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COMPOUND SCATTERING OF PROTONS AND DEUTERONS

BY

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Introduction.

In his analysis of the scattering of α-rays leading to the discovery of the atomic nucleus, RUTHERFORD was the first clearly to distinguish between single scattering and compound scattering. While the former is due to rare, violent collisions, the latter results from the accumulated effect of a large number of small deflections. The theory of the compound scattering was further developed by BOTHE[1] in 1921, and later a detailed discussion of the phenomenon was given especially by WILLIAMS[2]. A general representation of the theory with special emphasis on the intimate connection between scattering and stopping effects was published recently by BOHR[3].

Lately, the theory has been used to estimate the energy of particles stopped in photographic emulsions[4], but so far only few experiments have been made[5, 6] which are suited for a more direct test of the theory.

In experiments performed at this institute by MADSEN and VENKATESWARLU[7] with the purpose of measuring the energy lost by protons penetrating thin foils, a considerable decrease in the current of protons striking the target was observed when the foils were inserted in the beam. This effect was caused just by the scattering of the protons in the foils, and it seemed possible to measure the corresponding angular distributions rather accurately by very simple means.

[1] W. BOTHE, Zs. f. Phys. **4**, 300 (1921) and Zs. f. Phys. **13**, 368 (1923).

[2] E. J. WILLIAMS, Proc. Roy. Soc. A **169**, 531 (1939) and Phys. Rev. **58**, 292 (1940).

[3] N. BOHR, Dan. Mat. Fys. Medd. **18**, no. 8 (1948).

[4] e. g. Y. GOLDSCHMIDT-CLERMONT, D. T. KING, H. MUIRHEAD, and D. M. RITSON, Proc. Phys. Soc. Lond. **61**, 183 (1948).

[5] H. GEIGER, Proc. Roy. Soc. **83**, 492 (1910).

[6] Recently, the case of compound scattering of electrons has been studied experimentally by L. A. KULTICHITSKY and G. D. LATYSHEV, Phys. Rev. **61**, 254 (1942).

[7] C. B. MADSEN and P. VENKATESWARLU, Phys. Rev. **74**, 648 (1948).

Experimental Method.

The distributions of the scattering angles were studied by means of the deflection chamber shown in fig. 1. In this chamber, the divergent beam emerging from the scattering foil F was swept over the slit S by means of an oscillating magnetic field through the coils C. The current distribution in a cross-section of the beam taken at the position of the slit could then be determined by measuring the current to the collector plate P as a function of the magnetic field. These measurements were made by means of an oscillograph, the plates of which gave an X-deflection proportional to the current in the coils, and an Y-deflection proportional to the collector current.

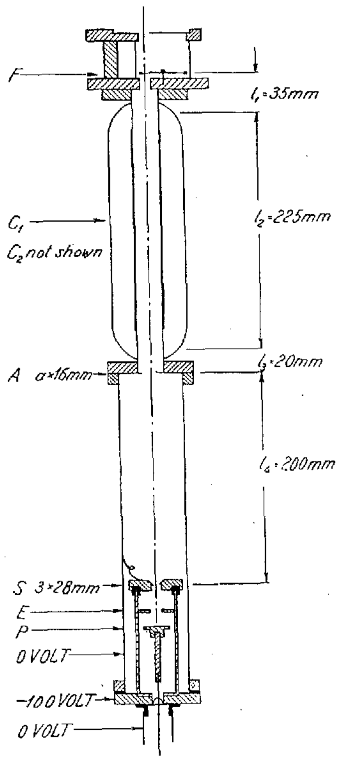

Fig. 1. Sketch of deflection chamber; of the two coils only the back one is shown.

The fluctuations in the current and the position of the beam demanded that each distribution curve was measured within about one millisecond. The photographic writing speed of the oscillograph was just high enough to permit the recording of a single trace made at this speed; consequently, it was sufficient to intensify the writing of the oscillograph only during half a period of the oscillation.

