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STUDIES ON THE ORIGIN OF THE SOLAR SYSTEM

BY

D. TER HAAR

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TABLE OF CONTENTS

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INTRODUCTION

In this introduction, we shall give in Part A a statement of the problem of the origin of the solar system and of the facts which have to be explained. In Part B, we shall take a necessarily short survey of sundry theories which have been proposed, together with reasons why we feel that they cannot be accepted as final solutions of this fascinating problem.

A. Statement of the Problem.

The nearest neighbours of our earth in the universe are the moon, the sun and the other members of the solar system. The sun and the moon are by far the brightest objects in the sky and the other members of the solar system are also among the brightest. It is not, therefore, surprising that astronomers throughout the ages have devoted special attention to the solar system.

Moreover, the system shews so many regularities in its dynamic and physical properties that its formation was certainly not due to chance. The fact alone that the direction of orbital motion of all planets and asteroids is the same is sufficient to establish this.

Before we discuss some of the theories advanced as to the origin of the system, we shall point out some of these regularities.

The solar system consists of the sun, nine large planets, twenty-eight satellites belonging to six of these planets, more than 1500 asteroids, and the comets and meteors. We shall center our attention on the large planets and only speak occasionally about the other bodies.

The regularities shewn by the solar system may be divided into a few groups:

A. The first group is that of the orbital regularities (cf. Table I). Apart from the common direction of orbital motion, the eccentricities of the orbits are small and the orbital planes are practically coincident. Also, the rotation of the sun is in the same direction and its equator is only slightly inclined to the planetary orbits.

B. The second striking feature is that the mean distances of the planets from the sun very closely obey the so-called Titius-Bode law. If the mean distance of the n-th planet from the sun is denoted by r_n and if we count the group of the asteroids as one of the planets, we have:

$$r_n = a + b \cdot 2^n,$$

where $a = 0.4$ A. U. and $b = 0.3$ A. U.

We may remark here that orbital regularities and laws for distances, comparable to the Titius-Bode law, are also found for the satellite systems (compare Tables II, III, IV, V).

C. The next group is the fact that the planets can be divided into two groups. The inner planets which form the first group have relatively small masses, high specific densities, low rotational velocities, and few satellites. The outer planets, which form the second group, have large masses, low specific densities, a relatively fast rotation, and large satellite systems[1].

If a theory is able to withstand the attacks of serious criticism, it ought to be able to explain the above-mentioned facts. However, there are more features of the solar system which have to be considered. We may call the reader's attention to a few of these.

Between Mars and Jupiter, there is no other planet, but the system of asteroids, estimated by Baade to contain about 30000 bodies, of which only less than 2000 have been observed up to now. The total mass of the asteroid system is extremely small (about 0.0003 times the mass of the earth).

Saturn possesses a ring system.

The outer satellites of Jupiter and Saturn have retrogade motions.

The inclination of the equatorial plane to the orbital plane is increasing in the series of the outer planets. Also some orbits of satellites are much inclined to the equatorial plane of their primaries.

Pluto, as we have already remarked, does not fit in with the other outer planets.

[1] We leave Pluto out of this discussion. Pluto's orbit has a large eccentricity, and the planet itself is small and dense.

D. Finally, the distribution of the angular momentum in the solar system has proved to be a stumbling block for many theories. As it is, the sun, although possessing more than 99 per cent. of the mass of the system, possesses only 2 per cent. of its angular momentum. The puzzle is why the sun has so little angular momentum.

We may, perhaps, point out here the difficulties inherent in this distribution of the angular momentum.

If the origin of the solar system has to be ascribed to a catastrophe of some kind, this accident in itself could have been able to transfer angular momentum to the material which would condense subsequently into the planets.

If, however, one tries to build up a theory starting from the sun, perhaps surrounded by a gas cloud, it is difficult to understand how this distribution came about. If the sun had been surrounded from the beginning by a gas cloud, the difficulty is to understand why the angular momentum per unit mass in this gas cloud should be so much larger than the angular momentum per unit mass in the sun. If, on the other hand, the system started from the sun alone, with the material for the planets being provided for instance by eruptions from the sun, one certainly would expect the angular momentum per unit mass to be about the same for the solar as for the planetary material.

Fouché, in 1884, was the first to point out the extraordinary character of the actual distribution of the angular momentum.

We shall see how this question has played a great role in the evaluation of sundry theories.

The origin of the asteroids will not be discussed here. The generally accepted explanation involves the breaking up of a larger body. According to recent work of Brown (1), this process might also have given rise to the meteorites.

We shall also not enter extensively into a discussion of the irregularities mentioned above. As far as the satellite systems are concerned, the great resemblance between them and the planetary system seems to point to a formation of the satellite systems analogous to the formation of the planetary system itself, even though the distribution of the angular momentum is not quite so extreme as in the case of the planetary system (2).

The ring system of Saturn is probably due to the fact that its distance from Saturn is less than the limit of Roche, inside which no satellite is stable against a tidal action of the mother planet. To understand this qualitatively, imagine a satellite brought nearer and nearer to its primary. The tidal forces increase, but the gravitational forces of the satellite itself on its matter remain the same. And so at a certain moment, the satellite, if liquid, would break in two and so forth until the fragments would be so small that surface tension keeps them together. If the density of the planet were the same as that of the satellite, the critical distance at which the breaking up would begin would be 2.44 times the planet's radius, as shown by Roche in 1850. Since the ring system of Saturn lies completely inside this limit, it seems reasonable to accept the thesis that these rings are the remains of a satellite, broken up during its formation.

It has been established that the age of the solar system is of the order of 2 to 3.10^9 years by different, independent indications such as, for instance, the lead content of rocks, where the lead is the end product of a radioactive family and thus has a different atomic weight (206.0) from that of the familiar lead (207.1). Another determination of the age of the universe can be obtained from the redshift of extragalactic nebulae, giving the same result[1].

The sun is radiating at present at a rate of 4.10^{33} erg per sec, which corresponds to a loss of mass of 4.10^{12} g sec^{-1}. If the sun had radiated energy at the present rate during the 3.10^9 years of its probable existence, it should only have lost 0.0001 of its mass. We shall assume in the present paper that during the process, leading to the solar system as we find it at present, the physical state of the sun was as we observe it at present. It is possible that we neglect vital processes by this assumption.

B. Survey of Theories about the Origin of the Solar System.

We can only report here very incompletely on the various theories. For further details, and a detailed criticism of the older theories, we therefore refer the reader to the original papers and to the many textbooks written on this subject, especially the volumes by Russell, Dugan, and Stewart (3), Nölke (4), and Russell (5).

[1] Compare the considerations of Bok (39) and Unsöld (40).

In general it is possible to divide all theories into two groups, according to the question whether or not the author has assumed an interaction with other celestial bodies as an important factor in the development of the solar system. In the first case, we have an open system and, using the term introduced by Belot, we can call these theories *dualistic* (sometimes the adjective "catastrophic" is used). In the other case, we have to deal with a closed system and the theories are called *monistic* or *uniformitarian*.

I. *Monistic theories*: 1. Descartes' theory. The first theory proposed in modern times is that of Descartes, advanced in 1644. At that time, observational data were scarce and only the sun, 6 planets and 7 satellites (the moon, 4 Jovian and 2 Saturnian satellites) had been observed. Also Newton's law of gravitation, which was to be published in 1665, was still unknown. It is thus more surprising that Descartes was able to formulate a theory which could explain many of the observational data than that his theory had to be abandoned after Newton's severe criticism.

Descartes started from a large whirl of matter in which 14 large bodies were floating as pieces of wood in a river. As can be seen in actual whirls carrying pieces of wood, the larger bodies have a tendency to collect around them the smaller ones and in the same way the sun became surrounded by the 6 planets, while the earth, Jupiter and Saturn got respectively 1, 4, and 2 satellites. Since the movement in the inner regions of a whirl is faster than in the outer regions, one could also understand that the rotation of the inner planets was faster than that of the outer ones.

The great historical significance of this theory is that it was the first attempt to explain the observational data, starting from some simple hypothesis. As soon as Newton had found his gravitational law, it was, however, possible to shew that this theory could not be maintained.

Newton himself believed that God had created the solar system in its present state and that He would look after it if its future were endangered by mutual perturbations of the planets. His influence on his fellow-scientists was so large that the cosmogonical theories of Buffon and Kant remained practically unnoticed. This only changed when Laplace arrived with his theory.— Laplace who wrote about Newton: "Je ne puis m'empêcher d'observer combien Newton s'est écarté sur ce point de la méthode dont il a fait ailleurs de si heureuses applications."

2. KANT's theory. In 1755, IMMANUEL KANT in his "Allgemeine Naturgeschichte und Theorie des Himmels" gave a qualitative cosmogony, which was ultimately worked out more quantitatively by Du Ligondès in 1897.

Kant started his treatise by answering the theological objections to the proposal of a cosmogony by remarking that the laws of nature are created by God, so that it is not lack of reverence when we try to find out the effects to which their action leads.

Kant's idea is to start from a nebula in the centre of which the sun is placed. Due to gravitational forces the rest of the matter will rotate around the sun. Under the influence of mutual collisions, the nebula will pass into a disc, where all particles are rotating in circles around the sun. The next step is that there is a tendency of the matter in the disc to condense into some large bodies which become the planets. Since this condensation takes place gradually, the first result will again be a rotating nebula, but now on a smaller scale, from which the satellite systems ensue. The larger the planet, the larger its gravitational attraction, and the larger the number of satellites.

Kant also shews that the rotation of the planets around their axes will be in the same direction as their rotation around the sun. To understand this, we have to consider a particle moving in the same orbit as and behind the planet. Under the influence of the attraction of the planet, its velocity will increase and thus also the centrifugal force. The result is that it will move outwards and that if it collides it will give to the planet an angular momentum of the right direction.

Kant was able to explain the first group of regularities, mentioned in part A. He did not attempt to explain the other three. He was unaware of the difficulty of the distribution of the angular momentum, and even of the fact that angular momentum has to be preserved. The fact that the present distribution of the angular momentum was not explained in this theory was the reason why Kant's theory was not accepted as the final answer. In the following chapters we shall see that an extension of this theory seems to be able to give an explanation of C and perhaps of B.

3. LAPLACE's theory. In many textbooks and popular works, Kant's theory is mentioned together with the theory of Laplace of 1796. The view often held is that Laplace put Kant's ideas into scientific terms. As we shall see, this is far from correct. The theories are widely different. Moreover, Laplace when writing his popular book "Exposition du Système du Monde" was unaware of the existence of Kant's theory.

Laplace's idea was to start from a situation where the sun is surrounded by a hot gaseous atmosphere. This nebular atmosphere was gradually cooling and thus contracting. As it contracted, the rotational velocity necessarily increased by the preservation of angular momentum, and thus also the centrifugal force at the equator. Ultimately this force became larger than the gravitational force and a ring of matter was flung into space.

This process was repeated, giving rise to a system of concentric rings from which by a process not further explained the planets derived. Finally, the remains constituted the sun.

Laplace can easily explain A and perhaps B, but the crucial point here is again D. In fact, if all the mass and angular momentum of our solar system was concentrated in even as small a volume as that of the present sun, the centrifugal force at its equator would only be about five per cent. of gravity and it would be far from any danger of breaking up.

This failure to explain D alone suffices to disprove Laplace's theory. Another difficulty, which can only be overcome quite artificially, is that the Laplacian rings have no tendency to condense into planets (they might form a swarm of asteroids but not larger bodies). The only explanation is to suppose that the actual condensation should have begun already before the throwing off of the rings.

Faye's theory of 1885 was essentially the same as Laplace's and is also unable to explain D.

4. BIRKELAND's theory (6); BERLAGE's theories (7): In 1912, Birkeland gave a sketch of a theory in which the solar magnetic moment and the particles emitted by the sun played a role. His idea was that through the strong magnetic field of the sun the charged particles, which are for the most part emitted from the equatorial regions, should spiral down towards limiting circles. The radii of these circles would depend on the ratio of the charge to the mass of the particles.

Birkeland is thus able to explain both A and B. The problem D is not a problem in this case either, since, as shewn by Alfvén, due to currents in the surrounding matter evoked by the sun's magnetic field transfer of angular momentum from the sun to the surrounding matter is possible. The time needed for this transfer is small (10^5 years) as compared with the age of the solar system.

Nevertheless, this theory could not be maintained since the solar magnetic field is not strong enough to produce the desired effect. The orbits of the emitted particles are only slightly curved and they all leave the regions of the solar system.

Birkeland was the first author to consider electromagnetic effects. After him, Berlage, inspired by his ideas, tried to account for many features of the solar system by taking the solar electric field into account. Berlage's theories met with the same fate as Birkeland's. They remained practically unobserved. For instance, Alfvén who in 1942 again investigated the possible influence of the solar magnetic field does not mention either of them.

In his first theory Berlage assumed that the sun emits negatively charged solid particles and positively charged ions. Their emission is a consequence of the fact that radiation pressure on them exceeds gravitation. The result is a space charge around the sun and a positive charge of the sun itself.

The next assumption is that the sun, as in the theory of Kant, is surrounded by a gaseous disc. If we now roughly calculate the equilibrium position of an ion in the disc under the influence of the space charge, solar charge and solar gravitational field (Berlage neglects the centrifugal force), it can be shewn that for each ion there exists an equilibrium distance which increases with decreasing atomic number of the ions.

The result is that in the disc there will be formed concentric rings of ions, their radii depending on the ion in question.

These ion rings will act as the initial nuclei for condensation, and afterwards each of these rings will condense ultimately into one planet, as in Laplace's theory.

Since to each of these ion rings is ascribed a certain isotope of one of the elements, Berlage is able to estimate the masses of the planets. Also he finds decreasing densities of the planets with increasing distance from the sun, which—assuming that Jupiter

and Saturn possess a heavy nucleus surrounded by a lighter atmosphere (Jeffreys)—is in agreement with observation.

The distance of the rings from the sun can be shewn to correspond to the Titius-Bode law.

