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EXPANDING UNIVERSE AND THE ORIGIN OF GALAXIES

BY

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§ 1. According to the general theory of relativity, the time-behaviour of homogeneous isotropic Universe is described by the equation (1)

$$\frac{dR}{dt} = \sqrt{\frac{8\pi G}{3}\varrho R^2 - c^2}, \tag{1}$$

where G is the Newtonian constant, c the velocity of light, ϱ the total mean density in space, and R the curvature radius[1]. If l is the distance between any two material points in the expanding Universe standing in a constant ratio α to the curvature radius, we can rewrite (1) as:

$$\frac{dl}{dt} = \sqrt{\frac{8\pi G}{3}\varrho l^2 - c^2\alpha^2}. \tag{2}$$

As it is well known, this equation simply states the law of conservation of mechanical energy (kinetic plus potential) for the masses enclosed within a sphere of the radius l. Generally speaking, the total mean density ϱ is composed of two terms: the density of matter ϱ_{mat} which is inversely proportional to l^3, and the mass-density of thermal radiation given by aT^4/c^2. At the present state of the universe, the mass-density of thermal radiation is presumably negligibly small as compared with the density of matter which can be assumed to be of the order of magnitude of $10^{-30}\,\mathrm{g/cm^3}$.

Dividing (2) by l we get

$$\frac{1}{l}\frac{dl}{dt} = \sqrt{\frac{8\pi G}{3}\varrho - \frac{\alpha^2 c^2}{l^2}}. \tag{3}$$

[1] R. C. Tolman. Relativity, Thermodynamics, and Cosmology. Clarendon Press. Oxford. 1934 p. 396. We have assumed the cosmological constant Λ to be zero since the cosmological term is not needed in the expanding model, and since, in fact, the new value of Hubble's constant leads to the correct age of the universe without the help of cosmological terms.

The present value of $\frac{dl}{dt}/l$ is known as HUBBLE's constant, and is numerically equal to the inverse age of the universe. We have

$$\left(\frac{1}{l}\frac{dl}{dt}\right)_{\text{pres.}} = \frac{1}{3.5\cdot 10^{9}\,\text{years}} = \frac{1}{10^{17}\,\text{sec.}} = 10^{-17}\,\text{sec.}^{-1}. \qquad (4)$$

Substituting this value, along with that for the present mean density, in the equation (3), we find that, for the present epoch, the first term under the radical is negligibly small as compared with the second term, indicating that l is now increasing linearly with time. For the curvature radius the equation (3) leads to an imaginary quantity: $R_{\text{pres.}} = ic\,10^{+17} = i\,3\cdot 10^{27}\,\text{cm} = i\cdot 3\cdot 10^{9}$ light-years, meaning, geometrically, that the space of our universe is hyperbolic (infinite), and ever-expanding. Physically our result means that, just as in the case of a space rocket which had escaped from the terrestrial field of gravity, the galaxies are now flying away from each other without being hindered by the forces of mutual gravitational attraction. For that free expansion period we can apparently write:

$$\varrho_{\text{mat}} = 10^{-30}\left(\frac{10^{17}}{t}\right)^{3} = \frac{10^{21}}{t^{3}}\,\text{g/cm}^{3}. \qquad (5)$$

For earlier periods of time, when the deceleration by gravity could not be neglected, the equation (3) should be integrated analytically. The result of integration indicates, however, that, down to one hundredth of the present age, the deviations of calculated matter density from the simple expression (5) are less than by a factor of three. Since in future consideration we will not be interested in the behaviour of matter-universe for still earlier dates, and since the present value of density used in the derivation of (5) is not known anyway within a factor of three, we will not use this refinement in our calculations.