The required oscillating magnetic field was produced by discharging a battery of condensers through the coils by means of a mercury switch. Each of the two coils used had 125 turns and a resistance of 0.7 ohms, and the condensers had a total capacity of 28 μF. With the two coils connected in series, the discharging took place in the form of a damped oscillation having a frequency of 350 cycles per second and a logarithmic decrement of 0.50. The average amplitude of the magnetic field obtained

with the battery loaded to the permitted maximum of 750 volts was approximately 1000 Oersteds in the first period, as calculated by means of an effective area of the coils of about 100 cm^2.

The voltage across a resistance of 0.35 ohms connected in series with the coils was fed to the X-amplifier of the oscillograph. In order to diminish the influence of induced currents on the field, the vacuum chamber was made of a brass tube with a wall thickness of only 0.3 mm. Oscillograms showed that with this wall thickness the deflection was delayed by 2.5 $^0/_0$ of a period with respect to the current in the coils. This effect was to the first approximation compensated for by means of a filter in the leads to the X-amplifier.

The current to the collector plate was sent through the grid leak of the first tube of an a.c. amplifier, the output of which was fed to the Y-amplifier of the oscillograph. In order to avoid the influence of secondary electrons running from the collector plate to the slit, or vice versa, an electrode E, having a negative potential of 100 volts, was placed between the slit and the plate in such a way that it could not be hit directly by the proton beam. This electrode, and the tube in which it was mounted, were made of soft iron in order to screen the collector plate from the oscillating magnetic field.

The time-constants of the collector system and of the amplifiers had to be made small compared to the time of oscillation, so that their influence on the oscillograms could be neglected. They were measured by sending the alternating voltage across the coils through a filter to one set of the deflection plates of the oscillograph, and a proper fraction of the same voltage through the amplifier to the set perpendicular to the first one. The oscillograph was here intensified for a whole period of the oscillation so that the filter necessary to compensate the phase shift could be determined. The only important time-constant was found to be that of the collector system, and this constant was reduced to 30 micro-seconds by using a grid leak of 100 kilo-ohms. The corresponding noise level was found to be about $\pm 2 \cdot 10^{-10}$ ampères. The errors in the measured half-widths of the distribution curves should thus, as far as band-width and noise level of the pre-amplifier are concerned, be limited to a few per cent., even though, owing to the heat dissipated in the foils, total beam currents of the order of magnitude of only 10^{-7} ampères had to be used in the experiment. Errors from statistical fluctuations in the number of particles collected should in all cases be less than about $\pm 0.3\ {}^0/_0$.

The measurements of the time-constants also gave information about the magnitude of the amplifications and proved that the amplifiers were sufficiently linear. For linear amplification, the angle α, i. e. the projection of the scattering angle on a plane perpendicular to the slit, should be related to the corresponding distance δ on the screen of the oscillograph, by the equation

$$\alpha = \frac{C}{\sqrt{A_1 (E_0 - \Delta E)}} \cdot \delta, \qquad (1)$$

where A_1 is the mass number of the particles in the beam, E_0 their energy before, and $E_0 - \Delta E$ their energy after their passage through the foil. The energy straggling in the foil is here of no significance, because the energy loss ΔE enters only as a smaller correction.

The factor of proportionality C can be determined from the maxima of δ and the calculated maxima of the magnetic field, which gives

$$C = 0.030\ (\mathrm{MeV})^{\frac{1}{2}}/\mathrm{cm} \pm 5\ {}^0/_0. \qquad (2)$$

A more accurate and direct determination of C can be obtained from oscillograms like that shown in fig. 2, which was taken without a foil and with the slit parallel to the direction of the magnetic deflection. The half-widths of such curves are determined by the length of the slit, and the values for C found by this method were all inside the range

$$C = 0.0287\ (\mathrm{MeV})^{\frac{1}{2}}/\mathrm{cm} \pm 3\ {}^0/_0. \qquad (3)$$

The determination was performed at the two energies 0.550 MeV and 1.800 MeV, and with the three masses H_1^+, H_2^+ and H_3^+.

