Det Kongelige Danske Videnskabernes Selskab
Matematisk-fysiske Meddelelser, bind **27**, nr. 11

Dan. Mat. Fys. Medd. **27**, no. 11 (1953)

THE FUTURE ORBIT OF COMET 1898 VII (CODDINGTON–PAULY)

BY

ERIK SINDING

København
i kommission hos Ejnar Munksgaard
1953

Printed in Denmark
Bianco Lunos Bogtrykkeri

About 20 calculations back in time of nearly parabolic comet orbits have been carried out according to the principles set out by E. STRÖMGREN[1], in order to determine the eccentricities of these orbits when the comets were moving at a great distance from the Sun. In Publ. 146 from Cop. Obs. (1948) a list of the results of these calculations is given showing the changes in $\frac{1}{a}$, the reciprocal semi-major axis. It is seen that the great majority of the orbits have changed in the elliptical direction by the backward-computation, and on an average we find:

$$\varDelta\left(\frac{1}{a}\right)_m = +\,0.000552 \text{ with a standard error of } \pm\,0.000055,$$

considering the distribution as random. Including v. BIESBROECK's calculation for Comet 1908 III[2] we find:

$$\varDelta\left(\frac{1}{a}\right)_m = +\,0.000568 \pm 0.000054$$

and for an individual orbit:

$$\varDelta\left(\frac{1}{a}\right) = +\,0.00057 \pm 0.00025.$$

Considering the conditions in the solar system in a simplified way and using a Jacobian integral for the Sun, Jupiter, and a comet it is shown that on an average we get:

$$\varDelta\left(\frac{1}{a}\right)_m = \frac{1}{\bar{a}} - \frac{1}{a} = \frac{2\,m_1}{\varrho} - \frac{m_1}{r_1^3}\,(r^2 - \varrho^2).$$

Here $\bar{a}$ is the semi-major axis in the original orbit of the comet relative to the common centre of gravity of the Sun and Jupiter, and a is the same element of orbit in the osculating orbit relative to the Sun (at an epoch near the time of perihelion). Furthermore m_1 and r_1 are the mass and radius of orbit of Jupiter, Jupiter's orbit being considered as circular, and r and ϱ are the distances comet-Sun and comet-Jupiter, respectively, at the moment of osculation. Introducing an average value of ϱ equal to $r_1 = 5.203$, $r = 1$, and $m_1 = \frac{1}{1047}$, we find:

$$\Delta\left(\frac{1}{a}\right)_m = +0.000544.$$

These considerations show that the phenomenon that nearly parabolic orbits change systematically in the elliptical direction when epochs further and further back in time are considered is plausibly explained by the dominating influence of Jupiter.

In 1906 G. Fayet published a great investigation[3] showing the result of an approximate backward-calculation of 146 nearly parabolic cometary orbits. J. H. Oort has computed $\Delta\left(\frac{1}{a}\right)_m$ for these orbits[4] and found $+0.000500$. I have made a new computation excluding Comets 1844 III and 1897 I, as Fayet in those cases used erroneous elements. The result is:

$$\Delta\left(\frac{1}{a}\right)_m = +0.000498 \pm 0.000056,$$

corresponding to:

$$\Delta\left(\frac{1}{a}\right) = +0.00050 \pm 0.00067$$

for an individual comet. (This standard deviation is different from Oort's value.)

In comparison with the standard deviation found by the 22 rigorous calculations backwards we find that this is essentially smaller, and in Oort's opinion the reason must be that by chance no great perturbations take place for these 22 comets. The real reason is undoubtedly another. The 'original elements' of Fayet are determined relative to the Sun while the same elements in the case of the rigorous calculations are computed relative to the gravity centre of the system Sun-Jupiter-Saturn (possibly more planets), and this circumstance is the cause of the great standard deviation in FAYET's case.