Since in an adiabatically expanding thermal radiation the energy density changes as the inverse fourth power of linear dimensions, in contrast to the inverse third power in the case of matter density, we should expect that, for sufficiently early stages of expansion, mass density of thermal radiation plays a more important role than the density of matter. In this case the equation (3) can be rewritten in the form

$$\sqrt{\frac{8\pi G}{3}\frac{a}{c^2}}\,T^4 = \frac{1}{l}\frac{dl}{dt} = -\frac{1}{T}\frac{dT}{dt}. \tag{6}$$

The last equality holds because in an adiabatically expanding thermal radiation the temperature varies inversely proportionally to linear dimensions. Equation (6) can be integrated as

$$T = \sqrt[4]{\frac{3\,c^2}{32\,\pi a G}}\,\frac{1}{t^{1/2}} = \frac{1.5\cdot 10^{10}}{t^{1/2}}\,^\circ K, \tag{7}$$

giving, for the mass density of radiation during the radiation period of the expansion,

$$\varrho_{\text{rad}} = \frac{4.4\cdot 10^5}{t^2}\ \text{g/cm}^3. \tag{8}$$

Density functions given by (5) and (8) are shown by two straight lines marked "matter late", and "radiation early" in the logarithmic plot of Fig. 1. We see that these two lines intersect at the point corresponding to

$$\left.\begin{aligned} t &= 2.2\cdot 10^{15}\ \text{sec} \\ \varrho &= 1\cdot 10^{-25}\ \text{g/cm}^3 \\ T &= 320\ ^\circ K. \end{aligned}\right\} \tag{9}$$

Thus we may conclude that, during the first two hundredths of its history, the expansion of the universe was ruled by radiation, whereas during the remaining time the matter was of primary importance. During the radiation epoch, matter density was changing as $l^{-3} \sim T^3 \sim t^{-\frac{3}{2}}$, so that we can write

$$\varrho_{\text{mat. early}} = \frac{\varrho_0}{t^{3/2}} = \frac{10^{-2}}{t^{3/2}}\ \text{g/cm}^3, \tag{10}$$

where the numerical value of the coefficient is adjusted so that the line passes through the intersection point obtained before. During the matter epoch, radiation density is changing as $l^{-4} \sim t^{-4}$, and we have

$$\varrho_{\text{rad. late}} = \frac{2.5\cdot 10^{36}}{t^4}\ \text{g/cm}^3. \tag{11}$$

These variations are shown by lines marked "matter early" and "radiation late" in Fig. 1. It is interesting to notice that (11) leads to the radiation mass density $2 \cdot 10^{-32}$ g/cm^3, and the temperature $7\ ^\circ K$, for the present epoch.

§ 2. Some time ago[1] it was indicated by the author that the assumption of the predominantly radiative, and predominantly

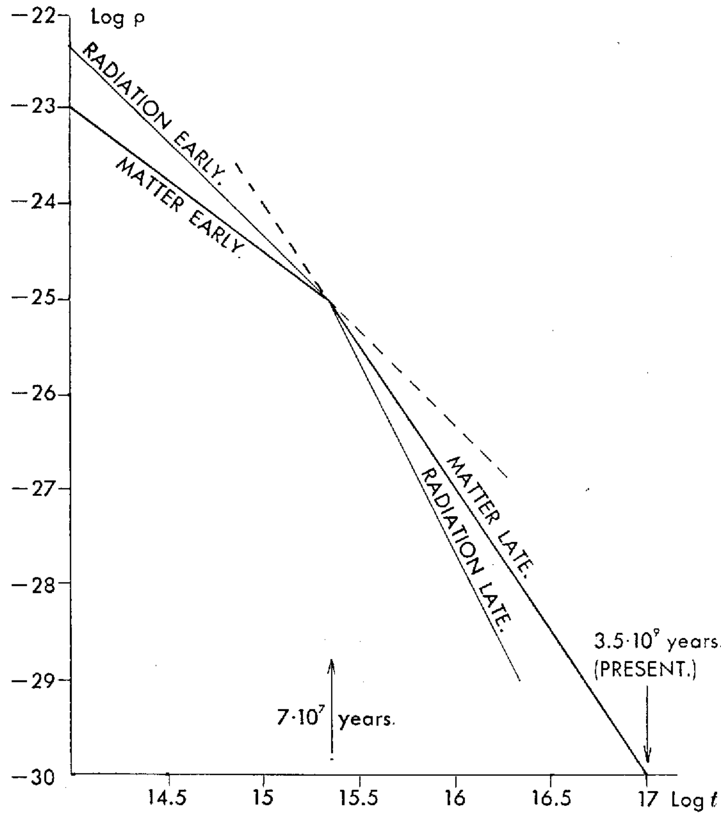

Fig. 1. Variation of matter—and radiation—densities in the expanding Universe.