We see that Berlage is able to explain here A, B and C: he does not attempt an explanation of D. This theory will not, however, stand criticism. Apart from the fact that it can easily be shewn that in the way Berlage suggests enough matter can never be collected to build up, for instance, Jupiter there is the fact that the basic assumption that the sun should emit negatively charged solid particles is shewn to be wrong by observation.

This was the reason why Berlage himself left this theory for a second attempt where he now used the fact that the sun emits positive ions and electrons. Considering the effect of space charge, radiation pressure and gravitation on the charged particles, but still neglecting centrifugal forces due to rotation, Berlage is able to calculate the electric field strength in the neighbourhood of the sun. It then appears that this field is of a periodic character. This means that there are concentric spheres on the surface of which the field strength is equal to zero.

If we now consider the gaseous disc which is again supposed to be surrounding the sun, we see that since the atoms will all be ionized for part of their life matter will be concentrated on the circles where the disc is intersecting the spheres of zero field strength. In this way Berlage now gets his rings of matter. The rest of the condensation then takes place in the same unexplained way as in Laplace's theory.

This theory explains A and B, but has to leave C and D unexplained. Berlage himself sees as a serious deficiency of this attempt that it is unable to explain the satellite systems. Another serious objection is that the degree of ionization in the gaseous disc will be so low that electrostatic effects are negligible (compare Chapter II, Section B).

In his latest theory, Berlage has completely left all electromagnetic considerations and considers in detail the history of a gaseous system which may be found around the sun. He thus follows Kant. First of all, he shews that this system will assume the form of a disc. He also gives an expression for the density in the plane of the disc as a function of the distance from the sun.

After that Berlage looks for a possibility that this disc may condense spontaneously into rings. Afterwards these rings have to condense into the planets. For that purpose he investigates whether a slightly different density function might be stable. This means that for this new density function, the total mass, angular momentum, and energy are the same as before, but the kinetic energy of the system is larger than initially. Berlage really finds such a tendency to form rings.

In this way he can explain A and B. His reasoning is, however, very loose as, for instance, his assumptions about the temperature distribution and the laminar motion in the disc. Also his assertion that rings will be formed does not rest on a firm foundation. Finally, there is still the difficulty of the condensation of the rings into planets which we met already in the discussion of Laplace's theory.

5. ALFVÉN's theory (8). The Swedish physicist Alfvén has given a very interesting theory in a series of three papers, taking into account the magnetic forces on ionized matter.

His reasons for advancing this theory are the following. To begin with, the force exerted by the sun's magnetic moment on ionized matter can be much larger than the gravitational force on the same matter. For instance, on a proton moving in the earth's orbit with the earth's velocity, the first force exceeds the second by a factor 60.000. In the second place, ALFVÉN has shewn in an earlier paper (9) that transfer of angular momentum from the sun to a surrounding ion cloud is possible. The rotating magnetic moment of the sun evokes currents in the cloud and an effect similar to that braking a metal between a magnet's poles takes place. This transfer of angular momentum can take place in an appreciable amount in as short a period as 10^5 years. In this way, D does not present any difficulty.

Now, Alfvén's idea about the formation of the outer planets is the following. Suppose that in its journey through space, the sun meets an interstellar gas cloud and becomes surrounded by it. If we may neglect the rotation and velocity of the cloud with respect to the sun, the atoms in the cloud will start falling towards the sun, and their kinetic energy will increase during that fall. Eventually this kinetic energy will become so large that ioniza-ion by collisions can take place.

The idea is now that collisions are so frequent that this ionization indeed takes place. Once an ion is formed, the movement towards the sun is stopped and the ion has to move along the magnetic lines of force until it reaches an equilibrium position. Alfvén shews that this equilibrium position is situated in the equatorial plane of the sun.

Assuming now that the ions are moving uniformly towards the sun and are all ionized at the same distance from the sun, and considering in detail the subsequent movement of the ions towards their equilibrium position in the equatorial plane, he gets the mass distribution in the equatorial plane. Alfvén takes the fact that this mass distribution agrees roughly with the mass distribution in the series of the outer planets as a support of his theory.

In this way Alfvén is able to account for the outer planets. This mechanism is, however, unable to explain the origin of the inner planets because even in the most favourable case the distance from the sun at which ionization occurs will be by far larger than the mean distance of Mercury from the sun. Also, one should expect from this mechanism to find lower densities for the inner than for the outer planets but the densities of the inner planets are higher than those of the outer ones.

Alfvén without any detail suggests the following process. The sun in its travel through space should have met an interstellar smoke cloud consisting of solid particles. Through the strong radiation of the sun those particles will sublimate as soon as they have come near enough. The resulting atoms become ionized but at a shorter distance from the sun.

Instead of the Titius-Bode law, Alfvén introduces a diagram where the ratios of the masses and of the distances from the primary are connected. His explanation of this diagram, however, seems to be extremely weak, and it does not seem to be possible to get the same result by valid reasoning.

But also his original idea is unable to stand a critical scrutiny. In the case of a gaseous cloud surrounding the sun from the beginning electromagnetic forces will not play any role at all because of the absence of ionization in the cloud (cf. chapter II, section B).

As far as Alfvén's suggestion about the heating up of an interstellar gas cloud is concerned, the atoms will certainly not save up their energy until they reach the immediate vicinity of the sun. It can easily be shewn that their mean free path is by far too small. However, one could imagine that the whole cloud was heated up while contracting. Apart from the fact that one has to assume zero angular momentum of the cloud around the sun, and the fact that the energy gained seems to be emitted by radiation before ionization takes place, it seems that the desired object still is not attained. Ionization will start all over the cloud, and since ions cannot approach the sun, only a very small fraction of the gas cloud, insufficient to form the planets, will be available for further condensation.

6. Von Weizsäcker's theory (10). In a paper, dedicated to Sommerfeld on the occasion of his 75th birthday, von Weizsäcker has advanced a new theory about the origin of the solar system. The greatest importance of this theory is in the fact that it provides us with a definite scheme for further calculations[1].

His theory can be divided into different parts, corresponding to the different stages in the development of the solar system. First, he discusses the formation of a gaseous disc around the sun, secondly, the formation of a system of vortices in this disc, finally, the condensation process, and the satellite systems.

The first part is practically identical with the similar parts in Kant's or Berlage's theory. The disc is supposed to contain about one tenth of the solar mass, and the over all density will be about 10^{13} atoms per cm^3.

The second part is the most interesting, but probably also the weakest point in this theory. Supposing that the orbits of mass elements in the disc may be assumed to be Keplerian, von Weizsäcker shews that a system of vortices can be built up from these Keplerian orbits. In fig. 1, we see such a configuration.

Von Weizsäcker is led to such a configuration for two reasons. The first is that gravitational forces are by far the most important forces in the disc. The second is that in a system of vortices, as shewn in fig. 1, the energy dissipation will be small. In the large vortices the dissipation will be negligible in a first approximation. However, along those circles where the rings of vortices meet there will be large viscous stresses. These will presumably

[1] It will be seen that the present paper is to a large extent a clarification and extension of von Weizsäcker's ideas.

give rise to secondary eddies on the circles sepa[...] vortices. These eddies are called the "roller bea[...] They will probably regulate the whole system. Howe[...] will be dissipated in these "roller bearings". Conditions [...] densation will be more favourable in these secondary [...] (compare Chapter IV), and so we may expect the planets [...] formed at distances from the sun corresponding to the radii [...]

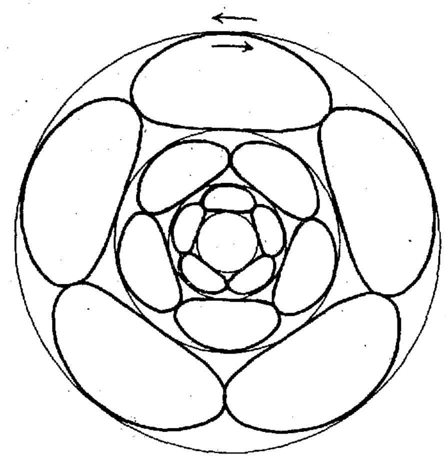

Figure 1. The outer arrow indicates the direction of rotation of the whole disc, while the inner arrow indicates the direction of rotation in the vortices. The sun is in the centre of the whole system.

the circles separating the main vortices. Now, von Weizsäcker gives reasons to believe that the number of large vortices in each ring is constant. This means that the ratio of two consecutive radii will be constant, thus giving us the Titius-Bode law for the distances of the planets from the sun (neglecting the constant term).

Another consequence of the condensation into planets in the "roller bearings" is that we will get a counter-clockwise rotation of the planets if the whole system is rotating in a counter-clockwise direction, in agreement with observation. The rotation in the large vortices is in the opposite clockwise direction.

During their formation and immediately thereafter the planets will be surrounded by extended atmospheres. In these atmospheres the satellite systems will be formed. Von Weizsäcker does not, however, enter into an extensive discussion of this question.

Due to the dissipation of energy, the disc will disappear gradually. Von Weizsäcker estimates its lifetime to lie between 10^7 and 10^8 years, which is of the same order of magnitude as the period necessary to build up bodies of the size of the planets.

Von Weizsäcker has given an explanation of A, B, and C. He also gives an explanation of D in the following way. The dissipation of the gaseous system is accompanied by a flow of atoms into interstellar space and a simultaneous flow of matter to the sun. He now assumes that the light elements leave the system, carrying with them the necessary angular momentum, while the matter falling onto the sun does not possess any angular momentum. In this way, he can at the same time explain the difference in constitution of the planets and the sun, and the distribution of the angular momentum. It is, however, difficult to see why this separation of the cloud according to angular momentum and atomic weight should take place. Also, this process cannot decelerate the sun sufficiently; there is a discrepancy of a factor 100.000.

Also his picture of the vortices seems difficult to maintain: Keplerian orbits are only a first approximation. Hydrodynamics has to be applied, but, as we shall see in Chapter III, it is as yet unable to give von Weizsäcker's configuration. However, the main merit of this theory is that it has revived again Kant's theory and that it has drawn attention to the importance of hydrodynamical considerations.. In the following chapters we shall see that a slightly different attack seems to give a reasonable explanation of A, C, and perhaps B, while D has as yet to remain unexplained.

II. Dualistic Theories: 1. Buffon, Chamberlin-Moulton, Jeffreys, Jeans. Ten years before Kant published his theory a dualistic theory had been advanced by Buffon. In those days fantastic ideas about comets were common and Buffon therefore proposed the collision of the sun and a comet as the source of our solar system. (Buffon estimated the mass of the comet of 1680 as 28000 times the earth's mass.)

Through the collision matter was torn out of the sun which matter later condensed into planets. The rotation of the sun might also have been caused by the collision.

Modern tidal and collision theories have the same foundation the only difference being that another star, instead of a comet, is the foreign body which produces the material.

Chamberlin and Moulton proposed that as a second star was passing the sun in a hyperbolic orbit by tidal action and eruptions material for the planets was provided. The first heavier eruptions would provide the material for the outer and the secondary eruptions that for the terrestrial planets.

After the second star had departed the gaseous matter would cool and condense. Part of it had fallen back on the sun and part of it escaped into open space, but the rest could be used for building up the planets. In the cooling process liquid drops (planetesimals) would be formed and even larger solid cores which were sufficiently large to hold the lighter gases. In the course of their rotation around the sun those cores swept up matter and so the planets grew out of this gas.

The orbits of the cores which had originally large eccentricities are "ironed out" by the resisting medium.

The theory proposed by Jeans and also by Jeffreys in his first paper is about the same. They do not introduce the solar eruptions since it is known that the radiation pressure responsible for prominences and similar phenomena is not large enough to cause eruptions as large as needed here. Tidal action produced a filament which breaks up into smaller gaseous fragments. In those fragments condensation takes place into liquid bodies and so on to planets.

The small eccentricities are again brought about by the resisting medium.

These theories have the advantage that they are able at first sight to explain the distribution of the angular momentum (D) without difficulty. They do not attempt to explain either B or C while for A they use the resisting medium.

The first difficulty lies in the explanation of the planetary rotation. The explanation put forward by Chamberlin is not convincing and therefore Jeffreys assumed later that it was not a close encounter, but an actual collision which took place. Taking into account the viscosity of the resulting ribbon torn out of the sun, he could then shew that rotation of the right order of magnitude would ensue.

The next and greater difficulty is as Nölke has shown the influence of the resisting medium. It seems to be doubtful whether this medium really can bring about the small eccentricities.

Another difficulty is the formation of the satellite systems. Although the original idea was that tidal forces caused by the sun were responsible for them, Jeans himself shewed that this notion would not work. In Jeffreys' later theory it is perhaps more easily explained, since (quoting Russell) "almost anything may have happened in the period of wild turbulence, which included the formation of the ribbon and its segregation into separate bodies."

Also, if the material comes from the sun, it will be extremely hot and the danger exists that it may fly away into open space before beginning to condense, as was pointed out by Spitzer (11).

Finally, the explanation of D is not as easy as it seems. At first sight one would think that during the collision sufficient angular momentum may have been imparted to the filament. Russell (5) has shown, however, that this transfer of angular momentum by the second star is not an easy job and that, if it was possible at all, which he doubts, one would expect large inner and small outer planets.

Russell also deals with some other hypotheses to save these theories, but ends his monograph by saying that we are as yet no wiser about the origin of the solar system than we were when Newton found his law of gravitation, a point of view shared by Nölke.

2. Binary hypotheses; Lyttleton (12, 13), Hoyle (14). During the last decade, several theories have been proposed involving the assumption that the sun was originally a member of a binary or multiple system.

The first theory of Lyttleson assumes that the binary companion of the sun undergoes a close encounter with a third star, similar to the encounter assumed in Jeffreys' theory. The encounter results in a disruption of the binary system and the production of a gaseous filament which may produce the planets. Although Luyten's manifold criticism does not seem to be valid, the formation of satellite systems and the small eccentricities, together with Spitzer's objection (11) seem to be too large stumbling blocks.

In his second theory, Lyttleton starts from a triple star. The separation of the two companions of the sun will decrease as part of the evolution of a binary system. The two stars will finally combine into one mass. This mass will, however, break up because of rotational instability. After this fission the two parts will leave the system producing a situation similar to that met in Lyttleton's first theory. The same objections apply, therefore, to this theory.