Even though the magnetic field was produced by a double coil, some inhomogeneities should of course be expected, and for this reason an independent determination of C is needed also for particles which have been moving close to the wall of the deflection tube. This is obtained by means of the

broad distribution curves for which the limiting aperture A (fig. 1), as illustrated by fig. 3, has caused a cut-off at the big deflections. This determination gives, by means of the known stopping powers of the foils, values for C which are all inside the range

$$C = 0.0293\ (\mathrm{MeV})^{\frac{1}{2}}/\mathrm{cm} \pm 3\ {}^0/_0. \qquad (4)$$

This result is obtained from measurements performed with three different foils, at the energy E_0 equal to 0.550 MeV, 0.800 MeV,

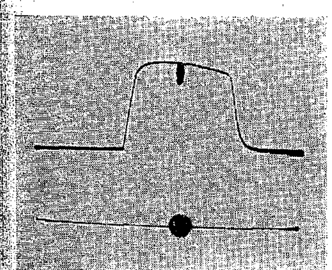

Fig. 2.

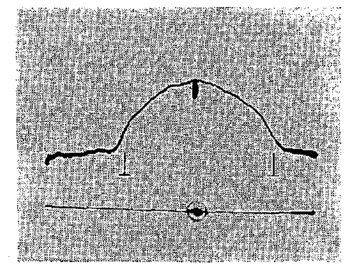

Fig. 3.

Fig. 2. Oscillogram taken with H_1^+, but without a foil and with the slit parallel to the direction of the magnetic deflection. $E_0 = 1.800$ MeV. Full length of sweep $Sw = 7.0$ cm on the screen. Distance 'D' between dots corresponds to $20 \cdot 10^{-8}$ ampères. The half-width is determined by the length of the slit, since the aperture A was put in the position $a = 8 + 8$ mm. This and the following reproductions are all made from oscillograms which have been slightly retouched.

Fig. 3. Distribution of H_1^+ after penetration of the 1.09 mg/cm² Cu foil. $E_0 = 0.550$ MeV. $Sw = 7.0$ cm $= 18.7$ degrees. $D = 1 \cdot 10^{-8}$ ampères. The aperture A in the position $a = 8 + 8$ mm has caused a cut-off at the deflections indicated by the two marks on the figure.

and 1.050 MeV, and with the width 'a' of the aperture varying from 12 mm to 16 mm. The errors should thus be expected to be only a few per cent. when the value

$$C = 0.0290\ (\mathrm{MeV})^{\frac{1}{2}}/\mathrm{cm} \qquad (5)$$

is used for all the distribution curves.

Some further examples of the angle distributions obtained are shown in figs. 4—6. The undeflected positions of the two beams of the oscillograph are visible as two dots on the oscillograms, because the intensity control was not turned down completely. The second beam was not deflected in the Y-direction. The line written by it, and the two dots, serve as a frame of reference. Such a frame is needed for the broadest distribution curves, like that shown in fig. 4, which allow only one of the movable sides of the aperture A to be put in a 'cut-off' position, whereby it is made more difficult to construct the zero-line for the distribution curve. In order to have the distribution curves passing outside the upper dot, a fraction of the pulse used for the intensification was sent through the pre-amplifier, thereby shifting the curves a bit upwards.

The measurements were performed with foils of one of the

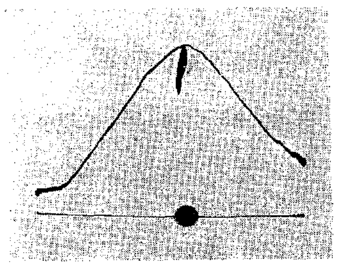

Fig. 4.

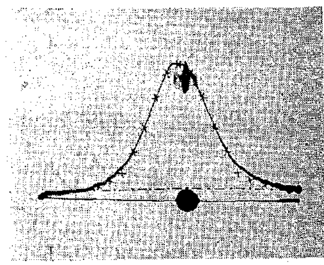

Fig. 5.

Fig. 4. D_1^+ after 0.84 mg/cm² Au. $E_0 = 1.050$ MeV. $Sw = 7.0$ cm $= 8.3$ degrees. $D = 1 \cdot 10^{-8}$ ampères. $a = 5 + 7$ mm.