material states of the universe can be very helpful for the understanding of the formation of "protogalaxies" (i. e. the initial gaseous galaxies before their condensation into individual stars) from the originally uniform gaseous material which presumably existed during the very early stages of the evolution. In fact, as long as thermal radiation was the predominant factor in the

[1] G. Gamow. Nature. Vol. 162, p. 680 (1948).

universe, the particles of matter submerged into it must have been constantly kicked around by light quanta, and should have maintained a uniform distribution through the space. As soon, however, as material density became the main factor, Newtonian forces between the elements of gas must have caused a break-up of the formerly uniform distribution into the individual gas clouds (protogalaxies) the size of which was determined by the well known JEANS' formula for gravitational instability. The present mean density of individual galaxies, being of the order of magnitude 10^{-24} or $10^{-25}\,g/cm^3$, suggests indeed that the separation process must have taken place near the intersection or radiation- and matter-curves in Fig. 1. Substituting the density and temperature values from (9) into the JEANS' formula

$$D_{\min} = \sqrt{\frac{5\,\pi\varkappa\,T}{3\,G m_H \bar{\mu}\varrho}} \tag{12}$$

for the minimum diameter of gravitational condensation[1], we obtain

$$D_{\min} = 5 \cdot 10^{21}\,\mathrm{cm} = 5\,000 \text{ light-years.} \tag{13}$$

For the minimum mass of the condensation we obtain

$$M_{\min} = 10^{40}\,\mathrm{g} = 5 \cdot 10^{6} \text{ sun masses.} \tag{14}$$

We can now compare this theoretical lower limit with HOLMBERG's data[2] concerning mass distribution among the galaxies, shown in Fig. 2. We see that the theoretical minimum mass-value, being certainly of galactic order of magnitude, falls short by a factor of ten from the observed lower limit of galactic masses. This may be due, of course, to the approximate nature of the theory, but may also be a real effect caused by not taking into account the possibility of turbulent motion in the primordial gas. In fact, according to recent calculations of S. CHANDRASEKHAR[3],

[1] The value for the mean molecular weight $\bar{\mu}$ may range from 2.7 for a half and half (by mass) mixture of molecular hydrogen and atomic helium, to 0.7 for the same mixture in completely ionised state. Evaluating (12) we have assumed $\bar{\mu} = 1$.

[2] HOLMBERG. Lund. Medd. Ser. II. No. 128 (1950). The curve shown in Fig. 2 is redrawn from HOLMBERG's original luminosity curve under the assumption that the luminosities of different galaxies are proportional to their masses.

[3] S. CHANDRASEKHAR. Proc. Roy. Soc. A. Vol. 210, p. 26 (1951).

JEANS' expression for minimum radius of gravitational condensation in turbulent medium must be multiplied by a factor

$$\left(1+\frac{1}{3}M^2\right)^{1/2}, \quad (15)$$

where M is the mean Mach number of turbulence. Thus, the assumption of Mach number 4 would bring the calculated lower limit of galactic masses to its observed value.

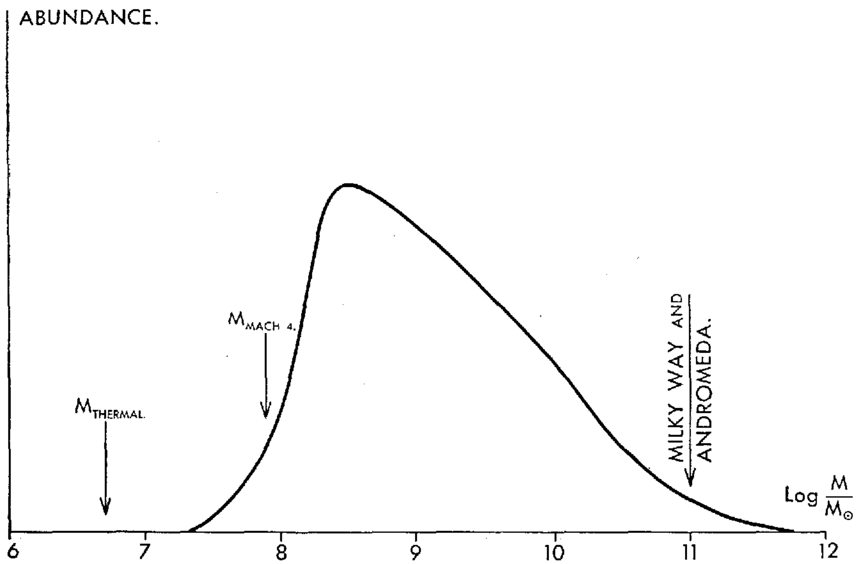

Fig. 2. Comparison of HOLMBERG's curve for the distribution of galactic masses with the calculated lower limit.

