thus seems interesting to try to find whether any simple correlation exists between the ft-values and the deviations of the magnetic moments from the Schmidt lines. Such correlations have actually been found by Trigg[12)] who evaluates the matrix elements from an interpolation between the Schmidt lines and the Margenau-Wigner lines for the magnetic moments.

In the following, we shall try to give an argument for an evaluation of the Gamow-Teller matrix element which is essentially an interpolation between the two Schmidt lines.

If one assumes that the deviation of the magnetic moments from the Schmidt lines arises from an interaction between the odd particle and some other particles in the nucleus, which take over part of the angular momentum, e.g., as in the model used by A. Bohr and B. Mottelson[7], one may write the magnetic moment

$$\begin{aligned}\mu &= g_s \langle s_z \rangle_{J_z = J} + g_l \langle l_z \rangle_{J_z = J} + g_R \langle R_z \rangle_{J_z = J} \\ &= (g_s - g_l) \langle s_z \rangle_{J_z = J} + g_l J + (g_R - g_l) \langle R_z \rangle_{J_z = J}.\end{aligned}$$

Here, $\vec{J} = \vec{s} + \vec{l} + \vec{R} = \vec{j} + \vec{R}$. s and l are the spin and orbital angular momenta of the single particle, and R is the angular momentum of the system of particles to which it is coupled. $\langle\,\rangle$ means average value and the g's are the gyromagnetic ratios.

The success of the shell model in predicting spins of the light nuclei leads one to believe that $\langle j_z \rangle_{J_z = J}$ is not very different from J, that is $\langle R_z \rangle_{J_z = J}$ is small compared with J. As the factor $(g_R - g_l)$ is small also compared with $(g_s - g_1)$, g_R being perhaps of the order $^1/_2$, one may in first order neglect the last term in (9). This result in fact also turns out if one uses the model by Aage Bohr[7].

For the Gamow-Teller matrix element, we get from (4)

$$\left|\int \vec{\sigma}\right|^2 = 4\,\frac{J+1}{J}\left|\langle s_z \rangle_{J_z = J}\right|^2.$$

$\langle s_z \rangle$ may be inserted from (9) after neglect of the last term and we thus get the following approximate formula

$$\left|\int \vec{\sigma}\right|^2 \approx 4\,\frac{J+1}{J}\left(\frac{\mu - g_l J}{g_s - g_l}\right)^2. \tag{10}$$

For the Fermi matrix element we find from (8)

$$\left|\int 1\right|^2 = 1.$$

Using the magnetic moments listed in Table 2, we can thus obtain the matrix elements listed under the heading "semi-empirical". The results are plotted in Fig. 3. The full-drawn lines