Fig. 5. H_1^+ after 0.52 mg/cm² Cu. $E_0 = 0.800$ MeV. $Sw = 6.8$ cm $= 13.1$ degrees. $D = 1 \cdot 10^{-8}$ ampères. $a = 8 + 8$ mm. The crosses indicate a Gaussian distribution coinciding with the measured distribution at maximum and at half-maximum.

three elements beryllium, copper, and gold. The beryllium foil, which was available only in a thickness of 0.61 mg/cm², had been produced by evaporation, whereas the copper and gold foils were commercial, hammered foils of thicknesses about 0.50 mg/cm² and 0.17 mg/cm², respectively. Small inhomogeneities in the thicknesses of the foils are of no importance for the present purpose, but the thickness of the hammered foils also varied systematically over larger areas. When two or more layers were used, they were, therefore, put together in such a way that the total thickness of the foils should be as constant as possible.

A small disk with six circular openings, each about 1 cm in diameter, was used as support for the foils. It could be turned around its axis from outside the vacuum by means of a per-

manent magnet. Only four of the openings were covered with the foils to be investigated. The fifth opening was covered with a mica foil, the glow of which was used to indicate the position and degree of focusing of the beam, and also the current in it. The beam analyzer, which was used to select the atomic beam from the others, was adjusted so that the mica foil, for a proper position of the disk, was hit in the center. By turning another foil into a corresponding position it was assured that the beam penetrated it near its center, where the thickness is ex-

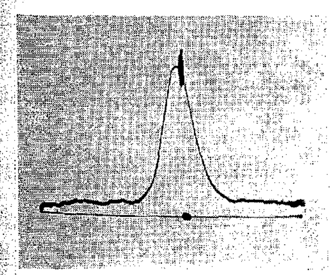

Fig. 6.

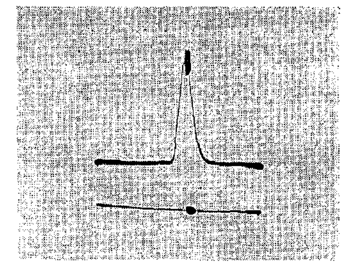

Fig. 7.

Fig. 6. H_1^+ after 0.61 mg/cm² Be. $E_0 = 1.800$ MeV. $Sw = 6.9$ cm $= 8.8$ degrees. $D = 1 \cdot 10^{-8}$ ampères. $a = 5 + 5$ mm.

Fig. 7. Same conditions as in fig. 6, but without foil. $Sw = 4.2$ cm. $D = 4 \cdot 10^{-8}$ ampères.

pected to be close to the average thickness determined by weighing.

The sixth opening was uncovered and was used to check the resolving power. A curve taken without a foil is shown in fig. 7. In order to have a detectable trace, the gain of the Y-amplifier was made smaller in this case. For the same reason also the sweep amplitude was made somewhat smaller by discharging the condensers before they had reached their full load. The half-width of the curve is determined mainly by the 3 mm width of the slit and the 1—2 mm diameter of the beam, and only to a lesser extent by the band-width of the pre-amplifier. The half-widths measured for the different foils must be corrected for the corresponding half-widths measured without the foils in order to give the true half-width $\delta_{\frac{1}{2}}$. The correction is, however, rather small, because the half-widths to the first approximation add up only geometrically.

The product $\alpha_{\frac{1}{2}} \cdot E$ of the true half-width $\alpha_{\frac{1}{2}}$ of the distribution of the projected scattering angle and the mean energy E of the particles scattered in the foil is, therefore, given with good approximation by

$$\alpha_{\frac{1}{2}} \cdot E \simeq \frac{C}{\sqrt{A_1 (E_0 - \Delta E)}} \cdot \delta_{\frac{1}{2}} \left(E_0 - \frac{1}{2} \Delta E \right) \simeq C \sqrt{\frac{E_0}{A_1}} \cdot \delta_{\frac{1}{2}}, \tag{6}$$

i. e. it does not depend explicitly on ΔE.