Apart from reasonable numerical agreement between the observed minimum value of galactic masses, and the value given by JEANS' formula, we want to stress the point that the shape itself of HOLMBERG's curve, with its steep descent on the side of lower masses, and a long tail extending into the region of exceptionally massive systems (such as our Milky way, and Andromeda), strongly supports the point of view that the formation of protogalaxies proceeded under the conditions characterised by the existence of a certain lower threshold for their mass.

§ 3. We have mentioned above the possible role of turbulence in determining the sizes and masses of gravitational condensations in the expanding primordial gas. As it was first suggested by WEIZSÄCKER[1], the assumption of turbulent motion in the primordial gas might be also absolutely necessary for the correct description of the condensation process itself. In fact, as it is in the case of many instability problems, JEANS' gravitational condensations will develop within a finite time only if there are present some rudimentary local compressions and expansions which can be further augmented by the action of gravity forces. It seems in fact that, unless we have extensive density fluctuations by a factor of two or three in the primordial gas, no gravitational condensations could develop within the time period permitted by the age of the universe. It is clear that no such fluctuations could be expected in the original gas on the basis of the simple statistical theory. On the other hand, since the RAYNOLD's number may become arbitrarily large in an infinite gas medium, we may expect the formation of large size turbulent eddies. Since, in this case, the velocity of turbulent streamings could well have been larger than the velocity of thermal motion (as it is the case for interstellar gas in our Galaxy), one could easily expect the formation of local compressions and rarefactions of all possible sizes. And the compression eddies exceeding the minimum mass given by JEANS' formula must have been prevented from subsequent expansion by the Newtonian forces between their parts. According to this point of view, HOLMBERG's distribution of galactic masses may reflect the state of turbulent motion in the primordial gas, and it would be interesting to see whether this distribution is indeed in accordance with KOLMOGOROFF's spectral law for isotropic homogeneous turbulence.

Much larger, and correspondingly much weaker, compression and rarefaction eddies would have no time to change considerably since the separation time, as would be noticeable at present only as certain inhomogeneities in the space distribution of individual galaxies. Such deviations from the uniform distribution of galaxies through the space of the universe were actually observed and studied by H. SHAPLEY[2], and C. D. SHANE[3]. In Fig. 3 we give, as an example, one of SHANE's diagrams showing the isolines for

[1] C. VON WEIZSÄCKER. Astrophys J. Vol. 14, p. 165 (1951) and earlier publications.
[2] H. SHAPLEY. Proc. Nat. Acad. Sc. Vol. 37, p. 191 (1951), and previous publications.
[3] C. D. SHANE. Proc. Amer. Phil. Soc. Vol. 94, p. 13. (1950).