This is easily shown by a simple reasoning. H. SEELIGER has derived the following Jacobian integral for the motion relative to the Sun[5]:

$$V^2 = 2\gamma + \frac{2k^2}{r} + \frac{2k^2 m_1}{\varrho} + 2n_1 k\sqrt{p}\cos i - \frac{k^2 m_1}{r_1^3}(r^2 - \varrho^2)$$

or:

$$\frac{1}{a} = -2C - \frac{2m_1}{\varrho} - \frac{2n_1}{k}\sqrt{p}\cos i + \frac{m_1}{r_1^3}(r^2 - \varrho^2).$$

V is the velocity of the comet, 2γ and $2C$ constants, n_1 is the mean motion of Jupiter, and p and i the parameter and inclination, respectively, of the orbit of the comet. If we consider now the motion of the comet at greater and greater distances from the Sun the term $\frac{2m_1}{\varrho}$ is decreasing towards zero, while the last term is oscillating about zero with increasing amplitude. A great material with elements relative to the Sun will on an average yield:

$$\Delta\left(\frac{1}{a}\right)_m = \frac{2m_1}{\varrho} - \frac{m_1}{r_1^3}(r^2 - \varrho^2),$$

i. e. the same expression as given above but with a greater scattering.

The expression may be used as a matter of course, if we take into consideration the changes in $\frac{1}{a}$ in nearly parabolic orbits in forward-calculations. Now it is a well-known fact that not one decidedly hyperbolic orbit is found in backward-calculations, and that consequently all these comets originate in the solar system. But the result of the forward-calculations is that among the known, near perihelion nearly parabolic orbits there are some which are hyperbolic at a great distance from the Sun, and consequently these comets are leaving the solar system.

In accordance with the principles used for the above-mentioned 146 approximate backward-calculations FAYET has carried out forward-calculations for 36 nearly parabolic orbits[6]. A computation of $\Delta\left(\frac{1}{a}\right)_m$ in this case yields:

$$\Delta\left(\frac{1}{a}\right)_m = +0.000409 \pm 0.000110,$$

corresponding to:

$$\Delta\left(\frac{1}{a}\right) = +0.00041 \pm 0.00066$$

for an individual comet, in good agreement with the expected result. Among these comets FAYET found 7 with hyperbolic orbits. He made a new calculation of these 7 orbits, computing by numerical integration perturbations of the first order in the eccentricity caused by Jupiter 15—20 years forward in time, in conclusion referring the eccentricity to the centre of gravity of the system Sun-Jupiter and obtaining the same result.

In order to check FAYET's approximate calculations I have carried out a rigorous calculation of the perturbations for Comet 1898 VII. The definitive orbit has been determined by C. J. MERFIELD[7] on the basis of 414 observations during the time 1898 Jun. 11—1899 Dec. 6, and perturbations by Venus, the Earth, Mars, Jupiter, and Saturn have been taken into account.

Osculation: 1898 Jun. 21.0 G. M. T.

$$T = 1898 \text{ Sept. } 14.0442056 \text{ G. M. T.}$$

$$\left.\begin{aligned} \omega &= 233^\circ 15' 18''.66 \\ \Omega &= 74\ \ 0\ 58.17 \\ i &= 69\ 56\ \ 0.37 \end{aligned}\right\} 1900.0$$

$$\begin{aligned} \log q &= 0.2308587 \pm 0.0000009 \\ e &= 1.0010336 \pm 0.0000164 \\ \frac{1}{a} &= -0.00006074 \pm 0.0000096 \end{aligned}$$

Reducing to 1950.0 we find:

$$\left.\begin{aligned} \omega &= 233^\circ.26204 \\ \Omega &= 74.71189 \\ i &= 69.93455 \end{aligned}\right\} 1950.0$$

The corresponding equatorial constants are as follows:

$$\left.\begin{aligned} P_x &= +0.1075006 & Q_x &= +0.4092631 \\ P_y &= -0.2963618 & Q_y &= +0.8831024 \\ P_z &= -0.9490065 & Q_z &= -0.2294208 \end{aligned}\right\} 1950.0$$

The orbit was traced forward through 27 years by direct numerical integration of the rectangular co-ordinates, and attractions by the Sun, Jupiter, and Saturn were taken into account. To some extent perturbations by Mercury, Venus, and the Earth have been taken into account, including the masses of these planets in the constant of attraction. The calculation was carried out to 7 decimal places and Comrie's 'Planetary Co-ordinates' was used. My thanks are due to Mr. P. Naur, M. Sc., for some checking.