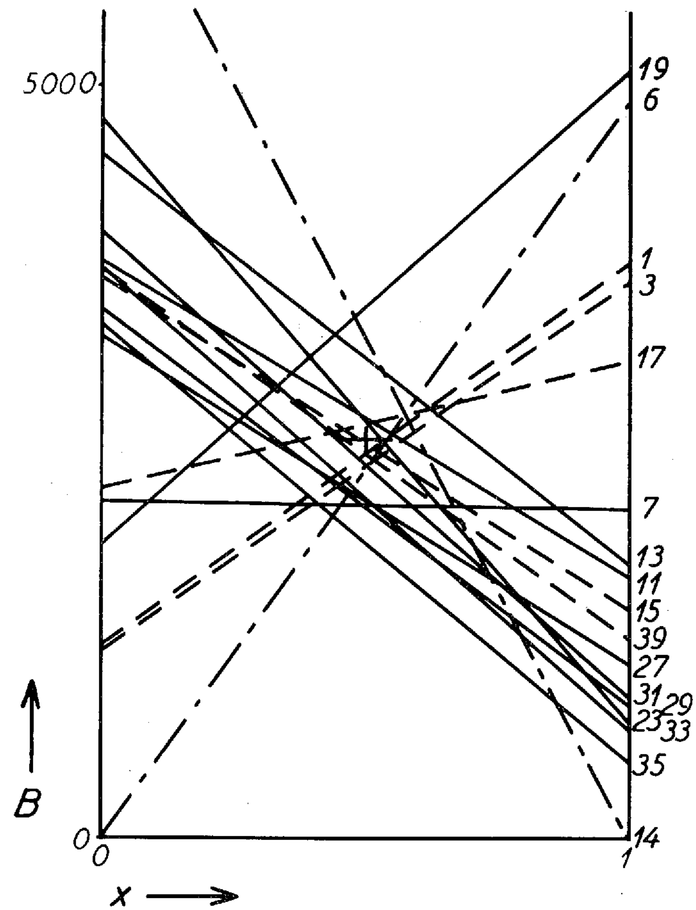

Fig. 3. $B(x)$ lines for mirror transitions with adjusted matrix elements and for He^6 and O^{14} decay. Mass numbers are indicated.
— — — closed shell ± 1 transitions.
——— other mirror transitions.
— · — He^6 and O^{14} decay.

are essentially the same transitions as those plotted in Fig. 2. The dotted lines are those transitions which were used in Fig. 1 for the determination of B_0, x_0. The use of formula (10) instead of

(5) has changed these lines only by small amounts. The semi-empirical method does not work, however, in the case of the triton decay ($|\langle s_z\rangle| > 1/2$) where we have used the value 3†).

The deviations of the full-drawn lines from the B_0, x_0 value are, in several cases, larger than the experimental uncertainty. We have tried to evaluate the magnitude of the term $(g_R - g_l)\langle R_z\rangle_{J_z = J}$ which would be necessary to explain this deviation. It turns out that $|g_R - g_l| \approx 1/2$ and $|\langle R_z\rangle_{J_z = J}| \lesssim 1/2$ will suffice to explain the deviations in all cases. One might thus hope that a theory which in detail explains the magnetic moments will at the same time explain the matrix elements for mirror transitions.

c) Even-A transitions and unfavoured transitions.

Among the allowed even-A transitions only those which are of the type $0 \rightarrow 0$ no are very simple. For transitions of this type, the Gamow-Teller matrix elements vanish. The Fermi matrix element may be calculated from formula (8) which is based on the assumption of charge independence of nuclear forces only. Until now, two cases of $0 \rightarrow 0$ no transitions are known with some certainty, namely $C^{10} \rightarrow B^{10*}$ and $O^{14} \rightarrow N^{14*}$. The experimental data are listed in Table 3.

TABLE 3.

Decay	E_{max} (MeV)	t	Branching ratio	ft-value	$\|\int 1\|^2$	$\|\int \vec{\sigma}\|^2$
$C^{10} \rightarrow B^{10*}$	1.15 ± 0.10	$19.1^s \pm 0.8$	0.021 ± 0.006	$6000 + 3000$	2	0
$O^{14} \rightarrow N^{14*}$	1.8 ± 0.1	$76.5^s \pm 0.2$	1	3300 ± 900	2	0
$He^6 \rightarrow Li^6$	3.50 ± 0.05	$0.823^s \pm 0.013$	1	815 ± 70	0	6

The O^{14} decay has been used by BLATT[9)] to determine the coupling in β-decay, but, with the present experimental uncertainty, both this transition and the C^{10} decay are consistent with our values for B_0 and x_0.

†) A more detailed estimation of the Gamow-Teller matrix element has been given recently by BLATT[8)] who finds $|\int \vec{\sigma}|^2 \approx 2.84$. As mentioned by BLATT, this result is rather uncertain. The value 3 which we have used is, according to BLATT, an upper limit.

Also the decay of He^6, the experimental data of which are listed in Table 3, has been used for the determination of the coupling[10]. It seems, however, that the Gamow-Teller matrix element for this transition is rather ambiguous (the Fermi matrix element is certainly 0). The matrix element obtained by using j—j coupling is definitely too small. The matrix element quoted in the table is obtained by L—S coupling. Although this value is probably an upper limit; we have used it for the $B(x)$ line for He^6 in Fig. 3.

A few other even-A transitions with "superallowed" ft-values exist, but matrix elements for these are even more uncertain than for the He^6 decay.