The result for protons and deuterons of different energies penetrating different thicknesses of the three foil materials are given in fig. 8, where $\alpha_{\frac{1}{2}} \cdot E$ is plotted as a function of E. The quantity $\alpha_0 \cdot E$ has been used as a unit, because this is convenient for the comparison with theory. The angle α_0 is given by the expression

$$\alpha_0 = Z_1 \frac{Z_2}{\sqrt{\frac{1}{2} A_2}} \cdot \frac{I}{E} \cdot \sqrt{\frac{t}{t_0}}. \tag{7}$$

Z_1 is the atomic number of the scattered particles, Z_2 and A_2 are the atomic number and the mass number of the hit particles, and t is the thickness of the foil measured in mass per unit area. The constant 'I' is the Rydberg energy 13.6 eV and the constant t_0 is the nuclear mass unit M_0 divided by the cross-section πa_0^2 of the hydrogen atom, i. e.

$$t_0 \equiv \frac{M_0}{\pi a_0^2} = 1.89 \cdot 10^{-5} \text{ mg/cm}^2. \tag{8}$$

The uncertainty in the determination of $\alpha_{\frac{1}{2}}$ is estimated to be about $\pm 5\,\%$. However, since the determination of the factor of proportionality C is somewhat indirect, a more direct measurement of the angular distribution was also made in the case of the 0.52 mg/cm² copper foil bombarded with protons of an energy E equal to 0.950 MeV. This was done in the following way.

A photographic plate was placed in the wide tube (fig. 1) just before the slit and under an angle of 4 degrees with the axis. The entrance stop to the chamber containing the disk with the

foils was exchanged with a stop having an aperture of only 1 mm. Then, this chamber was placed between the narrow and the wide tube (position A, fig. 1), and the whole apparatus joined to the lower end of the acceleration tube, but this time in

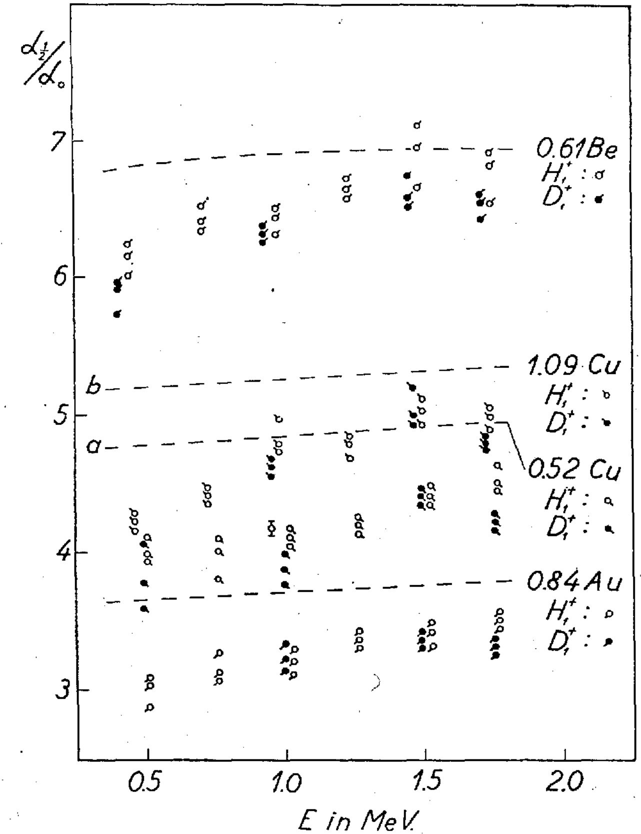

Fig. 8. The measured half-widths $\alpha_{\frac{1}{2}}$ as a function of the mean energy E of the scattered particles. The unit α_0, which is given by formula (7), is proportional to $1/E$. The dashed curves give the values to be expected from the theory summarized in the text. The single point at 0.95 MeV belongs to the 0.52 mg/cm² Cu foil and is obtained by means of a photographic plate.