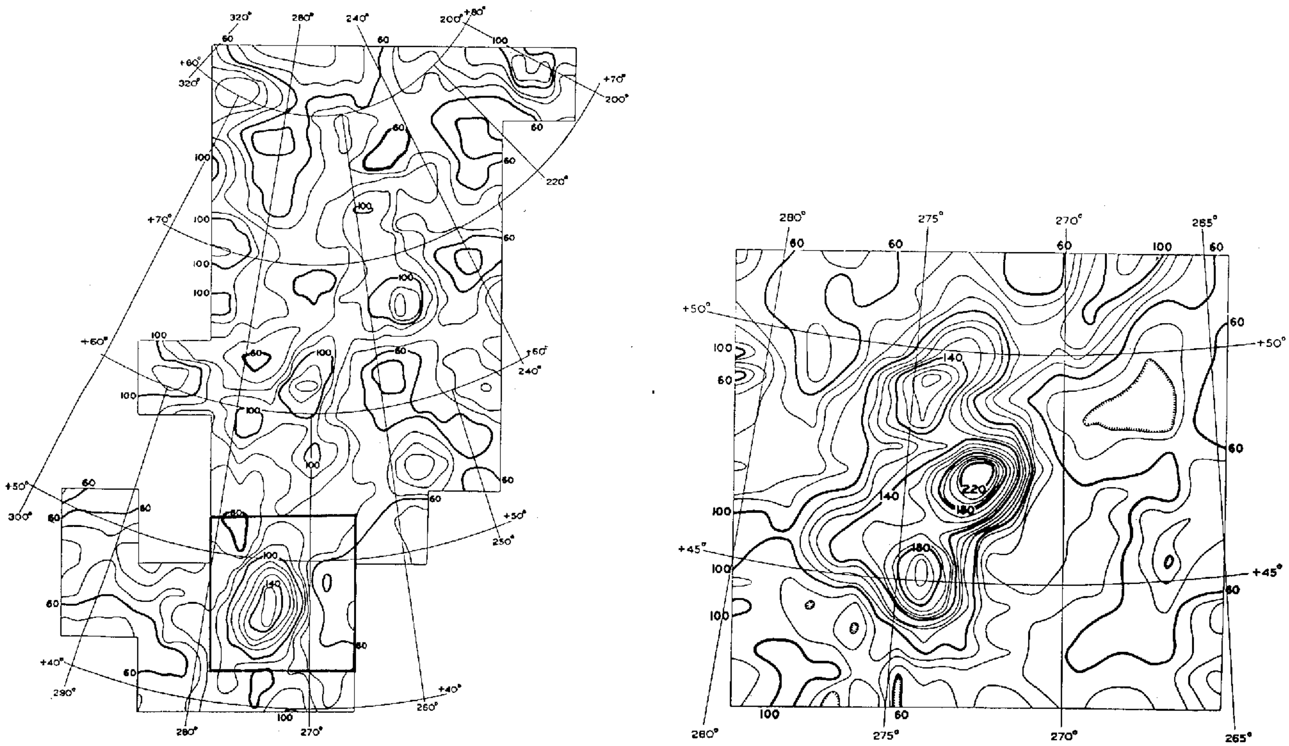

Fig. 3. Irregularities in the distribution of galaxies according to C. D. SHANE. Numbers indicate the number of galaxies per square degree of the celestial sphere. Picture on the right represents the result of more detailed study of the region squared in the left picture. Notice the appearance of smaller condensations within the original large condensation, in accordance with the expectations of the theory of turbulent motion. (The author is grateful to Dr. SHANE for the permission to reproduce these, as yet unpublished, diagrams.)

the projected distribution density of galaxies in a certain region of celestial sphere. The distribution shown in this diagram does indeed resemble a density distribution which might be expected in the case of turbulent motion in a compressible fluid.

The study of the observed space distribution of galaxies from the point of view of the theory of turbulence is now being carried out by the author in collaboration with Dr. F. N. FRENKIEL and Mrs. VERA RUBEN, and may lead to some interesting conclusions concerning the early state of the expanding universe, and the problem of the formation of galaxies.

§ 4. Accepting the ideas expressed in the previous section, we can now inquire about the conditions which must have existed very early in the history of the universe, close to the singular point at $t = 0$. For the temperature variations, and the variations of matter density during these very early stages, we may use the expressions (7) and (10) derived above.

During the first few seconds of expansion, the temperature of space must have been of the order of billions of degrees, corresponding to kinetic energy of thermal motion of the order of millions of electron volts, and the density of matter was comparable to that of the atmospheric air. Thus we may expect that at this time matter must have existed in completely dissociated state, being composed entirely of neutrons, protons, and electrons. As the temperature was dropping in the process of expansion, nucleons forming this primordial material, or *ylem*, must have started to aggregate, forming composite atomic nuclei of various degrees of complexity. The process must have come to an end after about half an hour when most of the neutrons present in the original mixture have either decayed or been captured by protons and other nuclei, and the temperature dropped to $4 \cdot 10^8$ °K (60 Kev), being too low for most thermonuclear reactions between charged particles. The result of the process must have depended critically on the assumed value of the coefficient in (10). If the assumed density is too low, most of the neutrons will decay into protons before being captured by other particles, and the resultant material will be almost exclusively hydrogen. If, on the contrary, the density is taken too high, everything would be built into helium and heavier elements and no hydrogen will be left. The problem was first investigated by the author[1] with the result that, in order to obtain the observed half-and-half hydrogen-helium ratio, one should choose $\varrho_0 = 0.7 \cdot 10^{-2}$. Subsequent more detailed calculations were carried out by FERMI and TURKEVICH[2], who took into account all possible thermonuclear reactions up to the formation of He^4. The result of their calculations, carried out with $\varrho_0 = 1.7 \cdot 10^{-3}$, are shown in Fig. 4,