For all other allowed transitions the ft-value is a factor 50—100 larger than the ft-value for mirror transitions; they are so-called unfavoured transitions. The "unfavoured factor" may partly be understood by noting that, in all other cases than the mirror transitions, the quantum number T is different for the ground states of mother and daughter nuclei, i.e. the Fermi matrix element vanishes[6]. This will, however, only explain a part of the unfavoured factor and probably one has to take into account other differences between the nuclei than the states for the odd nucleons[7].

3. Cross terms[11].

Until now we have assumed that the products $g_1 g_2$ and $g_3 g_4$ vanish. In this section, we shall investigate in how far this assumption can be verified experimentally.

a) Spectrum shape.

As seen from Eq. (1), the cross terms are energy dependent, i.e. they will show up in a β-spectrum. If we consider a Kurie plot

$$K = \sqrt{\frac{P}{F(Z, E)\,pE}} = C\,(E_{\max} - E)\sqrt{1 + b/E}, \qquad (11)$$

the most important effect of the cross term b/E for low and medium maximum energies is a change of the slope of the plot. This change is, however, not detected since it is equivalent to a change of C. What remains is a curvature which appears as a small deviation from a straight line the slope of which is adjusted to the experimental points.

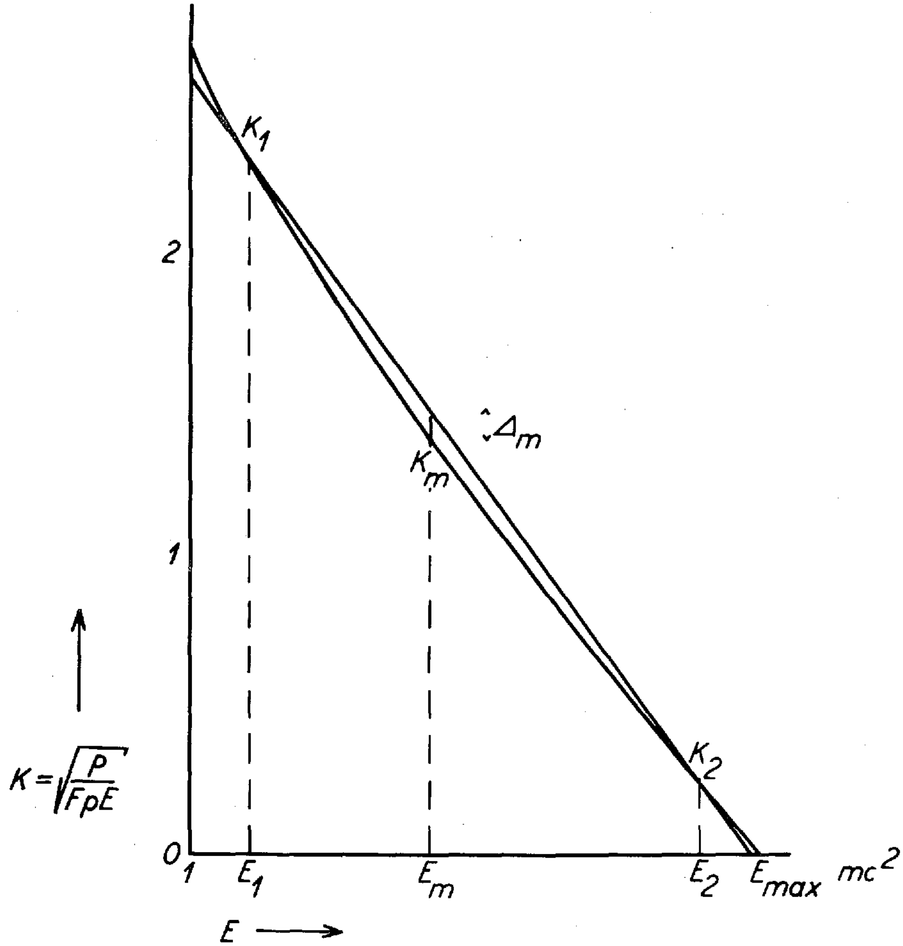

Fig. 4. Kurie plot with cross terms compared with straight line adjusted to the points E_1 and E_2.