The last development in this direction is given by Hoyle. According to Hoyle, a supernova outburst of the second component will account for the breaking up of the binary system and for the material from which the planets are formed. It seems, however, that this theory meets the same difficulties.

In his last paper, Hoyle considers the condensation process in detail and arrives at estimates of the original rotational periods of the planets. His reasoning develops along lines parallel to those which will be discussed in Chapter IV. It seems, however, that he arrives at wrong conclusions because he neglects the exhaustion of the gaseous system and all hydrodynamical effects. His proof of the direct rotation of the planets is essentially the same as that given by Kant or Alfvén.

III. Final Remarks: We have not included all theories in our survey. Manyof them as, for instance, those by Arrhenius and See are merely variations on themes discussed here. Other theories like the "Welteislehre" by Hörbiger-Fauth, which has been dealt with conclusively by Nölke (4), or the recent theory by Haldane (15) who seems to drive the consequences of the expanding universe rather far need not to be taken seriously.

However, there exists one recent theory which seems to be, at present anyhow, only an outline of a theory but which must be mentioned briefly. It is Whipple's attempt (16) to produce a planetary system from a large smoke cloud. He starts from a smoke and gas cloud with a radius of about 30000 A. U. containing about one solar mass. The contraction of this cloud should produce both the sun and the planetary system.

The original cloud is assumed to possess negligible angular momentum so as to account for the low angular momentum of the sun. The planets are assumed to be formed in a stream in the cloud so that those initial condensations which have to develop into the planets have already from the beginning the necessary angular momentum. The solution of D is thus put into the theory from the beginning. The planets (or better the condensations which will later form the planets) will now spiral towards the sun because their accretion of matter of zero angular momentum.

Whipple gives a rather half-hearted explanation of A, but does not attempt to explain B or C. Furthermore, his discussion of the planetary rotations seems to be difficult to follow and lacks quantitative evaluation. Altogether, there seems to be very little reason as yet to accept this theory as a final solution.

Recapitulating, we can say that there seems at present to be no theory which can explain satisfactorily the various properties of our solar system. Especially the differences between the outer and the inner planets and the present distribution of the angular momentum seem to have presented unsurmountable difficulties.

Dr. L. Spitzer has kindly drawn my attention to the recent papers by A. Gasser (Helv. Phys. Acta, *18*, 226, 1945), J. Sourek (Memoirs and Observations Czechoslov. Astron. Soc., Nr. 7, 1946), A. C. Banerji, Proc. Nat. Inst. Sc. India, *8*, 173, 1942) and G. Armellini (Rendic. Reale Accad. d'Italia, serie 7, vol. *4*, no. 11). It has not, however, been possible to include these in the review in this introduction.

Chapter I.

Summary.

In view of the fact that as yet no acceptable solution for the origin of the solar system appears to exist, it seems justifiable to investigate again a few aspects of this old question. There are several reasons why this should be done. First of all, it seems that as yet no sufficient attention has been paid to the physical properties of a gaseous system from which the planets should condense. Secondly, up to now nobody seems to have drawn any conclusions from the remark of JEFFREYS (17) that the initial steps in the condensation process will be the same as in the case of a supersaturated vapour. HOYLE (14) has discussed this problem rather extensively, but his discussion lacks quantitative reasoning and he neglects a few important aspects of the problem and therefore arrives at the wrong conclusions. Finally, in an as yet unpublished paper which was dedicated to Prof. Niels Bohr on the occasion of his sixtieth birtday[1], VON WEIZSÄCKER (18) has set forth new ideas about cosmogonies which might be used for a discussion of the origin of the solar system. Our discussion will, however, run along lines slightly different from those of von Weizsäcker's own theory (10) about the origin of the solar system, because of the difficulties encountered there.

Before discussing the new ideas which we wish to present in the present paper and the reasons why we are discussing just those points which we shall look into, we shall briefly discuss this second paper by von Weizsäcker.

Von Weizsäcker starts from a situation in which the universe

[1] I wish to express my sincere thanks to Prof. Bohr for giving me an opportunity to see this manuscript. This paper has in the meantime been published.

is filled with gas. The composition of this gas is supposed to be roughly the same as that of the sun or of the interstellar gas, i.e., mainly hydrogen. Also there is a velocity distribution which may be described apart from its fluctuations as the expansion of the universe. The origin of these velocities and of the distribution of the elements in the gas are not discussed and are supposed to belong to earlier periods. Now, von Weizsäcker investigates the development of this gaseous system. Because of its large dimensions, turbulence will be present. The consequence is that there will be regions of higher density. Matter entering such denser regions will lose the energy gained in the gravitational field because of viscous interaction and will be captured. In this way we shall get conglomerations of matter. These conglomerations are the first stage of galaxies.

In such a proto-galaxy, the same process will start afresh on a smaller scale, and the condensations will now be the proto-stars. The next step should be the formation of planets in the gaseous system doomed to become a star, and the last step might be the formation of the satellite systems.

The formation of the star from the gaseous rotating system will be accompanied by the dissipation of the system. The rotation is due to the whirling movement of the matter, and we may expect the linear velocities at the outskirts of the system to be of the order of magnitude of the turbulent velocities. Due to the concentration of matter in the centre, the outer parts will try to move with velocities given by Kepler's third law. This means that different parts of the system will move with different velocities and viscous stresses will result. These forces try to accelerate the outer parts and decelerate the inner parts of the system in an attempt to bring about a uniform rotation like that of a rigid body. Also these viscous forces entail a loss of energy. So we have a situation where there is at the same time a dissipation of energy and a transfer of angular momentum from the inside to the outside of the system. Von Weizsäcker assumes that these two processes are possible because mass with higher than average angular momentum disappears into interstellar space while at the same time the rest of the mass with low angular momentum will become concentrated in the centre of the system thus providing us with the necessary energy. In this way we get a slowly rotating central

mass (the star) surrounded by a faster rotating surrounding gaseous cloud. This implies that although in the initial stages the rotational velocities in the centre were much higher than at the outskirts the second stage presents us with a slowly rotating star and a faster rotating gaseous cloud. As soon as the density in the cloud is below a certain limit the rotational velocities in the cloud will be determined by the central mass and follow the third Keplerian law.

The equilibrium shape of such a rotating gaseous cloud will be a lens shape or disc. In this disc there will still be turbulence. However, it is still the question whether the configuration of vortices will really be as regular as the one given by von Weizsäcker.

Accompanying the disappearance of the solar gaseous envelope, condensation will take place in it. There will be many centres of condensation and during the lifetime of the disc these condensations will grow to become as large as the present planets. Together with their formation the planets will become surrounded by extended atmospheres. The evolution of these atmospheres will probably be analogous to the evolution of the solar envelope. In this way we have a mechanism for the formation of the satellite systems.

Now, the question discussed in the present paper is in how far this qualitative scheme may account for the various properties of the solar system. Before starting to discuss the various aspects of the problem quantitatively we shall give a brief survey of the contents of the following chapters.

In Chapter II we shall first of all discuss the shape of the solar gaseous envelope. We shall try to take into account the dissipation of the disc by assuming this disc shape to vary slowly. After that we shall discuss the various physical properties of this disc. The most important property is the temperature in the disc since the temperature is important in determining the shape of the disc. First, it is shewn that ionization in the disc is negligible. As was first shewn by Eddington (19), ionization by stellar (or solar) radiation will result in a much higher temperature of the gaseous system because the electrons will leave the atom with kinetic energies corresponding to the surface temperature of the star. These high velocity electrons will, by interactions with the gas atoms, set up a high temperature. Then, we have to calculate the optical depth of the disc in order to determine whether much radiation energy is captured in the disc. This, however, appears not to be the case. After that, we can determine the temperature in the disc. This temperature ranges from 75°K in the neighbourhood of Neptune, to 700°K in the vicinity of Mercury.

Next, it is shewn that the radiation density will be approximately a diluted Planck radiation, that radiation pressure can be neglected, and that there will be no appreciable separation of elements, due to either gravitational separation, thermal difffusion, or other sources. Finally, we compute the densities of various molecules in the disc.

In this way, we have a more or less definite physical picture of the disc.

In Chapter III we shall discuss the hydrodynamical aspects of a gaseous disc in general.

We shall try to estimate the lifetime of the disc, and the transfer of angular momentum, not necessarily due to a flow of matter, from the central body to the disc during the lifetime of the disc. We shall also discuss the question whether it is possible to explain the Titius-Bode law.

In Chapter IV the condensation process is discussed. This discussion will resemble very closely the discussion of Hoyle (14) or von Weizsäcker (10) but some new features will be revealed. We shall discuss the three stages in the condensation process. These are the formation of condensation nuclei, the growth of these nuclei, and finally the stage of rapid gravitational capture.

In Chapter V we shall apply the results of Chapters II to IV to the solar envelope. We shall see that we are now able to explain the differences between the outer and the inner planets as far as mass and density are concerned.

In Chapter VI we shall discuss the satellite systems and the rotations of the planets. It will be seen that we can divide the satellites into two groups which we shall call the "regular" and the "irregular" satellites. It is proposed that the "regular" satellites are formed out of the planetary envelopes. The "irregular" satellites, however, are supposedly captured by the planets.

If we now compare the results of the present paper with the requirements of a successful theory discussed in the introduction

we see that we have been able to explain some hitherto unexplained points of the group C, and, possibly, shed some light on the difficulties connected with the explanation of B and D.

We have been able to account for the fact that the planets fall into two definite groups (C) by looking carefully into the condensation process.

Although the Titius-Bode law (B) has still to remain unexplained there seem to be indications that a thorough investigation of the hydrodynamical problems connected with the evolution of gaseous systems, such as we have studied here, might give a clue to this property of the solar system.

A regular system of vortices would at the same time give us an easy explanation of the circular orbits. The direct rotation of all the planets in one plane follows immediately from the fact that the condensation takes place in a rotating disc.

The present distribution of the angular momentum (D) still cannot be explained but some indications are given as to the direction in which the solution might possibly be found.

We have not discussed at all the way in which the sun should have been formed from an original nebula. This formation may have an important bearing on the explanation of the present distribution of the angular momentum but falls outside the scope of the present paper.

Altogether, the present paper gives a program for future investigations of many points rather than a complete solution.

Chapter II.

Physical Properties of the Solar Envelope.

We shall consider here a gaseous system in the centre of which the sun is situated. The radiation of the sun is assumed to be the radiation of a black body of 6000° K. The dimensions of the sun are supposed to be the same as at present ($r_0 = 7.10^{10}$ cm). The constitution of the envelope will be assumed to be about the same as the constitution of the sun, i. e., mainly hydrogen and helium, corresponding to a mean molecular weight of about 3.

A. Shape of the envelope. In this section we shall follow VON WEIZSÄCKER (10) with a few alterations. We shall start from the equations of motion:

$$\text{grad } U + \frac{1}{\varrho} \text{ grad } p - \omega^2 \vec{s} = 0, \tag{2.1}$$

where U is the gravitational potential energy, ϱ the density of the gas, p its pressure, and ω its angular velocity. Finally, $\vec{s}$ is the vectorial distance from the rotational axis (z-axis).

We take for U:

$$U = -\frac{\gamma M_0}{r}, \tag{2.2}$$

where γ is the gravitational constant, M_0 the solar mass (we neglect the gravitational action of the gaseous envelope), and r the distance from the centre of the sun.

For the pressure we use the ideal gas law:

$$p = \varrho R T, \tag{2.3}$$

where R is the gas constant per gr., and T the absolute temperature.

For this temperature we shall use:

$$T = a \cdot r^{-\frac{1}{2}}, \tag{2.4}$$

which follows if the temperature is determined by an equilibrium between the absorbed solar radiation, and emitted black body radiation by the gas. In the next sections of this chapter we shall derive this formula for the temperature.

Combining equations (2.3) and (2.4), we have

$$p = b \varrho r^{-\frac{1}{2}}. \tag{2.5}$$

Normalizing b so that $T = 6000°$ for $r = 7.10^{10}$ cm (solar radius), we get: $b = 4.10^{16}$ cm$^{\frac{5}{2}}$ sec^{-2}. [1]

Introducing:

$$b \log \frac{\varrho}{\varrho_0} = \sigma, \tag{2.6}$$

where ϱ_0 is an arbitrary constant, and writing equation (2.1) out in the two directions parallel and perpendicular to the rotational axis, we have

$$\frac{\partial \varrho}{\partial z} = \left(-\frac{\gamma M_0}{r^3} + \frac{b}{2 r^{\frac{5}{2}}} \right) r^{\frac{1}{2}} z, \tag{2.7}$$

$$\frac{\partial \sigma}{\partial s} = \left(\omega^2 - \frac{\gamma M_0}{r^3} + \frac{b}{2 r^{\frac{5}{2}}} \right) r^{\frac{1}{2}} s. \tag{2.8}$$

Equation (2.7) can be solved, and gives us

$$\sigma = \frac{2 \gamma M_0}{r^{\frac{1}{2}}} + \frac{b}{2} \log r + \tau(s), \tag{2.9}$$

where τ is independent of z, and has to be solved from the following equation, obtained by substituting equation (2.9) into equation (2.8):

$$\frac{d\tau}{ds} = \omega^2 r^{\frac{1}{2}} s. \tag{2.10}$$

[1] Von Weizsäcker's normalization giving 300° K. for $r = 10^{13}$ cm (mean distance of Venus from the sun) is derived from the observational data about Venus' temperature. The surface temperature of Venus is, however, lower than the equilibrium temperature required here by a factor 1.4 because of the fact that the sun can only heat up that part of the surface which faces the sun.

Since τ is independent of z we have for the case of equilibrium:

$$\omega^2 = f(s)\, r^{-\frac{1}{2}} = \mu^2 \frac{\gamma M_0}{s^3} \left(\frac{s}{r}\right)^{\frac{1}{2}}, \tag{2.11}$$

where μ is still a function of s.