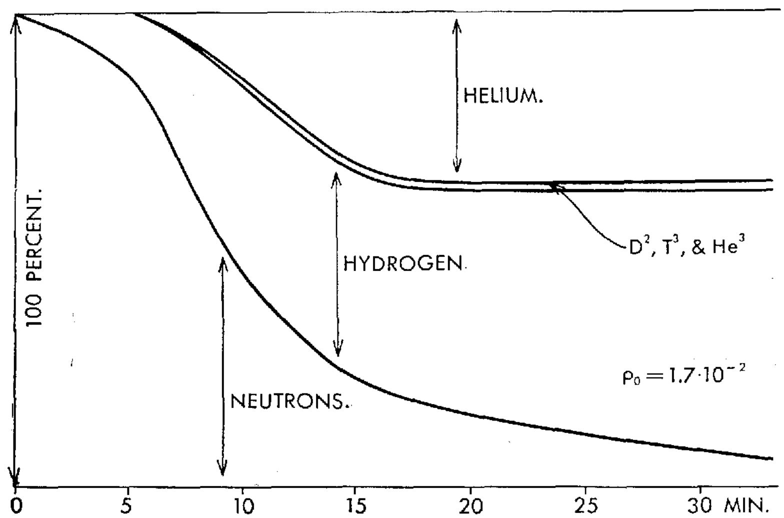

Fig. 4. Variation of relative abundance of lightest elements (by weight) during first half hour of expansion, calculated by FERMI & TURKEVICH.

and are in good agreement with the observed relative amount of Hydrogen and Helium in the universe. Attempts by the same authors to carry the detailed calculations of the element formation beyond He^4 have failed, mostly because of the non-existence of any nucleus with mass 5, and this difficulty is not as yet removed.

On the other hand, a general theory of the formation of heavier elements by the process of neutron capture, developed by the author in collaboration with ALPHER[3], and later extended by ALPHER and HERMAN[4], shows that the amount of heavy ele-

[1] G. GAMOW, *l. c.*

[2] E. FERMI and A. TURKEVICH. Unpublished. For more details on these calculations, see the review by R. A. ALPHER and R. C. HERMAN. Rev. Mod. Phys. vol. 22, p. 153 (1950).

[3] ALPHER, BETHE, and GAMOW. Phys. Rev. vol. 73, p. 803 (1948).

[4] R. ALPHER and R. HERMAN. Phys. Rev. vol. 84, p. 60 (1951).