In Fig. 4 we have illustrated an example where the straight line is adjusted to the curved Kurie plot[11)] in the points E_1 and E_2. The maximum deviation Δ_m will appear at a point $E_m \approx \sqrt{E_1 E_2}$. To illustrate the magnitude of the curvature, we have plotted in

Fig. 5 Δ_m/K_m as a function of the maximum energy for different values of b in the case where $E_1 = 1.2$ and $E_2 = E_{\max} - 0.2$.

It is seen from Fig. 5 that even large values of b ($-1 \leqslant b \leqslant 1$) give only small deviations and that, consequently, it is difficult to obtain narrow limits for b. In fact, an analysis of the published

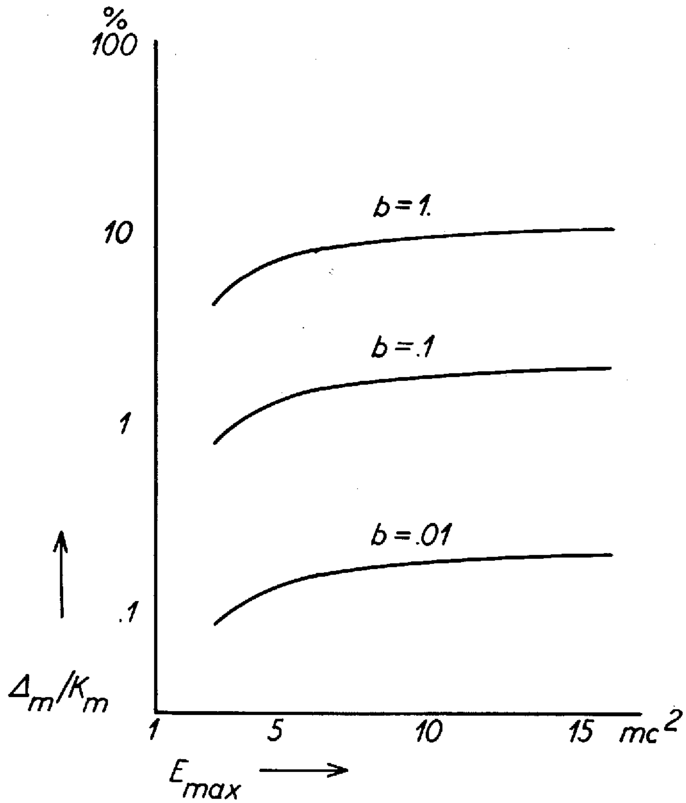

Fig. 5. Maximum deviation Δ_m/K_m of curved Kurie plot from straight line for different values of cross terms b.

β-spectra indicates that, in no case, b-values as large as 0.4 can be excluded. In such experimental comparison it should, of course, be remembered that b has opposite sign for positon and negaton emission*).

*) Recently, Mahmoud and Konopinski[21] have given a careful analysis of the shapes of some allowed β-spectra in order to set a limit on the cross terms. Their result $|b| < 0.2$ is based on a statistical treatment of all the experimental data.

Furthermore, it should be noted that experimenters usually apply the straightness of the Kurie plot as a control on their spectrometers.

The information about the coupling constants which can be obtained from a determination of b is contained in Eq. (1a). Since b depends on the nuclear matrix elements, it is of course most valuable to determine b in cases where the ratio $|\int 1|^2/|\int \vec{\sigma}|^2$ is known. As an example, we have in Fig. 7 plotted the dependence of b on the ratio g_4/g_3 in the simple case where $|\int 1|^2$ is known to vanish.

b) *ft*-values.