Now, the pressure gradient is everywhere in the system small compared with the gravitational force (even for $r = 10^{15}$, the term with b in equation (2.8) is only about 1/200 of the gravitational term). It seems therefore to be permissible to neglect in equation (2.1) the term $\partial p/\partial s$, and determine ω from the equation:

$$\frac{\partial U}{\partial s} \approx \omega^2 s,$$

or

$$\omega^2 \approx \frac{\gamma M_0}{r^3}, \tag{2.12}$$

which corresponds to Kepler's third law.

We might try to take into account the dissipation of the disc, which will result in a steep density gradient, and therefore a steep pressure gradient. (Von Weizsäcker here introduces an artificial boundary.) One way of introducing this is by putting

$$\mu^2 = 1 - a \cdot s, \qquad (2.13)^1$$

where a may increase with time. As long as a^{-1} is large as compared with the dimensions of the solar system, equation (2.12) will approximately be valid in the equatorial plane of the sun.

Using equations (2.6), (2.9), (2.10), (2.11), and (2.13), we get for the density in the envelope:

$$\varrho = \varrho_0 \left(\frac{r}{r_0}\right)^{\frac{1}{2}} \cdot e^{\varkappa\left(\frac{1}{\sqrt{r}} - \frac{1}{\sqrt{s}}\right) - \varkappa a \sqrt{s}}, \tag{2.14}$$

where $\varkappa$ is given by

$$\varkappa = \frac{2\gamma M_0}{b} = 10^{10} \text{ cm}^{\frac{1}{2}}. \tag{2.15}$$

We see from equation (2.14) that the density falls off rapidly in directions perpendicular to the equatorial plane. If we take

[1] Any μ^2, decreasing with increasing s, will give a slowly decreasing density in the equatorial plane. Equation (2.13) is one of the simplest ways of introducing such a decreasing μ^2.

for the height of the disc the distance over which the density is decreased by a factor 2, we get for this height h:

$$\frac{h}{r} = 2\sqrt{\log 2} \cdot \left(\frac{r}{\varkappa^2}\right)^{\frac{1}{4}} \sim \frac{1}{30}. \qquad (2.16)^1$$

The density in the equatorial plane decreases because of the term with $\varkappa a \sqrt{s}$ in the exponential. Since $\varkappa^2$ is very large compared with the dimensions of the solar system, it is possible to find values of a such that $\varkappa a \sqrt{s}$ is large as compared with one, and still a^{-1} large as compared with the dimensions of the solar system. In this way, we should have an appreciable decrease in density in the equatorial plane, thus getting for the shape of our envelope a lens shape.

The density in the equatorial plane can be written in the form:

$$\varrho = \varrho_m \left(\frac{s}{s_m}\right)^{\frac{1}{2}} e^{1 - \left(\frac{s}{s_m}\right)^{\frac{1}{2}}} \tag{2.17}$$

where ϱ_m is the maximum density in the disc, and s_m the distance from the sun where that maximum density is attained. We find s_m from:

$$s_m = (\varkappa a)^{-2}. \tag{2.18}$$

The advantage of the density function given by equation (2.14) over the one given by von Weizsäcker lies in the fact that it is now no longer necessary to introduce an artificial boundary as was done by von Weizsäcker.

In using equation (2.17), we shall often assume:

$$\varrho_m = 2.10^{16} \text{ atoms per cm}^3; \; s_m = 1{,}6.10^{12} \text{ cm}, \tag{2.19}$$

corresponding to a total mass of the system of about one tenth of the solar mass. The value of s_m is taken so that we can expect a maximum planetary mass in the approximate neighbourhood of Jupiter (cf. Chapter V, Section B).

B. Degree of ionization. There are two possible causes for ionization, viz., the solar radiation or the collisions between the atoms. In order to get an idea about the degree of ionization due

[1] Strictly speaking h/r depends on r but only as $r^{1/4}$. The value given in equation (2.16) is an average value for the disc.

to the solar radiation, we may suppose for a moment that we have to deal with a spherical gaseous envelope with a density of about 10^{16} hydrogen atoms per cm^3. This certainly will give us an upper limit since there is a possibility of the loss of energy by oblique emission from the disc which possibility is not present in the case of a sphere.

Using STRÖMGREN's equation (20):

$$^{10}\log S_0 = -0.44 - 4.51\,\theta + \frac{1}{2}\,^{10}\log T + \frac{2}{3}\,^{10}\log R - \frac{2}{3}\,^{10}\log N, \quad (2.20)$$

where S_0: radius of the sphere containing the H II region (i. e., the region where the hydrogen is ionized), in parsecs (1 pc $= 3.10^{18}$ cm);
R: radius of the central star, in solar radii;
T: temperature of the central star;
θ: $\frac{5040°}{T}$;
N: number of hydrogen atoms per cm^3.

Using $T = 6000°$, $N = 10^{16}\ cm^{-3}$, we get from equation (2.20):

$$S_0 = 3.10^5 \text{ cm},$$

which is even far less than the solar radius. This means, of course, that we may safely assume all the hydrogen in the disc to be neutral. Since the ionization potentials of oxygen and nitrogen are larger than that of hydrogen they will also be neutral.

The next element is carbon. We then have the equation:

$$\left.\begin{aligned} ^{10}\log S_0 = -6.17 - \frac{1}{3}\,^{10}\log \alpha - \frac{1}{3}\,\theta\chi + \frac{1}{2}\,^{10}\log T + \\ + \frac{2}{3}\,^{10}\log R - \frac{2}{3}\,^{10}\log N, \end{aligned}\right\} \quad (2.21)$$

where α is the absorption coefficient at the absorption edge, and χ the ionizational potential. Using $\alpha_C = \alpha_H\,(\chi_H/\chi_C)^3 = 10^{-17}\ cm^2$ (cf. (21)), $\chi = 11{,}22$ ev, we get:

$$S_0 = 10^8 \text{ cm}.$$

We shall finally investigate Mg, Na, K. Their abundance and ionization potentials decrease in this order. Using again equation

(2.21), we get the following table, where for the absorption coefficients of Na and K we use the values given by RUDKJÖBING (21) and LAWRENCE and EDLEFSEN (22), and for Mg:

$$\alpha_{Mg} = \alpha_{Na}\,(\chi_{Na}/\chi_{Mg})^3:$$

Table 2. I.

	Mg	Na	K
Abundance relative to hydrogen	3.10^{-5}	3.10^{-6}	10^{-6}
Absolute abundance	3.10^{11}	3.10^{10}	10^{10}
Ionization potential in electrovolts	7.61	5,12	4,32
α in cm^2	5.10^{-20}	$1{,}6.10^{-19}$	3.10^{-20}
S_0 in cm	7.10^{10}	3.10^{12}	7.10^{12}

We see that the only element which might be ionized would be potassium. We have not, however, taken into account that the effect of recombination processes leading to excited states, followed practically always by cascading to the ground state, will decrease the degree of ionization as pointed out by STRÖMGREN (20). Furthermore, the fact that the radiation emitted after the recombination can leave the disc obliquely also diminishes the degree of ionization.

One might be afraid that the radiation density in the ultraviolet might be higher than corresponding to a black body radiation of 6000°. Recent V-2 rocket experiments (23) show, however, that the radiation density in the ultraviolet follows a black body radiation of 3800° more closely than one of 6000°.[1] This factor also shows that we have overestimated the degree of ionization. Using equation (2.21) with T = 3800°, we get for K for instance:

$$S_0 = 10^{12} \text{ cm},$$

which is far less than the mean distance of Mercury from the sun.

Altogether, it seems safe to conclude that the ionization due to the solar radiation is certainly absent in the region of the major planets and almost certain also in the region of the inner planets.

The next step is to investigate the degree of ionization due to collisions between the atoms, i. e., the ionization equilibrium of

[1] This may no longer be true in the far ultraviolet.

the different elements at temperatures ranging from 700° K to 75° K. We use the normal Saha equation:

$$\frac{n_{A^+} n_{el}}{n_A} = q \left(\frac{2\pi mkT}{h^2}\right)^{\frac{3}{2}} e^{-\chi/kT}, \tag{2.22}$$

where q is a weight factor, m the electron mass, A^+ the ion, and A the neutral atom.

Even for potassium (low density, low ionization potential), at 700° K (highest temperature), only one atom in 10^{10} is ionized. Hence, we can safely conclude that this source of ionization can also be neglected.

Since the solar radiation is unable to ionize even potassium, we may safely assume that the highly diluted radiation from other stars is also unable to produce any appreciable amouni of ionization except, perhaps, in a very thin boundary layer.

C. Optical depth; temperature of the disc. If the intensity of the radiation passing through matter is decreased by a factor $e^{-\tau}$, τ is called the optical depth of this matter. It is difficult to estimate accurately the optical depth of the disc since we ought to take into account the fact that the scattered radiation can leave the disc obliquely so that the radiation has not to pass all the mass before leaving the system.

We may, perhaps, obtain an estimate by smoothing out all matter in the disc over a sphere around the sun with the same linear dimensions as the disc. We obtain an upper and lower limit for this optical depth by considering two cases, viz. either a density varying according to equation (2.17), or a constant density.

The selective absorption starts at 4.3 eV (ionization potential of K) and the maximum intensity of the solar radiation occurs for 2.6 eV. Therefore, we may treat the scattering as Rayleigh scattering on H atoms.

The total optical depth τ is given by:

$$\tau = \int \delta \varrho(r)\, dr, \tag{2.23}$$

where δ is the cross section for Rayleigh scattering ($\delta = 10^{-27}\,\text{cm}^2$), and $\varrho(r)$ is the number of hydrogen atoms per cm³.

The lower limit is obtained by putting $\varrho(r)$ as constant. This density will be about $3.10^{10}\,\text{cm}^{-3}$ for a total mass in the disc of about 0.1 M_0. Then we get:

$$\tau = \delta l \varrho \simeq 0.03,$$

where l is the total path ($l \sim 10^{15}$ cm).

For the upper limit we use equation (2.17) for the density, with $\varrho_m \sim 10^{14}$, corresponding to a smoothing out of the total mass over the sphere. We then get:

$$\tau \sim 2.$$

The actual τ, giving us an estimate of the total scattering of light in the disc, will probably be somewhat smaller than unity, which means that the disc is rather transparent and that we may assume that the energy which a gas volume receives from the sun will be proportional to the inverse square of the distance from the sun.

We can then calculate the temperature in the way already indicated in Section A. The sun is considered to be the only source of energy. Equilibrium reigns if every gas volume in the disc emits as much energy as it absorbs. If temperature equilibrium should exist, the total energy emitted by a gas volume would be proportional to T^4 (Stefan-Boltzmann's law), which should still be valid for a H and He atmosphere because of the principle of detailed balancing. Since the energy received from the sun will be proportional to r^{-2}, we have:

$$T = a \cdot r^{-\frac{1}{2}}. \tag{2.4}$$

Normalizing T to 6000° for $r = 7.10^{10}$ cm (solar radius), we get the following table for the temperatures of the cloud at the present position of the planets:

Table 2. II.

Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
650° K	480° K	400° K	330° K	170° K	130° K	90° K	75° K

These temperatures may be lower limits in the neighbourhood of the inner planets (ionization of potassium giving rise to high energy electrons), while the temperatures in the regions of the outer planets may be regarded as upper limits since there will be a decrease in the intensity of solar radiation due to the Rayleigh scattering. This might perhaps give rise to a factor two, Table 2. II giving too high values[1].

D. Radiative conditions: seperation of elements. If we could completely neglect absorption in the disc, the radiation would be a diluted black body radiation, in as far as we may treat the solar radiation as a black body radiation. This means the energy density corresponding to a certain frequency (or energy) is given by the well-known Planck formula multiplied by a factor g, the so-called dilution factor:

$$\varrho(\nu) = g \cdot \frac{8\pi h \nu^3}{c^3} \cdot \left(e^{\frac{h\nu}{kT}} - 1\right)^{-1}.$$

The dilution factor g is given by

$$g = \left(\frac{r_0}{r}\right)^2, \tag{2.24}$$

where r_0 is the solar radius and r the distance from the sun.

However, there will be an appreciable absorption in the ultra-violet region ($h\nu > 4.32$ eV). For those wavelengths the dilution factor may well be as small as 10^{-12}—10^{-16}. In the rest of the spectrum, the dilution factor will probably be given by equation (2.24), perhaps with an additional factor of the order $^1/_2$ corresponding to the loss of scattered light (see Section C).

Since the disc is chiefly made up of hydrogen, and since Baade and Pauli (24) have shown that for hydrogen, at the surface of the sun, the radiation pressure is negligible as compared with the gravitational force, we may safely neglect the radiation pressure, the more so since the radiation pressure will presumably decrease more rapidly (due to the absorption) than the gravitational force. If there were no absorption both would decrease as the inverse square of the distance from the sun.

In the next chapter we shall see that all particles are part of

[1] Dr. L. Spitzer has kindly pointed out to me that the opacity of the disc might be larger than calculated in the beginning of this section, due to the absorption and scattering by small solid particles.

the turbulent motion in the disc so that it is difficult to imagine a process separating the different elements. The gravitational separation discussed by Eddington (25), e. g., will not take place since the centrifugal potential will balance the gravitational potential (cf., e. g., equation (2.12)). Other effects such as thermal diffusion, are very small and, as remarked before, will probably be annihilated by turbulence. Even if this should not be the case, it can be shown that this should only slightly affect the ratio of the heavier elements to hydrogen, and since anyhow hydrogen is the main element and the ratios in question uncertain, it seems that we may neglect all separation effects.

E. Molecular densities. As the last feature in the disc, we want to give a list of approximate densities of various compounds in the disc. Of course these densities vary from point to point, due to the different pressure and temperature, but in order to get a picture, we may take a density of the hydrogen of 10^{16} at cm^{-3} and a temperature of a few hundred degrees Kelvin.

We are far removed from an equilibrium situation, since the temperature of the radiation is different from the temperature in the disc and the radiation is diluted. It seems therefore dangerous to use the (quasi) equilibrium formulae of either Swings and Rosenfeld (26) or Rosseland (27). We have instead to look into the different possible processes, as was done for the interstellar space by Kramers and the present author (28)[1].

As an example we may discuss the case of CH and use the same considerations as in BAN 371. The numerical constants are, however, different. We now have: $T_g \sim 400°$, $T_{\rm rad} \sim 6000°$, g as given in Section D. (We shall use the same notation as in BAN 371 and refer to that paper for this notation).