Perturbed, equatorial co-ordinates of Comet 1898 VII.

	x	y	z
1898 May 10.5	−0.744165	−2.164022	−0.521978
20.5	0.678990	2.066691	0.631351
30.5	0.612088	1.964098	0.739110
Jun. 9.5	0.543456	1.855944	0.844768
19.5	0.473115	1.741948	0.947759
29.5	0.401126	1.621875	1.047434
Jul. 9.5	0.327594	1.495550	1.143059
19.5	0.252678	1.362889	1.233831
29.5	0.176597	1.223930	1.318886
Aug. 8.5	0.099637	1.078857	1.397338
18.5	−0.022149	0.928023	1.468316
28.5	+0.055460	0.771966	1.531016
Sep. 7.5	0.132743	0.611396	1.584756
17.5	0.209232	0.447178	1.629032
27.5	0.284468	0.280281	1.663554
Oct. 7.5	0.358030	−0.111727	1.688264
17.5	0.429548	+0.057471	1.703333
27.5	0.498733	0.226366	1.709131
Nov. 6.5	0.565371	0.394113	1.706190
16.5	0.629331	0.560000	1.695150
26.5	0.690550	0.723456	1.676716
Dec. 6.5	0.749028	0.884045	1.651614
16.5	0.804809	1.041458	1.620556
26.5	0.857976	1.195491	1.584225
1899 Jan. 5.5	+0.908633	+1.346031	−1.543251

			x	y	z
1899	Jan.	15.5	+ 0.956898	+ 1.493032	− 1.498210
		25.5	1.002902	1.636506	1.449621
	Feb.	4.5	1.046774	1.776503	1.397945
		14.5	1.088641	1.913101	1.343588
		24.5	1.128631	2.046399	1.286910
	Mar.	6.5	1.166861	2.176509	1.228222
		16.5	1.203446	2.303550	1.167799
		26.5	1.238490	2.427644	1.105879
	Apr.	5.5	1.272095	2.548915	1.042671
		15.5	1.304350	2.667483	0.978355
		25.5	1.335343	2.783469	0.913090
	May	5.5	1.365151	2.896986	0.847014
		25.5	1.421501	3.117049	0.712893
	Jun.	14.5	1.473920	3.328497	0.576788
	Jul.	4.5	1.522851	3.532070	0.439305
		24.5	1.568672	3.728433	0.300918
	Aug.	13.5	1.611710	3.918180	0.161989
	Sep.	2.5	1.652244	4.101838	− 0.022802
		22.5	1.690515	4.279879	+ 0.116420
	Oct.	12.5	1.726734	4.452724	0.255506
	Nov.	1.5	1.761085	4.620751	0.394318
		21.5	1.793729	4.784296	0.532751
	Dec.	11.5	1.824808	4.943663	0.670722
		31.5	1.854446	5.099127	0.808165
1900	Jan.	20.5	1.882755	5.250936	0.945030
	Feb.	9.5	1.909835	5.399313	1.081281
	Mar.	1.5	1.935773	5.544463	1.216886
		21.5	1.960651	5.686572	1.351825
	Apr.	10.5	1.984538	5.825811	1.486083
	May	20.5	2.029600	6.096284	1.752518
	Jun.	29.5	2.071407	6.356984	2.016153
	Aug.	8.5	2.110334	6.608847	2.276992
	Sept.	17.5	2.146694	6.852676	2.535069
	Oct.	27.5	2.180753	7.089163	2.790434
	Dec.	6.5	+ 2.212736	+ 7.318912	+ 3.043151