The cross terms will also have some influence on the *ft*-values as the new *f*-value will be given by

$$f = f_0 (1 + b\delta),$$

where f_0 is the usual Fermi integral and

$$\delta = \int F(Z, E)\, p\, (E_{\max} - E)^2\, dE / f_0.$$

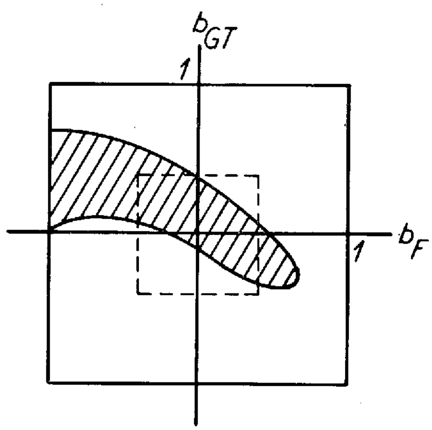

Fig. 6. Area in b_F, b_{GT} plane which is allowed according to the assumption of consistent $B(x)$ lines for closed shell $\pm$ 1 transitions.

To see what influence this will have on our considerations in section 2, we derive from Eq. (1) that

$$\frac{1}{t} = \frac{m^5 c^4}{2\pi^3 \hbar^7 \ln 2} f_0 \left[g_F^2 (1 \mp b_F \delta) |\int 1|^2 + g_{GT} (1 \mp b_{GT} \delta) |\int \vec{\sigma}|^2 \right].$$

With the notation (2a) we get

$$B = f_0 t \left[(1-x)(1 \mp b_F \delta) \left| \int 1 \right|^2 + x (1 \mp b_{GT} \delta) \left| \int \vec{\sigma} \right|^2 \right]$$

which shows that the $B(x)$ lines will still be straight lines, only they will be shifted by an amount depending on b_F and b_{GT}. This now provides us with another tool for the determination of the cross terms. If we assume that the $B(x)$ lines for the simple closed shell ± 1 nuclei have to pass through a common point within their experimental uncertainty, we can estimate limits for b_F and b_{GT}. In Fig. 6, we have indicated the area in the b_F, b_{GT} plane which is allowed according to this condition.

c) Recoil experiments.

If cross terms exist, the angular correlation between β-particle and neutrino for allowed transitions is given by[1)]

$$W(\theta_{\beta\nu}) = 1 + a p/E \cos \theta_{\beta\nu} + b/E$$

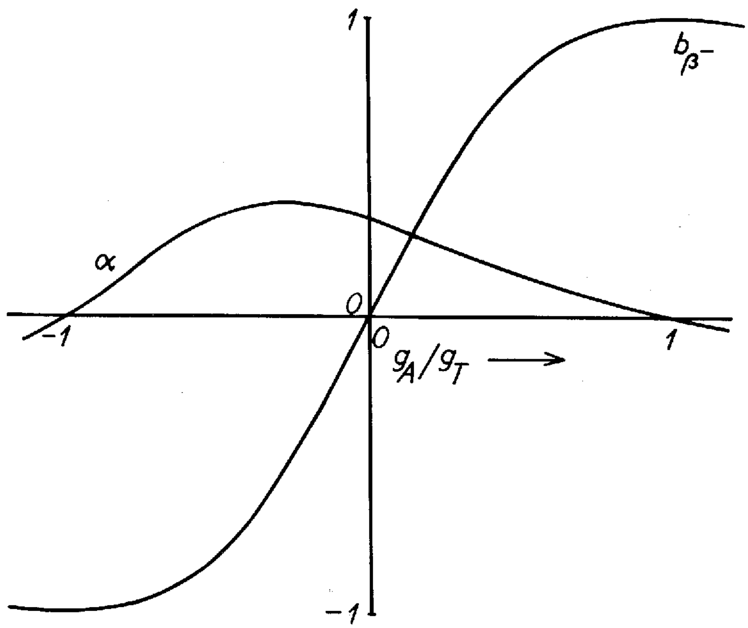

Fig. 7. The experimental angular correlation parameter a and the cross term parameter b_{β^-} for allowed GT transitions.

where

$$a = -\frac{(g_1^2 - g_2^2)\,|\int 1\,|^2 - {}^1\!/_3\,(g_3^2 - g_4^2)\,|\int \vec{\sigma}\,|^2}{g_F^2\,|\int 1\,|^2 + g_{GT}^2\,|\int \vec{\sigma}\,|^2}$$

and b is given in (1a).