The first processes which are of interest are the radiation captures (processes α and η). The number of these processes is given by

$$N_\alpha = Q_{\rm rad}\, \varrho_{C^+}\, \varrho_H, \quad N_\eta = Q'_{\rm rad}\, \varrho_C\, \varrho_H, \tag{2.25}$$

where $Q_{\rm rad}$ is given by

$$Q_{\rm rad} = 4\pi f \int_0^\infty A(r)\, r^2\, e^{-\frac{U(r)}{kT}} \left[F\left(\sqrt{-\frac{U'(r)}{kT}}\right) - F\left(\sqrt{-\frac{U(r)}{kT}}\right)\right] dr, \tag{2.26}$$

[1] We quote this paper in the following as BAN 371.

where

$$F(x) = \frac{4}{\sqrt{\pi}} \int_0^x e^{-x^2} x^2 \, dx. \tag{2.27}$$

$U(r)$ and $U'(r)$ are the potential energy curves of the molecule in the two electronic states between which the radiative transition can take place ($U(r)$ is an excited state and $U'(r)$ the ground state). The transition probability at a certain distance r is given by $A(r)$ and f is the probability that the upper state is realized when the two atoms meet.

In the case in which we are interested, the temperatures are so low that we can replace $F(x)$ by

$$F(x) = 1 - \frac{2}{\sqrt{\pi}} x \cdot e^{-x^2}, \tag{2.28}$$

and since $U'(r)/kT \ll U(r)/kT < 0$, we can write with fair accuracy instead of equation (2.26):

$$Q_{\text{rad}} = \frac{4\pi f}{\sqrt{kT}} \int_0^\infty A(r) \, r^2 \sqrt{-U(r)} \, dr. \tag{2.29}$$

We see that for low temperatures Q_{rad} is inversely proportional to the square root of the temperature since the integral is independent of T.

In the case of CH, we get from equation (2.29) by numerical integration for $T = 400°$:

$$Q_{\text{rad}} \simeq 2.10^{-17} \text{ cm}^3 \text{ sec}^{-1}. \tag{2.30}$$

For CN, numerical integration gives us:

$$Q_{\text{rad}} \simeq 10^{-17} \text{ cm}^3 \text{ sec}^{-1}. \tag{2.31}$$

We have assumed that three body collisions can be neglected as a means for the formation of molecules. For a density of 10^{16} hydrogen atoms per cm^3, we get for the Q corresponding to that process:

$$Q \sim 10^{-18} \text{ cm}^3 \text{ sec}^{-1}. \tag{2.32}$$

For the rate of formation of those molecules which cannot

be formed through a radiation capture accompanied by an electronic transition, we can use equation (2.32).

Larger molecules will be assumed to be formed by radiation capture, and we shall use for the capture cross sections (cf. BAN 371):

$$Q_{3\text{-}at} \; 10^{-16} \text{ cm}^3 \text{ sec}^{-1}; \quad Q_{n\text{-}at} = 10^{-19+n} \text{ cm}^3 \text{ sec}^{-1}. \tag{2.33}$$

For the processes involved in the CH equilibrium, we get the following table (we refer to BAN 371 for the meaning of the various processes):

Table 2. III.

$N_\alpha = 4.10^{-17} \varrho_{C^+} \varrho_H$;	$N_\beta = 2.10^{-12} \varrho_{el} \varrho_{CH^+}$;	N_γ = negligible
N_δ = negligible ;	N_ε = negligible ;	$N_\zeta = 2.10^{-13} \varrho_{el} \varrho_{CH^+}$
$N_\eta = 2.10^{-17} \varrho_C \varrho_H$;	$N_\vartheta = 10^{-16} \varrho_H \varrho_{CH}$;	$N_{\vartheta'} = 10^{-16} \varrho_H \varrho_{CH^+}$

(We have taken here $T = 100°$ K.)

Since $\varrho_{el} \sim \varrho_{C^+} \sim 0$, we see that the only processes of any importance are η and ϑ (i. e., radiation captures leading to CH, resp. CH_2), for the determination of ϱ_{CH}. The concentration of CH^+ will be negligible.

By equalizing N_η and N_ϑ we finally get:

$$\varrho_{CH} \sim 2.10^{14} \text{ cm}^{-3}.$$

For a few other compounds we get the following densities, using the above values for the formation cross sections. We want to stress that all values in Table 2. IV are very uncertain and may well be higher or lower by a few powers of ten.

Table 2. IV.

H_2 : 10^{15} cm^{-3}	CH : 2.10^{14} cm^{-3}	CH_4 : 2.10^{13} cm^{-3}	C_4H_{10} : 2.10^{10}
H_2O : 10^{12}	CN : 10^{11}	NH_3 : 10^{13}	O_2 : 10^{10}
HCN : 10^{10}	CO_2 : 10^7	C_2 : 10^{10}	SiC : 10^8
BaO : 10^5	SO_2 : 2.10^4	CO : 10^{10}	NO : 10^{10}

Chapter III.

Hydrodynamical Properties of a Gaseous Disc.

In this chapter, we shall be interested in the evolution of a gaseous disc in the centre of which a large mass is concentrated. We saw in Chapter II, Section A, that the angular velocities in the disc follow Kepler's third law closely. We shall assume that we may use equation (2.12) for the velocities in the disc.

We shall treat the problem as a two dimensional problem, i. e., we shall neglect all effects in directions perpendicular to the plane of the disc. For the height of the disc we shall assume:

$$h = a\,r, \quad a \sim 1/15 \tag{3.1}$$

in accordance with equation (2.16).

The density in the disc may be given by equation (2.17). We shall here use ϱ measured in g cm^{-3}.

In the disc we have a velocity gradient and an energy gradient. The energy content per unit mass is given by:

$$\varepsilon = -\frac{\gamma\, M_0}{2\, r}, \tag{3.2}$$

giving the energy of matter, moving in a Keplerian orbit round a mass M_0 at a distance r. The kinetic energy $\left(\frac{3}{2}\frac{\varrho}{m}kT\right)$ may be neglected with respect to ε, given by equation (3.2).

In Chapter I we saw that due to the velocity gradient in the disc viscous stresses will be set up which together with the escape of matter at the boundaries in the course of time may bring about a profound transformation of the disc. This transformation of the disc is accompanied by three phenomena, viz.

a loss of mechanical energy, a flow of matter from the disc, partly to the sun in the centre and partly to interstellar space, and finally a transfer of angular momentum in outward direction. In this chapter we shall try to estimate the rate at which the various processes take place.

A. Dissipation of energy. We can use here the formula given for instance by Lamb (29) for the dissipation of mechanical energy due to viscous forces. We have the equation:

$$\delta E = \int \eta\, s^2 \left(\frac{d\,\omega}{ds}\right)^2 dx\, dy\, dz,$$

where η is the viscosity coefficient and where we have assumed that the velocity is everywhere in the plane of the disc and perpendicular to the radius vector. The angular velocity will still depend on the distance from the sun in the way given by equation (2.12).

If we now consider a ring of height h, radius s, and thickness ds, we see that the total loss of energy per sec in that ring is given by:

$$\delta E = 2\,\pi\, h s^3\, \eta \left(\frac{d\,\omega}{ds}\right)^2 ds, \tag{3.3}$$

and the total loss of energy in the disc is given by:

$$\frac{d\,E}{dt} = \int_{r_0}^{s_0} \delta\, E = \frac{9}{2}\,\pi a \eta \gamma\, M_0 \log\frac{s_0}{r_0}, \tag{3.4}$$

where r_0 is the solar radius, s_0 the radius of the disc, and where we have supposed η to be constant throughout the disc.

In the case of laminar motion, η is the normal viscosity coefficient, but in the case where the motion is turbulent, we can still use the above equations. The quantity η is then, however, defined by the equation:

$$\eta \cong \frac{1}{3}\,\varrho\, v\, \lambda, \tag{3.5}$$

where λ is the mean free path or the so-called "mixing length".

B. Lifetime of the disc. We see that we have a steady loss of mechanical energy in the disc. The energy for this dissipation process is provided by matter falling towards the centre and so gaining gravitational energy.

We can estimate the total amount of energy available by assuming that a fraction β of each volume element in the disc falls onto the central body and that the rest of the mass disappears into space. In section D we shall see that β is given by:

$$\beta = r_0/s. \tag{3.6}$$

For the total energy available, we now get, using equations (3.2) and (3.6):

$$E_0 = -\int \varrho \frac{\gamma M_0}{2s} 2\pi shds + \int \varrho\beta(s) \frac{\gamma M_0}{r_0} 2\pi shds$$

$$\simeq \int_{r_0}^{s_0} \varrho \frac{\gamma M_0}{2s} 2\pi shds = \frac{\gamma M_0 M}{60 s_m},$$

where M is the total mass in the disc and s_m the distance at which the maximum density in the disc occurs.

The lifetime of the disc, τ, will now be determined by dividing E_0 by dE/dt of equation (3.4), and in this way we get:

$$\tau^{-1} = 90\pi a \frac{\varrho v \lambda s_m}{M} \log \frac{s_0}{r_0}. \tag{3.7}$$

The derivation of equation (3.6) is very tentative. Thus β might easily be larger, giving rise to an estimate of τ larger than that given by equation (3.7) by, say, a factor 10 or 100.

C. Transfer of angular momentum. Due to the velocity gradient there will be a transport of momentum through any area perpendicular to the radius vector. This transport of momentum will be accompanied by a transport of angular momentum and energy. Those three quantities are given by:

$$\delta P = s\eta \frac{d\omega}{ds}, \quad \delta\theta = -s^2\eta \frac{d\omega}{ds}, \quad \delta E = -s^2\omega\eta \frac{d\omega}{ds}. \tag{3.8}$$

The total transport of angular momentum per sec through a cylinder of height h and radius s will be given by:

$$\frac{d\theta}{dt} = -2\pi hs^3\eta \frac{d\omega}{ds} = 3\pi a\eta s^{\frac{3}{2}} \sqrt{\gamma M_0}. \tag{3.9}$$

We are especially interested in the angular momentum transferred from the central body during the lifetime of the disc. If in equation (3.9) we put s equal to r_0, we get the transfer of angular momentum per sec in a situation where the velocities are perpendicular to the radius vector and given by equation (2.12). As soon as the central body is slowed down, the velocity pattern in the neighbourhood of the central body will become changed and it is difficult to predict exactly what will happen.

In order to get an idea of the magnitude of the transfer, we might compare $d\theta/dt$ for $s = r_0$ with θ_0/τ, where θ_0 is the angular momentum of the central body in the case where its angular velocity corresponds to Kepler's third law:

$$\frac{\tau \frac{d\theta_0}{dt}}{\theta_0} \sim \frac{0.0003\ M}{M_0}, \tag{3.10}$$

where we have used $s_0/r_0 \sim 10^4$, and $\theta_0 = \frac{\omega_0 M_0 r_0^2}{5} = \frac{1}{5} M_0 \sqrt{r_0 \gamma M_0}$.

Although the above-mentioned phenomenon of transfer of angular momentum will slow down the solar rotation, it is clear at first sight from (3.10) that this can hardly account for the present slow rotation (present $\theta \sim 0.005\ \theta_0$).

The present slow rotation of the sun has perhaps to be explained by an investigation of the earlier steps in the process leading to the formation of the sun. This investigation, however, falls outside the scope of the present paper.

D. Estimation of the increase of the solar mass during the dissipation process. Before discussing the possibility of a regular system of vortices, we wish to look into the question of the dissipation of the disc. We shall try to estimate the quantity $\beta(s)$, i. e., the fraction of the mass which will fall onto the sun. In order to calculate this rigorously one should have to solve the hydrodynamical equations, preferably with the terms involving the viscosity. Also, one should have to consider a velocity component different from zero in the direction of the radius vector. These calculations should give us at the same time the transfer of angular momentum and, perhaps, the formation of a regular system of vortices.

However, we can try to get a first estimate of the magnitude of β in the following way.

If we consider a cylindrical ring with height h, between the radii s and $s + ds$, this ring will lose per sec angular momentum at the rate:

$$\delta\theta = \frac{d}{ds}\left(2\pi hs^3 \eta \frac{d\omega}{ds}\right) ds = \frac{9}{2}\pi a\eta \sqrt{\gamma M_0 s}\, ds.$$

The total angular momentum of the ring is given by:

$$\theta = 2\pi hs^3 \varrho\omega\, ds = 2\pi a\varrho \sqrt{\gamma M_0 s^5}\, ds.$$

If a kind of over all equilibrium in the disc should reign, this loss of angular momentum would correspond to a loss of mass given by:

$$\frac{\delta m}{m} = \frac{\delta\theta}{\theta},$$

where m is the mass of the ring, and given by:

$$m = 2\pi hs\varrho\, ds.$$

The energy loss per sec is given by equation (3.3):

$$\delta E_1 = 2\pi hs^3 \eta \left(\frac{d\omega}{ds}\right)^2 ds = \frac{9}{2}\pi a\eta \frac{\gamma M_0}{s} ds, \qquad (3.3)$$

and if a fraction β of the original mass of the ring falls onto the sun, this loss of energy is compensated by the gain of energy by this matter, again assuming a quasi equilibrium situation throughout the disc:

$$\delta E_2 = \beta\, \delta m \frac{\gamma M_0}{r_0} = \beta \frac{\delta\theta}{\theta} m \frac{\gamma M_0}{r_0}. \qquad (3.11)$$

Putting $\delta E_1 = \delta E_2$ we can determine β, and in this way we get:

$$\beta(s) = r_0/s. \qquad (3.6)$$

The total mass which will fall onto the sun is now given by:

$$\beta M = \int_{r_0}^{s_0} 2\pi hs\varrho\beta(s)\, ds = \frac{r_0}{5 s_m} M. \qquad (3.12)$$

The reasoning in this section is only tentative. It would be desirable to complement it by a direct estimate of the amount of matter which escapes from the boundaries of the disc.

E. Possibility of regular systems of vortices. We saw that VON WEIZSÄCKER (10) introduced a regular system of vortices in his theory and that he was able in that way to explain the Titius-Bode law. In this section we should like to look very briefly into this question.