a horizontal position, i. e. with the axis perpendicular to the beam. On a level with the apparatus an 0.17 mg/cm² gold foil was placed in the proton beam at an angle of 45 degrees with it, so that single scattering of the protons penetrating this foil caused the copper foil to be bombarded with a secondary proton beam which was 1 mm wide and practically parallel. With a tension of 1 million volts on the electrostatic generator and a beam current of about 0.1 micro ampère, this secondary proton beam will have an energy of 0.975 MeV and carry a current of only $3 \cdot 10^{-16}$ ampères. This means that, after an exposure of the order of 1 minute, the single tracks in the emulsion can still be

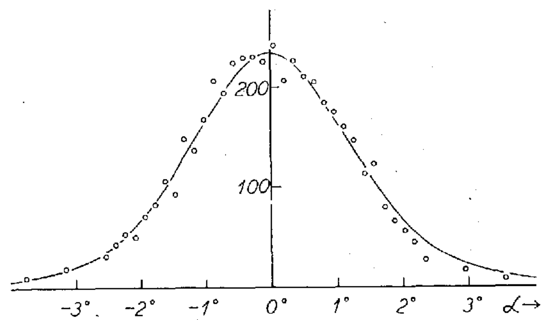

Fig. 9. Distribution of the projected deflection angle α determined by means of a photographic plate. Energy: $E = 0.950$ MeV. Foil: 0.52 mg/cm² Cu. Half-width of theoretical curve adjusted to give the best fit to the experimental points under the condition that the area below the curve equals the experimental value.

easily counted under the microscope, and the distribution of tracks in the plate thus be determined.

Under these conditions the distribution shown in fig. 9 is obtained. The half-width determined in this way has also been plotted in fig. 8, but with the following corrections. It has been decreased 1 % because of the finite width of the beam which was measured on a plate exposed with the copper foil removed; for reasons to be discussed in the following section, it has been increased by 3 % because the small angle between the plate and the beam implies that this measurement corresponds to an electrical measurement with a much smaller length of the slit. The total number of tracks counted was about 5000, so the uncertainty in the measurement should be of the order of ±2 %. The agreement with the electrical measurements is seen to be satisfactory.

Summary of the Theory.

The distribution of scattering angles for the beam of particles, after it has penetrated a foil, will in general consist of a central peak, the compound scattering proper, having tails extending to large scattering angles which are caused primarily by single violent collisions.

If the compound scattering results from a large number of individual small deflections, the peak itself will be of approximately Gaussian type and will have a standard deviation σ given by

$$\sigma = \alpha_0 \, [\log \bar{n}]^{\frac{1}{2}}, \tag{9}$$

where $\bar{n}$ may be said to represent the average number of effective nuclear collisions which a particle suffers by penetrating the foil.

One has to the first approximation[1],

$$\bar{n} \cong \frac{t}{t_0} \frac{\varkappa^2}{A_2 s^2} \quad \text{for} \quad \varkappa \lesssim 1 \text{ (Born's approximation)} \tag{10}$$

$$\bar{n} \cong \frac{t}{t_0} \frac{1}{A_2 s^2} \quad \text{for} \quad \varkappa \gtrsim 1 \text{ (classical approximation)}$$

with

$$\varkappa^2 = A_1 Z_1^2 (2\, Z_2)^2 \frac{M_0}{\mu} \cdot \frac{I}{E}, \tag{11}$$

where μ is the electron mass.

The quantity s is the so-called screening parameter, which characterizes the shielding of the nuclear field by the atomic electrons. As a simple estimate of s in the present case one may take

$$s \cong Z_2^{\frac{1}{3}}, \tag{12}$$

although, especially for light atoms, this expression represents only a rather crude approximation.

The above expressions cover the following experimental conditions:

$$\left.\begin{array}{l} A_2 \gtrsim A_1 \quad \bar{n} \gg 1 \quad \sigma^2 \ll 1 \\ Z_1 Z_2 \cdot s \cdot I \ll E \ll A_1 M_0 c^2, \end{array}\right\} \tag{13}$$

[1] N. Bohr, loc. cit.

where c is the velocity of light. These requirements are all fulfilled in the present experiment.