In angular correlation experiments, one usually determines the ratio between the cosine-dependent term and the constant term

$$\alpha = a/(1 + b/E),$$

where E is the energy of those electrons for which the angular correlation is measured.

To illustrate what information on the coupling constants can be obtained from a determination of α, we have plotted in Fig. 7 a α as a function of g_4/g_3 for $E = 2$ in the simple case where $|\int 1\,|^2$ is known to vanish. It is seen that a determination of α in general does not permit a unique determination of g_4/g_3. It thus seems valuable to combine angular correlation experiments with β-spectroscopic measurements, especially in those cases where the ratio $|\int 1\,|^2/|\int \vec{\sigma}\,|^2$ can be estimated.

4. Summary.

In the present paper, an investigation of nuclear matrix elements has been combined with the experimental ft-values in order to determine the coupling constants for Fermi- and Gamow-Teller interaction in β-decay. It is shown that the mirror transitions between nuclei with closed shells $\pm$ one nucleon are consistent with approximately equal amounts of the two couplings. Within the limits of error, this result agrees with similar calculations carried out by Trigg[12)], Bouchez and Nataf[13)], and Blatt[9)]. For the remaining mirror transitions, the matrix elements derived from the shell model are not consistent with this result. However, as pointed out by Trigg[12)], considerable improvement can be obtained by adjusting the wave functions to the observed magnetic moments. In the present paper, it is shown that, on the assumption of an interaction between the single particle and the nuclear core, an approximate correlation exists between the Gamow-Teller matrix elements and the magnetic moments of the nuclei involved in the transitions. The results for the adjusted matrix elements thus obtained are, within the uncertainties and the approximation, in agreement with the above mentioned coupling constants.

The possible existence of cross terms in the β-decay coupling is also discussed. It is shown that the present experimental data do not provide very narrow limits for these terms, and further experiments on this matter are therefore desirable.

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References.

1. S. R. de Groot and H. A. Tolhoek, Physica **16**, 456, 1950.
2. E. Wigner, Gruppentheorie. Braunschweig 1931, p. 264.
3. E. U. Condon and G. H. Shortley, The Theory of Atomic Spectra. Cambridge 1935, p. 169 ff.
4. O. Kofoed-Hansen and A. Winther, Phys. Rev. **86**, 428, 1952.
5. E. Wigner, Phys. Rev. **51**, 106, 1937.
6. E. Wigner and E. Feenberg, Rep. Prog. Phys. 8, 274, 1941.
7. A. Bohr and B. Mottelson, Dan. Mat. Fys. Medd. **27**, no. 16 (1953).
8. J. Blatt, Phys. Rev. **89**, 86, 1953.
9. J. Blatt, Phys. Rev. **89**, 83, 1953.
10. S. A. Moszkowski, Phys. Rev. **82**, 118, 1951.
11. O. Kofoed-Hansen and A. Winther, Phys. Rev. **89**, 526, 1953.
12. G. L. Trigg, Phys. Rev. **86**, 506, 1952.
13. R. Bouchez and R. Nataf, C. R. **234**, 86, 1952.
14. Nuclear Data, N. B. S. Circ. 499.
15. C. W. Li et al., Phys. Rev. **83**, 512, 1951.
16. C. W. Li, Phys. Rev. 88, 1038, 1952.
17. F. J. Boley and D. J. Zaffarano, Phys. Rev. **84**, 1059, 1951.
18. E. Feenberg and G. L. Trigg, Revs. mod. Phys. **22**, 399, 1950.
19. D. R. Hamilton et al., Phys. Rev. **83**, 215, 1951.
20. L. M. Langer and R. J. D. Moffat, Phys. Rev. 88, 689, 1952.
21. H. M. Mahmoud and E. J. Konopinski, Phys. Rev. 88, 1266, 1952.
22. G. Alaga, O. Kofoed-Hansen and A. Winther, in preparation.
23. M. Mizushima and M. Umezawa, Phys. Rev. **85**, 37, 1952.

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