Of course one cannot use von Weizsäcker's treatment since the mean free path in the disc is far too small to allow for unperturbed Keplerian orbits. However, one might hope to be able to deduce from the hydrodynamical equations a similar set of rings of vortices.

The first important point is that, as we already saw in the previous chapter, gravitational forces are by far the most important. They are not only more important than the pressure gradient, but also than the viscous forces. (Reynold's number $\left(= \frac{\varrho v l}{\eta}\right.$, where l is a length of the order of the dimensions of the system), which measures the ratio of the inertial forces to the viscous forces is very large in our disc). This might give rise to systems like the one pictured by von Weizsäcker (cf. p. 15).

The system which we consider is different from the common hydrodynamical systems because of the absence of a wall. But the fact that the mean free path increases with decreasing density may have the same effect as a wall. And also it might be that during the development of the gaseous system which will become a galaxy the other turbulence elements may have acted somewhat restrainingly on the whirl which would develop into the sun and the solar system. We are thus tempted to compare this with normal hydrodynamical systems although we are aware of the danger attached to this procedure. There are, however, some signs that this might not be as far from the actual truth as one might fear.

The idea is to assume for a moment that due to the preponderance of the gravitational force regular systems of vortices might be set up. Now, we can assume that the distance between two circles separating the various rings of vortices will be given by the mean size of the turbulence elements. In this way we might arrive at an estimate of the size of the turbulent elements in the solar envelope and in the planetary atmospheres from the differences of the observed mean distances of the successive planets (satellites) from the central body since these planets and satellites will probably have been formed on the circles separating the main vortices, as we shall see in the next chapter. In Table 3. I we have collected the data for the sun, Jupiter, Saturn, and Uranus, using only the data of the "regular" satellites (see Chapter VI). In the second row we have taken the observed planets and satellites only and in deriving the values for the last row we have assumed that due to some unknown reason there are gaps, corresponding in the series of the planets, e. g., to the asteroids. Finally, we assumed that the size of the turbulence elements is proportional to the distance from the primary:

$$l = a \cdot r, \tag{3.13}$$

and the values given in Table 3. I. are the mean values of a. If r_n is the mean distance of the n-th body from the centre, $l = r_n - r_{n-1}$, and $2r = r_n + r_{n-1}$.

Table 3. I.

	Sun	Jupiter	Saturn	Uranus
Mean value of a for "regular" satellites	0.56	0.56	0.42	0.36
Mean value of a with assumed gaps.	0.50	0.45	0.33	0.28
The number of gaps is inserted between brackets	(1)	(1)	(2)	(1)

We may compare this with von Kármán's formula (30) for the mean size of a turbulence element. This was first done by Tuominen (31), who shewed that the Titius-Bode law for the planets follows within a factor 2 from the size of vortices given

by equation (3.14). He did not, however, compare the planets with the satellites. We shall follow his argumentation here with a few alterations.

Von Kármán had remarked that for the cases which he investigates the mean size of the turbulence elements is given by:

$$l = k_0 \left| \frac{dv/ds}{d^2v/ds^2} \right|, \tag{3.14}$$

with a constant k_0 (~ 0.4). If the velocity is given by equation (2.12), we get for the size of the turbulence elements:

$$l \sim 0.27\, r. \tag{3.15}$$

If we now look at Table 3. I., we see one striking point, viz., that a is decreasing with decreasing mass of the primary, i. e., with decreasing influence of the gravitational force and that a approaches the value of equation (3.15). This might prove to be an important point in a discussion of the hydrodynamical properties of the disc and the planetary envelopes.

We want to point out a few more points connected with these regular systems of vortices.

The first is that the energy dissipation in such a regular system might be less than in the case of an irregular turbulent situation. In this way, we should get a longer lifetime than that corresponding to an energy loss, calculated under the assumption that we may use equations (3.4) and (3.5) with a λ equal to the dimensions of the vortices. This might amount to as much as a few powers of ten. There are also other indications that the lifetime of the disc might well have been much longer as we shall see in Chapter V. This might then also be an indication that regular systems of vortices have once been established. In order to bring about a regular series for the distances of the planets or satellites it is not necessary that the system remained the same during the whole lifetime of the disc. As was already shewn by von Weizsäcker it is only necessary that these regular systems lasted for about 10 years, which is of the order of magnitude of the period of rotation of the outer parts of the disc. In that period the condensation products become so large that they can no longer be displaced appreciably by turbulence in the disc.

The second point is that we can easily calculate the velocities on the outskirts of the large vortices which will form the systems of vortices. These velocities will be the turbulent velocities, and we may take for those the mean fluctuations of the velocities in a gas kinetic system with a velocity gradient as was done also by PRANDTL (32) in a similar case.

If v is the mean velocity given by equation (2.12), we have for the turbulence velocity u:

$$u = \lambda \left| \frac{dv}{ds} \right|, \tag{3.16}$$

where λ should be the mixing length which is equal to the mean size of the vortices and given by equation (3.14).

We see that u decreases with increasing distance from the central body which means that if the large vortices are rotating themselves in a counter-clockwise direction the motion in these vortices will be clockwise.

Between the rings of large vortices there will be large viscous stresses along the circles separating the main vortices. We may therefore here expect secondary eddies like the "roller bearing" eddies of von Weizsäcker. Those "roller bearings" will again show direct (counter-clockwise) rotation. Since the planets will probably be formed in those "roller bearings", as we shall see in the next chapter, we are here presented with an explanation of their direct rotation. It is a tempting thought to assume that the size of the "roller bearings" will be determined by the fact that the velocities at the outside will be equal to the turbulent velocities given by equation (3.16). This would mean that we should be able to determine in that way the size of the planetary atmospheres since the velocities in these atmospheres are determined by Kepler's third law (cf. Chapter II, Section A). We may remark here that the considerations of this paragraph also remain valid if there should not be a regular system of large vortices.

Finally, we may remark that the size of the large vortices will only on an average be proportional to the distance from the sun, i. e., that equation (3.13) will only approximately be fulfilled. We should therefore expect rather than be disappointed by the fact that the Titius-Bode law or similar laws for the satellite systems do not hold rigorously from planet to planet or from satellite to satellite.

Chapter IV.

The Condensation Process.

Condensation processes in astrophysics can be divided into two different phases. The first phase is the formation of nuclei on which the further condensation can easily take place. The second phase is this subsequent growth of the nuclei. These nuclei will grow in the beginning because impinging atoms or particles will stick to them, but later this growth will be much more rapid because of the possibility of gravitational capture,

A. Formation of nuclei for condensation. We shall use here a model given in an earlier paper (33), in the following quoted as BAN 361.

If we want to investigate the possibilities of condensation, it seems to be a fair approximation to treat the condensed particles as heteropolar crystals. We are interested in the condensation in a gas with density ϱ and kinetic energy corresponding to a temperature T_1, while the radiation density is assumed to be a diluted black body radiation with temperature T_2 and dilution factor g.

The first question to be investigated is the temperature of the condensed particles. We can find this temperature from the energy balance.

In as much as there are only slight deviations from harmonic binding between the atoms in the crystal, the particles will emit and absorb radiation practically as one large harmonic oscillator, and only the fundamental frequency contributes. If ω is the frequency of the oscillator and $kT \ll h\omega$, we have for the emitted and absorbed energy of the particle

$$E_{abs} = \frac{\pi\varepsilon^2}{M}\varrho(\omega), \quad E_{em} = \frac{8\pi^2 h\omega^3\varepsilon^2}{Mc^3}\cdot e^{-\frac{h\omega}{kT}}, \qquad (4.1)$$

where M and ε are the mass and charge of the oscillator, and $\varrho(\omega)$ the radiation density. If the particle, which is assumed to be small compared with the wavelength of light considered, consists of i atoms, we have: $\varepsilon = i\cdot e$ (e is an effective charge), $M = i\cdot m$ (m: mass of one atom), $T = T_i$ (temperature of a particle consisting of i atoms), and $\omega = \omega_0$ (fundamental frequency).

On the other hand, we have energy conveyed to and from the particles by colliding atoms which do not stick to the surface. As was pointed out in BAN 361, these are mainly hydrogen atoms. The energies in question are given by:

$$E_{\text{on}} = c_1\sigma\varrho v i^{\frac{2}{3}} kT_1, \quad E_{\text{off}} = c_2\sigma\varrho v i^{\frac{2}{3}} kT_i, \qquad (4.2)$$

where c_1 and c_2 are numerical constants of the order 1, σ the surface of one atom, v the mean velocity of the colliding atoms, and ϱ their density.

We have now the following equilibrium condition:

$$E_{\text{on}} + E_{\text{abs}} = E_{\text{off}} + E_{\text{em}} \qquad (4.3)$$

or

$$A\varrho T_1 i^{\frac{2}{3}} + Bgi\left(e^{\frac{h\omega_0}{kT_2}} - 1\right)^{-1} = C\varrho T_i i^{\frac{2}{3}} + Die^{-\frac{h\omega_0}{kT_i}}. \qquad (4.4)$$

For given values of ϱ, T_1, T_2, and g, we can determine from equation (4.4) the temperatures of the particles, T_i.

Inserting numerical values, we have (cf. BAN 361):

$$\left.\begin{array}{l} A \sim C \sim 4.10^{-26} \text{ erg degree}^{-1}\text{ cm}^3; \\ B \sim D \sim 2.10^{-13} \text{ erg}; \quad \dfrac{h\omega_0}{k} \sim 1400^\circ. \end{array}\right\} \qquad (4.5)$$

In all cases, the term with A is large as compared with that with B, but according to whether the term with C is small or large as compared with that with D, we have the following two cases:

(α) $C \ll D$, which will be realized in interstellar space, where we have low gas densities and low radiation density.

(β) $C \gg D$, which will be realized in all other cases in astrophysics such as condensation in nova shells, condensation in the corona, or condensation in a gaseous disc such as we consider in the present paper.

In case α, equation (4.3) reduces to

$$E_{\text{on}} = E_{\text{em}},$$

or

$$T_i = \frac{a}{\log(b\,i)}. \tag{4.6}$$

In case β, equation (4.3) reduces to

$$E_{\text{on}} = E_{\text{off}},$$

or

$$T_i = T_1. \tag{4.7}$$

The formation of nuclei can now be calculated in the way first indicated by Becker and Doering (34).

The main feature of the condensation is that in order to get an appreciable precipitation it is necessary that the vapour pressure of large particles is less than the pressure in the gas because in that case there will be more atoms condensing on than evaporating from the particles.

In case β, which is also the normal case in chemistry (where we have $T_1 = T_2$, $g = 1$), the vapour pressure of the particles will decrease with increasing size due to the influence of surface free energy. Finally it reaches the value of the saturated pressure for infinite size at the temperature present. Thus, if this saturated vapour pressure is smaller than the pressure of the gas, we can expect condensation. This is the well-known phenomenon of precipitation (or condensation) in a supersaturated vapour.

In case α, the decrease of vapour pressure with increasing size is due to the decreasing temperature (cf. equation (4.6))[1]. There are two possibilities, viz. that the temperature is already low enough for particles consisting of only a few atoms in which case the rate of precipitation depends only on the rate of formation of molecules of, say, 10 atoms because smaller particles cannot be considered to behave like crystals. If we denote the rate of precipitation by j, we have:

$$j \sim K\varrho^n, \tag{4.8}$$

[1] This case has been extensively discussed in BAN 361. We only give the main results here, and refer the reader to BAN 361 for details.

where n lies between 2 and 10 and depends on the number of atoms for which the capture in the "crystal" is difficult; K is a numerical constant.

This possibility is realized for extremely low densities. In that case, the energy conveyed to the particles will be very small so that their temperature will be low enough to allow for an easy condensation.

For higher densities which are still so low that we are in case α, the temperature of the small particles will be higher than corresponding to a vapour pressure equal to the gas pressure. However, the temperatures of larger particles will be low enough. However, since the temperature of the particles increases with increasing density (cf. equations (4.4) and (4.6)), the critical size, i. e., the size for which the temperature corresponds exactly to a vapour pressure equal to the gas pressure, will increase with increasing density. The rate of precipitation will correspondingly decrease:

$$j \sim K\varrho^a e^{-b}. \tag{4.9}$$

The so-called characteristic density, i. e., the density at which the transition between the two above-mentioned possibilities occurs and which also marks a maximum in j, is much lower than the density marking the transition from α to β.

In case β there will only be appreciable condensation if there is a state of supersaturation, i. e., if the gas pressure is higher than the saturated vapour pressure.

The vapour pressure of a crystal is given by:

$$p_v = \frac{m^{\frac{3}{2}}(kT)^{\frac{5}{2}}}{h^3}\, e^{-\chi/kT}, \tag{4.10}$$

where m and χ are the mass and sublimation heat (in ergs) of an atom of the crystal. If the density of the gas is ϱ atoms per cm^3, its pressure is given by the ideal gas law

$$p_g = \varrho kT. \tag{4.11}$$

The necessary condition for condensation is now

$$p_g \geq p_v \quad \text{or} \quad e^{\chi/kT} \geq \left(\frac{mkT}{h^2}\right)^{\frac{3}{2}} \cdot \frac{1}{\varrho}. \tag{4.12}$$

For a given temperature and density we have a critical sublimation heat χ determined by equation (4.12) with the equal sign. Compounds with a larger sublimation heat will condense, those with a smaller sublimation heat will not condense.

From equation (4.12) with the equal sign we can calculate χ for different values of T and ϱ, and we get the following table. We have given χ in eV and (between brackets) in Cal/mole.

Table 4. I.
Values of the critical sublimation heat.

ϱ	50°	100°	200°	400°	1000°	10000°
10^{10}	0.14	0.29	0.60	1.25	3.23	35
	(3.3)	(6.8)	(14)	(29)	(75)	(810)
10^{12}	0.12	0.25	0.53	1.09	2.84	31
	(2.8)	(5.8)	(12)	(25)	(65)	(720)
10^{14}	0.10	0.21	0.45	0.93	2.44	27
	(2.4)	(4.9)	(10)	(21)	(56)	(630)
10^{16}	0.08	0.17	0.37	0.77	2.04	23
	(1.9)	(4.0)	(8)	(18)	(47)	(540)

We see from Table 4. I, and equation (4.12) that χ depends only slightly on ϱ, but is mainly determined by T.