Formula (9) gives the scattering due to nuclear collisions only. Collisions with individual electrons will also give a certain contribution which is expected to increase the total scattering at most by a factor of the order of magnitude of $\sqrt{1+Z_2^{-1}}$ [1]. The accurate value of this correction is rather uncertain and, since more detailed considerations indicate that it may in fact be considerably smaller than the above simple estimate, we shall make

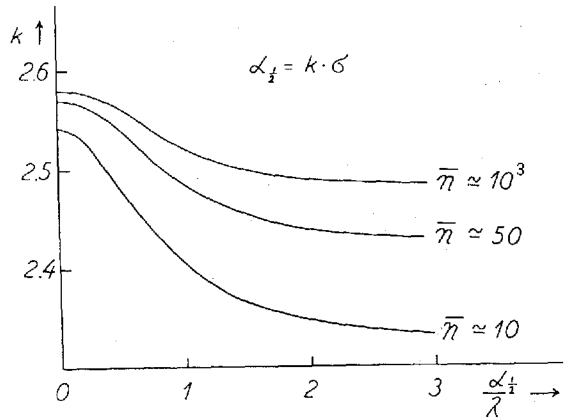

Fig. 10. Proportionality factor k as a function of the foil characteristics and $\alpha_{\frac{1}{2}}/\lambda$, where λ is the length of the slit divided by $l_1 + l_2 + l_3 + l_4$ (see fig. 1).

no correction for this effect in the theoretical formula. It may be added that only in the case of the Be-foil would the correction be of significance.

It should be stressed that σ refers only to the compound scattering proper and may differ considerably from the standard deviation for the total scattering distribution. Due to the contribution from the tails of the distribution, the total standard deviation would, for some of the foils investigated, be more than twice σ. With the present experimental techniques the tails cannot be followed very far due to the amplifier noise and the limited aperture of the apparatus. The total standard deviation is therefore not well suited for comparison with the above measurements.

A convenient measure of the compound scattering is the width of the peak at half-maximum. If the peak had a purely Gaussian

[1] N. Bohr, loc. cit.

form the half-width would equal 2.35 σ, but for the actual distribution it has a slightly different value. The relationship between σ and the measured half-width also depends to some extent on the length of the slit, since σ refers to the projection of the scattering angles on a fixed plane containing the beam, thus corresponding to the distribution measured with a slit of infinite length.

If one writes

$$\alpha_{\frac{1}{2}} = k\sigma, \tag{14}$$

where σ is given by the equations (9) to (12) and where $\alpha_{\frac{1}{2}}$ is the half-width calculated by the method of Williams[1], one gets for k the values shown in fig. 10. The theoretical curves in fig. 8 are drawn in accordance with these values.

Discussion.

For the measurements with the 0.84 mg/cm² gold foil, $\bar{n}$ is equal to 12 and the condition for compound scattering is therefore not very well fulfilled. Moreover, σ is rather sensitive to the value of $\bar{n}$ and, in view of the approximations involved in particular in the estimate of s, the 10—15 % discrepancy between theory and experiment, although outside the expected experimental uncertainty, presumably does not exceed the latitude of the simple theoretical formula used. The measurements confirm that for the value of $\varkappa$ in question, ranging from 18 to 35, the classical approximation must be used in the estimate of $\bar{n}$. In fact, the Born approximation leeds to values of σ almost twice as large as the values calculated by the classical formula and plotted in fig. 8.

For the Cu foils, the value of $\varkappa$ is between 7 and 19 and also here the classical approximation is expected to be valid. For the 0.52 mg/cm² foil $\bar{n} \cong 45$ and for the 1.09 mg/m² foil $n \cong 95$. As regards the agreement between theory and experiment, the situation here is much the same as for the gold foil. The difference between the two curves a and b in fig. 8 represents the expected deviation from the square root dependence of $\alpha_{\frac{1}{2}}$ on the thickness of the foil and is due to the influence of single scattering. This

[1] loc. cit., page 295.