	x	y	z
1901 Jan. 15.5	+ 2.242840	+ 7.542453	+ 3.293288
Feb. 24.5	2.271231	7.760248	3.540920
Apr. 5.5	2.298057	7.972712	3.786120
May 15.5	2.323446	8.180212	4.028962
Jun. 24.5	2.347512	8.383073	4.269521
Aug. 3.5	2.370355	8.581593	4.507868
Sep. 12.5	2.392064	8.776036	4.744074
Dec. 1.5	2.432389	9.153632	5.210326
1902 Feb. 19.5	2.469033	9.517536	5.668786
May 10.5	2.502436	9.869145	6.119922
Jul. 29.5	2.532966	10.209639	6.564162
Oct. 17.5	2.560926	10.540022	7.001896
1903 Jan. 5.5	2.586573	10.861160	7.433483
Mar. 26.5	2.610125	11.173798	7.859248
Jun. 14.5	2.631772	11.478590	8.279490
Sep. 2.5	2.651676	11.776109	8.694482
Nov. 21.5	2.669979	12.066858	9.104475
1904 Feb. 9.5	2.686808	12.351288	9.509700
Apr. 29.5	2.702273	12.629797	9.910370
Jul. 18.5	2.716477	12.902744	10.306681
Dec. 25.5	2.741453	13.433210	11.086943
1905 Jun. 3.5	2.762370	13.944921	11.851796
Nov. 10.5	2.779763	14.439737	12.602377
1906 Apr. 19.5	2.794093	14.919228	13.339689
Sep. 26.5	2.805757	15.384742	14.064621
1907 Mar. 5.5	2.815101	15.837451	14.777968
Aug. 12.5	2.822426	16.278385	15.480447
1908 Jan. 19.5	2.827990	16.708454	16.172704
Jun. 27.5	2.832018	17.128472	16.855329
Dec. 4.5	2.834697	17.539167	17.528858
1909 May 13.5	2.836188	17.941196	18.193783
Oct. 20.5	2.836626	18.335153	18.850552
1910 Mar. 29.5	2.836119	18.721576	19.499582
Sep. 5.5	2.834756	19.100956	20.141252
1911 Feb. 12.5	+ 2.832605	+ 19.473738	+ 20.775916

	x	y	z
1911 Jul. 22.5	+ 2.829718	+ 19.840329	+ 21.403899
Dec. 29.5	2.826131	20.201098	22.025502
1912 Jun. 6.5	2.821863	20.556382	22.641004
Nov. 13.5	2.816921	20.906484	23.250662
1913 Apr. 22.5	2.811302	21.251675	23.854712
Sep. 29.5	2.804992	21.592198	24.453372
1914 Mar. 8.5	2.797978	21.928261	25.046839
Aug. 15.5	2.790223	22.260047	25.635292
1915 Jan. 22.5	2.781725	22.587706	26.218895
Jul. 1.5	2.772469	22.911365	26.797791
Dec. 8.5	2.762459	23.231127	27.372112
1916 May 16.5	2.751713	23.547080	27.941976
Oct. 23.5	2.740270	23.859302	28.507496
1917 Apr. 1.5	2.728186	24.167867	29.068776
Sep. 8.5	2.715532	24.472853	29.625918
1918 Feb. 15.5	2.702389	24.774344	30.179025
Jul. 25.5	2.688842	25.072435	30.728200
1919 Jan. 1.5	2.674978	25.367229	31.273547
Jun. 10.5	2.660876	25.658840	31.815172
Nov. 17.5	2.646610	25.947387	32.353181
1920 Apr. 25.5	2.632242	26.232996	32.887682
Oct. 2.5	2.617824	26.515792	33.418778
1921 Mar. 11.5	2.603394	26.795905	33.946575
Aug. 18.5	2.588983	27.073458	34.471173
1922 Jan. 25.5	2.574605	27.348576	34.992672
Jul. 4.5	2.560268	27.621377	35.511166
Dec. 11.5	2.545968	27.891974	36.026746
1923 May 20.5	2.531689	28.160476	36.539502
Oct. 27.5	2.517410	28.426982	37.049515
1924 Apr. 4.5	2.503096	28.691584	37.556864
Sep. 11.5	2.488708	28.954364	38.061623
1925 Feb. 19.0	2.474196	29.215392	38.563858
Jul. 29.0	2.459507	29.474725	39.063630
1926 Jan. 5.0	+ 2.444586	+ 29.732404	+ 39.560990

Perturbed equatorial co-ordinates and velocities x, y, z, and $\frac{dx}{dt}$, $\frac{dy}{dt}$, $\frac{dx}{dt}$ for 1925 Feb. 19.0 are given below. The reductions ξ, η, ζ, and $\frac{d\xi}{dt}$, $\frac{d\eta}{dt}$, $\frac{d\zeta}{dt}$ to the centre of gravity of the system Sun-Jupiter-Saturn are also given together with co-ordinates and velocities $\bar{x}$, $\bar{y}$, $\bar{z}$, and $\frac{d\bar{x}}{dt}$, $\frac{d\bar{y}}{dt}$, $\frac{d\bar{z}}{dt}$ relative to the said centre of gravity.