In the next table we have for comparison collected the sublimation heats (in Cal/mole) for a number of inorganic and organic substances, and also their specific densities, σ.

Table 4. II.

Compound	χ	σ	Compound	χ	σ	Compound	χ	σ
CO	1.9	0.9	HNO_3	8?	2?	Mg	34	1.7
CH_4	2.3	0.5	SO_2	8.5	2	Ba	41	3.5
NO	3.8	1.6	HCN	8.5	1?	Ca	43	1.5
N_2O	5.8	1.6	H_2O	11.3	0.9	BaO	90	5.7
C_2H_6	6?	0.9?	N_2O_4	12.6	2.0	Fe	97	7.9
CO_2	6.3	1.6	NO_2	13	1.5	C	125	3.5
C_4H_{10}	7?	0.9	SO_3	12—16	2.4	Si	large	2.3
NH_3	7.5	0.8	K	21.8	0.9	SiO_2	large	2.3
$(CN)_2$	7.8	1.4?	Na	26	1.0	SiC	large	3.2

Comparing Tables 4. I and 4. II, and remembering that the temperature in the corona or nova shells is at least a few thousand degrees, we may safely conclude that in those cases there will be no condensation. This does not, however, exclude the possibility of the presence of molecules (cf. (35)). We also see that it is necessary to have a temperature which is at most 1000° in order to have an appreciable condensation. This is another difficulty encountered by theories like the one proposed by HOYLE (14).

B. Second and final stages of the condensation. After the first stage, the formation of nuclei for condensation, there are two more stages. The second stage is the normal condensation where the particles grow because impinging molecules stick to them. The final stage is that of the gravitational capture.

We may draw attention here to the fact that as soon as there is a state of supersaturation the nuclei will be formed in sufficient number (34), so that it is not necessary to consider that stage of the condensation process in any more detail.

The second stage closely resembles the process proposed by LINDBLAD (36) for the formation of interstellar smoke particles. For the sake of simplicity we shall assume that the particles are spherical with radius r and specific density ϱ_0.

If the density of the matter impinging on the particle and sticking to it is denoted by ϱ_1, and their mean relative velocity by v_1, we have for the increase of mass per sec:

$$\frac{dm}{dt} = 4\pi r^2 \cdot \frac{v_1}{4} \varrho_1. \tag{4.13}$$

Since

$$m = \frac{4}{3} \pi \varrho_0 r^3, \tag{4.14}$$

we have

$$r(-r_0) = \frac{\varrho_1}{\varrho_0} \frac{v_1}{4} t. \tag{4.15}$$

This is correct as long as gravitational effects can be neglected. If, however, we have reached the last stage, we get a much faster growth[1].

We can introduce a distance R (by Chandrasekhar called the

[1] For an extensive discussion of this stage of the condensation, we may refer to a paper by EAKIN and MCCREA (37).

"gravitational radius") such that the gravitational energy of matter at that distance from the centre of the particle is equal to its kinetic energy:

$$R = \frac{2\gamma m}{v_2^2}, \tag{4.16}$$

where v_2 is the mean velocity of the matter in the system and m again the mass of the growing particle.

The cross section for gravitational capture is now $\pi\delta^2 R^2$, where δ is of the order of magnitude 0.1. For the growth of the particle we have now

$$\frac{dm}{dt} = 4\pi\delta^2 R^2 \frac{v_2}{4}\varrho_2, \tag{4.17}$$

or, using equation (4.16):

$$\frac{dr}{dt} = \beta r^4, \quad \beta = \frac{16\pi^2}{9}\delta^2 \frac{\gamma^2 \varrho_0 \varrho_2}{v_2^3}, \tag{4.18}$$

with the solution:

$$r^3 = \frac{1}{3(\varepsilon - \beta t)},$$

where ε is an integration constant to be determined by $r = r_{\text{crit}}$ for $t = 0$. The quantity r_{crit} is the radius of a particle for which the gravitational cross section $\pi\delta^2 R^2$ equals the geometric cross section πr^2:

$$r_{\text{crit}}^2 = \frac{3 v_2^2}{8\pi\gamma\delta\varrho_0}. \tag{4.20}$$

If there should be no exhaustion of the gas, the lumps would become infinitely large in a finite time, given by:

$$t_\infty = t_{\text{crit}} + \frac{\varepsilon}{\beta} = \frac{4}{3} t_{\text{crit}}, \tag{4.21}$$

where t_{crit} is the time necessary to reach dimensions of the order r_{crit}, and given by (cf. equation (4.15)):

$$t_{\text{crit}} = \frac{4\varrho_0}{\varrho_1 v_1}\cdot r_{crit} = \frac{v_2}{v_1\varrho_1}\left(\frac{6\varrho_0}{\pi\gamma\delta}\right)^{\frac{1}{2}}. \tag{4.22}$$

As we mentioned in the previous chapter, condensation is more likely to take place in the "roller bearings" than in the large vortices, as was first shewn by von Weizsäcker, whose reasoning we follow here in the form presented by Chandrasekhar (10). This preference for the "roller bearings" is due to the fact that the mean free path for larger particles is greater than the size of the "roller bearings". The mean free path is in this case defined as the distance travelled through by the particle before its loss of momentum is of the same order of magnitude as its original momentum. This means that these particles will no longer be carried along by the "roller bearings" even though the large vortices are able to carry them along. Therefore the number of collisions between such particles and gas atoms or smaller condensation products will be enhanced in the "roller bearings".

The mean free path of a particle can be estimated in the following way. If ϱ_1 is the gas density, m the mass of the particle (for m we have equation (4.14)), r its radius and u_s its velocity relative to the medium, we have for the loss of momentum in an interval dt:

$$m du_s = -\pi r^2 \varrho_1 u_s^2 dt. \tag{4.23}$$

Using the definition of the mean free path, λ_p given above, we get for λ_p:

$$\lambda_p = \frac{m_s u_s}{\pi r^2 \varrho_1 u_s^2}\cdot u_s = \frac{4}{3}\frac{\varrho_0}{\varrho_1} r, \tag{4.24}$$

which gives us with $\varrho_0 = 3\text{ g cm}^{-3}$, $\varrho_1 = 10^{-5}\text{ g cm}^{-3}$:

$$\lambda_p = 4.10^9 r. \tag{4.25}$$

As long as λ_p is smaller than the size of a vortex, this vortex will carry the particle along. We see that hence there will be a range of particle sizes such that the large vortices can carry them along, but such that the "roller bearings" can no longer carry them along. Therefore, the probability of finding a condensation product is largest at the "roller bearing" circles in the regular system of vortices—if such a system has ever existed. It has not been proved that only *one* planet is formed on each circle. We may, perhaps, be allowed, as far as that is concerned, to express an optimism similar to von Weizsäcker's (10).

The time necessary to reach such dimensions that even the largest vortex is unable to move the particle essentially is found by combining equations (4.15) and (4.25).

As soon as the last stage, i. e. the stage of the gravitational capture, is attained, the bodies will collect an atmosphere around them. We can estimate the dimensions of these atmospheres in two different ways. Either in the way indicated at the end of the previous chapter, viz. that the velocities on the outskirts of the atmosphere should be equal to the turbulent velocities, or by using for the radius of the atmosphere the "gravitational radius". In formula we have, using equation (3.16) for the turbulent velocities and putting $a = {}^1/_2$ in equation (3.15),

$$R_1 = 16\,s\frac{M}{M_0} \qquad = 7.10^8\,m\bar{s} \qquad (4.26)$$

or

$$R_2 = \frac{2\,\gamma M}{v_2^2} = 2\,s\frac{M}{M_0} = 10^8\,m\bar{s}, \qquad (4.27)$$

where M and m are the planet's mass in grams and in the earth's mass as unit, and $\bar{s}$ its distance from the sun in astronomical units. For v_2 we have used again equation (2.12). We see that both equations, apart from a factor 8, give the same result. In the following chapters we shall use equation (4.26).

Chapter V.

The Planetary System.

In this and the next chapter we shall try to apply the results of the foregoing chapters to the solar system and the satellite systems of the major planets.

A. Densities of the planets. In Chapter II we saw that the temperature in the disc decreased with increasing distance from the sun. If we assume that the planets were formed at essentially those distances from the sun at which they are observed now, each planet corresponds to a certain temperature, as shewn in Table 2. II. According to the previous chapter, however, a given temperature corresponds to a certain critical sublimation heat given by equation (4.12). So we can assign to each planet a sublimation heat telling us which compounds will have taken part in the initial condensation process leading to dimensions of r_{crit} (see previous chapter). In Table 5. I we have given these sublimation heats. We have taken an average density of 10^{12} at · cm^{-3}. [1] Of course, we should for every compound calculate its density in the disc (cf. Chapter II, Section E) and investigate whether its sublimation heat is higher or lower than the critical sublimation heat for that density and the given temperature. So we should find for each temperature which compounds would

Table 5. I.

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
T..........	650°	480°	400°	330°	170°	130°	90°	75°
χ in Cal/mole	42	30	25	20	10	8	5	4

[1] This corresponds to about 0.1 per cent. of the gas condensing, and an average density of 10^{14} at cm^{-3} in the disc.

condense at the given temperature. Fortunately, however, the critical sublimation heat does not depend very strongly on the density, as we saw in Chapter IV, so that we can calculate the sublimation heats for an average density and we need not worry about the variation of density for various substances.

If we compare this table with the sublimation heats given in Table 4. II, we see that while in the outer regions compounds like HCN, H_2O, NH_3 can condense, in the regions of the inner planets only metals and other inorganic compounds can condense. This has two consequences. The inorganic compounds are less frequent and are heavier. Therefore, the first stage of the condensation will end in heavy bodies in the inner regions and lighter bodies in the regions of the outer planets. Since the dimensions of the inner planets are hardly larger than r_{crit}, we can expect higher densities for the inner planets than for the outer planets. The initial condensation stage brings this difference about, and the gravitational capture, practically only acting in the case of the outer planets, accentuates this difference. The dimensions of the inner planets are only just larger than r_{crit}, which is given by Equation (4.20), and gives us:

$$r_{crit} = \sqrt{\frac{3\,v_2^2}{8\,\pi\gamma\delta\varrho_0}} \sim 10^9\,\text{cm}, \qquad (5.1)$$

with $\delta \sim 0.1$, $\varrho_0 \sim 3$ g cm^{-3}, $v_2 \sim 10^6$ cm sec^{-1} (corresponding to Jupiter's distance from the sun). We see from this equation that, indeed, gravitational capture can only have played a minor part in the growth of the inner planets.

It seems even possible to account for the smaller differences in densities of the various planets, as was also shewn by BROWN (1). We shall not, however, enter into this question here.

B. Masses of the planets. The second consequence of the condensation picture is that there will be more condensation nuclei per cm^3 in the regions of the outer planets than in the inner regions because there are more compounds which can condense. This means that a greater fraction of the gas will take part in the condensation in the outer regions. This again is accentuated by the fact that gravitational capture has played a part in the building up of the outer planets. If we postpone for a moment the discussion of the problem why this gravitational capture has not been active in the case of the inner planets, we can try to estimate the masses of the planets under the assumption that a larger fraction of the matter took part in the building up of the major planets than in the case of the inner planets.

For the mass of the n-th planet we may write:

$$M_n = \Delta_n \varrho(r_n) A_n h_n, \qquad (5.2)$$

where Δ_n: fraction of the gas taking part in the condensation process;
r_n : mean distance of the planet from the sun;
$\varrho(r)$: gas density in the disc, given by equation (2.17);
A_n : area in the disc, involved in the building up of the planet; we may take $A_n = c \cdot r_n^2$ (c will be of the order of magnitude one);
h_n : height of the disc at a distance r_n; h_n is given by equation (3.1).

Equation (5.2) can now be written in the following form:

$$M_n = A\,\Delta_n \varrho(r_n)\, r_n^3 = B\,\Delta_n\, r_n^{\frac{7}{2}}\, e^{-(r_n/s_m)^{\frac{1}{2}}}, \qquad (5.3)$$

where A and B are constants. We now, for the sake of simplicity, take Δ to be constant throughout the regions of the inner planets, and also constant throughout the regions of the outer planets. For the ratio of Δ in the two regions we shall take 100, which takes into account the fact that gravitational capture has played a part in the formation of the outer planets, and the fact that lighter elements are more abundant than the heavier elements. We then get Table 5. II.

We see that the general agreement is quite good, especially in view of the fact that we have simplified the problem very much. We could probably get an even better agreement by a variation of s_m and the ratio of the Δ_n's in the two parts of the planetary system, but it does not seem worth while to do that. The only point is that the condensation picture presents us with a mass distribution in the solar system which agrees as well with the observational data as we can expect from necessarily rough considerations.

Table 5. II.

	$\frac{\Delta_n}{\Delta_{earth}}$	$\frac{\varrho}{\varrho_{earth}}$	$\frac{r_n}{r_{earth}}$	$\frac{M_{calc}}{M_{earth}}$	$\frac{M_{obs}}{M_{earth}}$
Mercury..	1	1.9	0.4	0.11	0.05
Venus....	1	1.4	0.7	0.5	0.8
Earth....	1	1	1	1	1
Mars.....	1	0.6	1.5	2.1	0.1
Jupiter...	100	0.045	5	640	318
Saturn ..	100	0.005	10	450	95
Uranus...	100	$1{,}4.10^{-4}$	19	100	15
Neptune..	100	6.10^{-6}	30	16	17
(Pluto....	100	6.10^{-7}	40	4	0.9)

The only serious disagreement seems to be a too small mass of Mars and the absence of a planet in the neighbourhood of the asteroids. We shall return to this point at the end of this paper.

There are, however, a few points which we still have to examine before we can accept the above considerations as giving us really an estimate of the planetary masses. These are the following:

(a) How great has the density to be in the disc in order to provide us with sufficient mass for the planets?

(b) What is the lifetime of the disc, and how does it compare with t_{crit}?