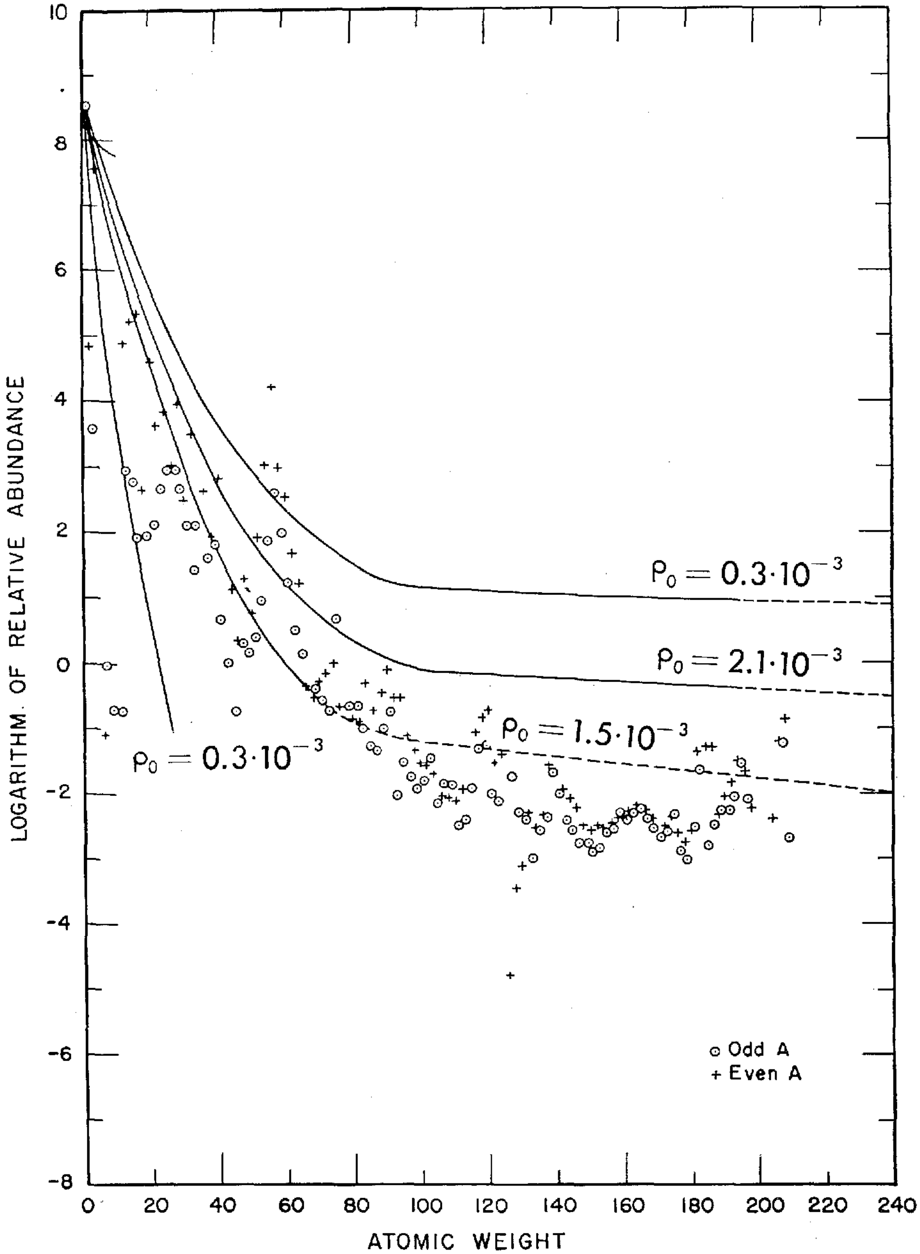

Fig. 5. Comparison of the observed relative abundance of elements (circles and crosses) with theoretical curves calculated with different values of ϱ_0 by ALPHER and HERMAN.

ments formed is also very sensitive to the assumed value of ϱ_0. Fig. 5 shows the theoretical abundance curves calculated for different ϱ_0's in the recent work by ALPHER and HERMAN[1]. The curves faithfully represent general variation of the relative abundances, and we find that the best fit with observed data is obtained by assuming $\varrho_0 = 1.2 \cdot 10^{-3}$, in good agreement with the results obtained by FERMI and TURKEVICH.

It is quite remarkable that the calculations pertaining to the formation of atomic species lead to about the same density as is obtained from purely cosmological considerations pertaining to the formation of galaxies. This agreement can be even improved if one assumes that the concentration of neutrons in ylem was lower than that (~ 100 per cent) assumed in the above calculations, since lower concentration of neutrons will call for higher total density necessary for the formation of elements.

[1] R. ALPHER and R. HERMAN. Phys. Rev. vol. 84, p. 60 (1951).

Conclusions.

Considering the very preliminary nature of the theory described in the present paper, and the exceptionally broad scope of the phenomena which it attempts to tie together into one consistent cosmogonical picture, one should agree that the proposed point of view may be accredited with a certain degree of success, even though there are still many difficulties standing in its way. In the author's opinion, the most promising feature of the theory is the possibility of binding together such seemingly non-related observational data as the relative abundance of chemical elements, on the one hand, and the sizes, masses, and space distribution of stellar galaxies, on the other. It seems to the author that the agreement between density functions obtained from direct observations at the present time ($t = 3 \cdot 10^{17}$ sec), from the conditions necessary for the formation of protogalaxies (at $t \simeq 2 \cdot 10^{15}$ sec), and from the theory of the origin of atomic species (at $t \simeq 10^{3}$ sec), cannot be entirely coincidental.

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