$x = +2.474196$	$y = +29.215392$	$z = +38.563858$
$\xi = +0.001637$	$\eta = +\ \ 0.006221$	$\zeta = +\ \ 0.002562$
$\bar{x} = +2.475833$	$\bar{y} = +29.221613$	$\bar{z} = +38.566420$
$\frac{dx}{dt} = -0.0145909$	$\frac{dy}{dt} = +0.2601723$	$\frac{dz}{dt} = +0.5009939$
$\frac{d\xi}{dt} = -0.0012790$	$\frac{d\eta}{dt} = +0.0000067$	$\frac{d\zeta}{dt} = +0.0000341$
$\frac{d\bar{x}}{dt} = -0.0158699$	$\frac{d\bar{y}}{dt} = +0.2601790$	$\frac{d\bar{z}}{dt} = +0.5010280$

From this we find the reciprocal semi-major axis with the aid of the following equation:

$$\bar{V}^2 = w^2 k^2 (1 + \Sigma m) \left(\frac{2}{\bar{r}} - \frac{1}{\bar{a}}\right)$$

or:

$$\frac{1}{\bar{a}} = \frac{2}{\bar{r}} - \frac{\bar{V}^2}{w^2 k^2 (1 + \Sigma m)}.$$

We get:

$$\frac{1}{\bar{a}} = -0.0007747.$$

Finally we get:

$$\bar{e} = 1.001321.$$

Fayet's result for 1924 Dec. 24 was:

$$\bar{e} = 1.001514$$

relative to the centre of gravity for the system Sun-Jupiter. The rigorous calculation thus confirms Fayet's result. *Hence it is certain that this comet will leave the solar system.*

The backward-calculation for this comet[8] shows that $\frac{1}{\bar{a}}$ in the original orbit (1886) was — 0.0000157. Considering the standard deviation of $\frac{1}{a}$, this hyperbolic residual is illusory and it may be taken for granted that the comet has its origin in the solar system.

A rigorous calculation of the change in the eccentricity when tracing a nearly parabolic orbit forward has been carried out also in the case of Comet 1904 I[9]. The main result was as follows. In the definitive orbit we had:

$$\frac{1}{a} = -0.0005040 \pm 0.0000079 \text{ (1904 May).}$$

With the aid of a calculation of the perturbations by Jupiter and Saturn we got:

$$\frac{1}{\bar{a}} = +0.0005096 \text{ for 1917 Apr.}$$

Hence this comet remains in the solar system. The backward-calculation for this comet[10] showed that in the original orbit (before perihelion passage):

$$\frac{1}{\bar{a}} = +0.0002165 \text{ (1891 Mar.).}$$

Hence the passage through the inner parts of the solar system has made the orbit more elliptical.

Even if FAYET's calculations are not absolutely convincing in the individual cases they show that rather a great diffusion of comets out of the solar system takes place. A rigorous calculation has to be carried out in the same way as the investigation of original orbits to provide evidence in the particular cases regarding the question whether a comet remains in the solar system or not. In accordance with the above considerations a special interest is connected with the orbits with $\frac{1}{a} <$ about -0.0005 at an epoch near the time of perihelion, on condition that the orbits have been well determined on the basis of observations distributed over several months. From the expression $q = a (1 - e)$ we deduce that this corresponds to $e > 1 + 0.0005 \cdot q$, where q is the perihelion distance.

References.

1. Publ. Cop. Obs. 19, 1914.
2. Publ. Yerkes Obs., **8**, 1943.
3. G. Fayet, Recherches concernant les excentricités des comètes, Paris 1906.
4. B. A. N., **11**, 102, 1950.
5. A. N., **124**, 209. 1890.
6. Ann. Bur. Long., **10**, B. 1, 1933.
7. A. N., **154**, 229, 1901.
8. Publ. Cop. Obs. 19, 1914.
9. Mat. Tidsskrift B, 124, 1945.
10. Publ. Cop. Obs. 105, 1935.

Indleveret til selskabet den 13. december 1952.
Færdig fra trykkeriet den 20. februar 1953.