(c) Why has the gravitational capture not played a role in the building-up process of the inner planets?

(a) If we assume that a fraction 10^{-4} has taken part in the building-up process of the inner planets and a fraction 10^{-2} in the building-up process of the outer planets, we have the following conditions, if there has been enough mass available to build up the inner, respectively the outer planets:

$$10^{-4}\int_{r_0}^{s_{\text{Mars}}} \varrho\, 2\pi s \cdot h ds \geq 10^{28}\text{ g.},$$

and

$$10^{-2}\int_{s_{\text{Jup}}}^{\infty} \varrho\, 2\pi s \cdot h ds \leq 2.10^{30}\text{ g.}$$

Using equations (2.17) and (3.1), we get for the maximum density ϱ_m the condition:

$$\varrho_m \geq 7.10^{15}\text{ at cm}^{-3}\text{ and }\varrho_m \geq 3.10^{16}\text{ at cm}^{-3}.$$

This tallies very well with our assumption of $\varrho_m = 2.10^{16}$ at·cm^{-3}, corresponding to a total mass of the gaseous disc of about one tenth of the solar mass at the stage where equation (2.17) is valid.

(b) From equations (3.7) and (4.22), we can calculate the lifetime of the disc and t_{crit}.

If we should assume a laminar motion in the disc, equation (3.7) would give us:

$$\tau = 3.10^{14}\text{ years},$$

which is obviously by far too large.

Even if we take into account the uncertainties involved in the derivation of equation (3.7), it will stay too large. We should in that case expect still to see the remnants of the disc at the present time.

However, if we assume turbulence, equation (3.7) presents us with a lifetime given by:

$$\tau \approx \frac{10^{11}}{\varrho v \lambda}\text{yrs} \sim 10^2\text{ yrs}, \tag{5.4}$$

where we used $\eta = \frac{1}{3}\varrho v \lambda$, $\varrho \sim 10^{-9}$ g cm^{-3}, $v \sim 10^6$ cm sec^{-1}, $\lambda \sim 10^{12}$ cm.

On the other hand, t_{crit} as given by equation (4.22) gives us:

$$t_{crit} = \frac{v_2}{v_1 \varrho_1}\left(\frac{6\varrho_0}{\pi\gamma\delta}\right)^{\frac{1}{2}} \sim \frac{10^{-3}}{\varrho_1 \Delta}\text{ yrs} \sim 10^8\text{ yrs}, \tag{5.5}$$

with $\Delta \sim 10^{-2}$, $\varrho \sim 10^{-9}$ g cm^{-3}.

Before looking into this question more carefully, and taking into account the change of t_{crit} with distance from the sun, we see immediately that t_{crit} is much larger than τ. This means that the lifetime of the disc should be too short to allow for even the building-up of the inner planets. We may remark here that these considerations are not restricted to the gaseous disc which we are considering, but may also play an important part in the discussion of whatsoever other theory one wants to propose.

If, however, some kind of regular system of vortices has existed, the dissipation of energy might well have been far less, perhaps even so much less that τ in that case should have been of the same order of magnitude as t_{crit}. In that case, it will not be unreasonable to assume a density distribution like the one given by equation (2.17) for the estimate of the planetary masses. The last part of the growth of the planets which is at the same time the part of the rapid growth happened just at a time when the dissipation of the envelope began to be felt. That dissipation, which will be strongest on the outskirts of the system is taken into account by the decreasing density given by equation (2.17).

(c) If we compare the ratio of τ and t_{crit} for the different planets, we get

$$\frac{t_{crit}}{\tau} \sim c \cdot \frac{v\lambda}{\Delta}. \tag{5.6}$$

Now, v is proportional to $s^{-\frac{1}{2}}$, λ to s (cf. Equation (3.14)) so that the numerator increases by a factor of the order 5 from the inner to the outer planets. The denominator, however, increases by a factor of the order 100. This means that it is possible that t_{crit} can be of the same order of magnitude as τ in the regions of the inner planets while being appreciably smaller in the regions of the outer planets. This entails that it is very possible that the size of the inner planets was restricted to r_{crit} because of the dissipation of the disc before they could grow larger. But the outer planets grew faster and were able to grow beyond the critical dimensions until also there the supply of matter ran out.

We are quite aware of the fact that the above considerations are very incomplete but in view of the many uncertain factors entering, it seems hardly worth while to start a more detailed investigation. It is, for instance, easy to see from equation (4.17), taking into account the decrease of ϱ with time ($\varrho \sim e^{-t/\tau}$), that if $t_{crit} > \tau$, the condensation products will not reach even the critical dimensions. However, if $\tau > t_{crit}$, the growth of the bodies can go on until all matter is used up. A change in the ratio t_{crit}/τ of only a few per cent. changes the picture completely in the region where that ratio is about 1. It also seems to be very dificult to take the exhaustion of the gas due to the condensation process itself adequately into account.

Chapter VI.

The Satellite Systems.

In this chapter we shall discuss the properties of the satellite systems and the rotational periods of the planets.

We want to stress the point that we cannot expect here a too close agreement with observational data. On the one hand, the observational data are not too accurate, and on the other, the situation in the planetary atmospheres will have been even more complicated than in the solar envelope. For instance, the fact that the dimensions of the atmospheres are of the same order of magnitude as the height of the disc will cause our two dimensional considerations to be certainly only rough approximations.

A. "Regular" and "irregular" satellites. If we look into the data about the satellites of the solar system (see Tables II—V), we see that we can divide them into two groups. The first group is made up of the first five Jovian satellites, the first eight Saturnian satellites, the four Uranian satellites, and Triton, Neptune's satellite. This group has orbits which are all approximately in the equatorial plane of the primary and whose eccentricities are small. We shall call these satellites the "regular" satellites.

The second group, that of the "irregular" satellites, consists of the moon, the two Martian satellites, the six outer Jovian satellites, and the outermost Saturnian satellite. Apart from the Martian satellites, the "irregular" satellites have orbital planes, highly inclined to the equatorial plane of the primary, and great orbital eccentricities[1].

We shall show here that there is also another difference between the two groups, viz., that the "regular" satellites may have been formed inside the planetary atmospheres. The "ir-

[1] We follow von Weizsäcker's classification (10). The origin of the moon is a problem lying outside the scope of the present paper. The Martian satellites are perhaps wrongly classified. See, however, the discussion on p. 68.

regular'' satellites, however, are probably condensation products captured at a later stage by the planets.

In order to prove the probability of this point, we have collected the next table. In the first row we have the mean distances of the outermost "regular" satellite. In the second row we have inserted the radius of the planetary atmosphere as given by equation (4.26). In the third row we have inserted the mean distances of the first "irregular" satellite. Finally, in the last row, we have given the ratio between the radius of the atmosphere and the radius of the planet itself.

Table 6. I.

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
s_{regul} in cm.	..	..	..	..	2.10^{11}	4.10^{11}	6.10^{10}	4.10^{10}
R_1 in cm ...	10^7	4.10^8	7.10^8	10^8	12.10^{11}	7.10^{11}	2.10^{11}	4.10^{11}
s_{irr} in cm ...	..	..	4.10^{10}	9.10^9	12.10^{11}	13.10^{11}	..	..
R_1/R_{plan} ...	0.06	0.6	1.1	0.3	170	120	80	160

We see that, indeed, the values of the second row are everywhere between those of the first and third row in agreement with our assumption of the origin of the "regular" and "irregular" satellites.

We note here finally that for the mean distances of the "regular" satellites from their primaries exponential laws like the Titius-Bode law seem to exist:

$$r_n = r_0 \varepsilon^n. \tag{6.1}$$

The value of ε decreases from 1.78 for Jupiter to 1.44 for Uranus. The value for the solar system is 1.86 if we exclude Pluto as an "irregular" planet. We have commented on these exponential laws in Chapter III and shall not discuss them here.

B. Densities and masses of the satellites. Since all satellites are smaller than the critical dimensions, gravitational capture has not played a role in their building-up process. Since (apart from the moon) all satellites are formed in the regions of the outer planets, we should expect densities of the satellites lower than those of the inner planets, but higher than those of the outer planets, since the outer planets have been able to pick up light gases during the stage of gravitational capture. This agrees within the observational uncertainties with the observed data.

We shall not estimate here the masses of the satellites in the same way as we have done in the case of the planets. We can, however, use equation (5.2) the other way round, and try to find the density function in the original planetary atmosphere from the observed masses of the satellites. We take the fraction of the matter taking part in the condensation, Δ, to be constant in each atmosphere.

The result is that we find a density function resembling very closely the density distribution in the solar envelope, i. e., a function with a maximum at a distance from the primary equal to about 10 planetary radii. However, it is impossible to arrive at any more definite conclusions.

We may draw the reader's attention to one more point connected with the condensation process of the satellites, viz. that we have to assume that the building-up process of the satellites started before the planets with their atmospheres were left in the regions of the solar system like islands in an empty space. The lifetime of the planetary atmospheres as given by equation (3.7) is at least 100 times smaller than the lifetime of the solar envelope, but the dimensions of the satellites are of the same order of magnitude as the critical dimensions so that we see that they could not have been formed during the time when the atmospheres were left to themselves.

This means that we have to imagine the following picture of the complete solar system, accepting for a moment the idea of regular systems of vortices. In the initial stages of the process, when the central mass had just become of the order of magnitude of the present solar mass, the concentration of matter in our galaxy in the neighbourhood of the solar envelope was still large enough to regulate to some extent the motion in the solar envelope. The result was a regular system of vortices, and between them "roller bearings". Originally these "roller bearings" were probably much smaller than the large vortices. However, after the planets had grown considerably they could keep larger gas masses around them. In that way the planetary atmospheres started. In the first stages of their development, there was still a disc of matter present which in its turn regulated the motion in the planetary atmospheres, resulting in a regular system of vortices in these atmospheres. In the "secondary roller bearings" the satellites started to grow.

Finally the whole disc started to disappear, and we were left with the system as we observe it at present. Of course, as soon as the dimensions of the atmospheres had become so small that there was no longer any turbulence, the planets were able to retain the atmospheres. These atmospheres are the ones we can observe now. Their lifetime is much longer than the probable age of the solar system.

C. Rotational periods of the planets. We have seen that there are so many features which are the same for the planetary system and the systems of the "regular" satellites that it seemed unavoidable not to arrive at the conclusion that their origin was analogous. These features were the nearly circular orbits lying practically in the equatorial plane of the primary, the distribution of mass in the system, viz. the largest bodies in the middle of the system, and exponential laws for the mean distances from the primary. Also the ratio of the total mass of the planetary system, respectively satellite systems to the sun, respectively mother planets, is about constant, i. e. about one thousandth. The question we are interested in now is why the outer planets have still a fairly rapid rotation while the sun is rotating so slowly.

Before considering the outer planets, we shall devote a few sentences to the inner planets. There are two reasons why we should expect low rotational velocities for the inner planets. First, they have had practically no atmospheres around them during their growth (see Table 6. I). This means that the interaction with the gas in the disc could not have followed a regular pattern. Secondly, the tidal action of the sun has been much larger for the inner planets than for the outer planets, as was shewn by STRATTON (38). This easily accounts for the fact that Mercury's rotational period is equal to its period around the sun.

Altogether it seems that the low rotational velocities of the inner planets constitute no serious difficulty. It is not, perhaps, irrelevant that the earth with the highest rotational velocity has also had the largest atmosphere.

In the case of the outer planets tidal action seems to have been negligible. We want to investigate how the atmospheres of the outer planets could have influenced their rotation. At the beginning, the large vortices will have supplied angular momentum to the planetary atmospheres. The result was probably that the angular velocities in these atmospheres corresponded to Kepler's third law (cf. Chapter II, Section A). In particular, the rotational velocities of the planets will have been given by that law.

However, as soon as the disc started to dissipate, the same process started for the planetary atmospheres, and the planets were decelerated because of the transfer of angular momentum accompanying the dissipation.

In the next table, we have inserted the rotational periods corresponding to Kepler's third law, the observed rotational periods, and the percentage change in angular momentum from the first to the second:

Table 6. II.

	T_{Kepl}	T_{obs}	$\delta\theta/\theta$
Jupiter	3 hrs	10 hrs	0.70
Saturn	4 hrs	10 hrs	0.60
Uranus	3 hrs	11 hrs	0.73
Neptune	2 hrs	12 hrs	0.83

We see that though the rotations of the planets are still fairly fast, they probably must have been slowed down considerably.

Final remarks.

In the last two chapters we saw that we could explain the differences between the outer and the inner planets as far as mass, density, and rotational velocities are concerned by looking carefully into the condensation process. This then presents us with an explanation of group C.

There are also indications given in Chapters III and V that the motion in the disc has once shewn regularities which might easily account for both the orbital regularities (A) and the exponential laws like the Titius-Bode law (B).

We have not entered into a discussion of the many irregularities which can be observed in the solar system. Some of them have been commented upon by von Weizsäcker (10). For instance, the fact that the eccentricity of Mercury's orbit is so large may well have been due to the regularity of the vortex system being disturbed in the immediate neighbourhood of the sun.

We want to remark here that there is one point which seems to deserve a thorough investigation. It is the fact that Mars is so much smaller than the earth, that Mars has only two very small satellites, and that instead of another planet between Mars and Jupiter we find the asteroids which together possess only a very small mass. This is an especially interesting point since there is also other evidence that in that neighbourhood some catastrophe has occurred. Recent investigations by Brown (1) indicate that the meteorites might be the remnants of a planet of the size of Mars which was broken up by some unspecified process.

A question which might be asked is how much chance is there to find a planetary system surrounding a certain star. It seems that planetary systems will be much more frequent than corresponding to e. g. Jeans' tidal theory. However, there are still a few requirements which have to be met. One of them is

that the temperature of the central star has to be below a certain value. Otherwise condensation will be out of the question. This can, for instance, be seen from equation (5.6). If the temperature in the disc is much higher than in the disc considered in this paper, the fraction of the gas taking part in the condensation will be much smaller and t_{crit} will be larger than the lifetime of the disc, thus leaving us without condensation products. A higher temperature of the central star results not only in higher temperatures in the disc because of greater energy output, but also in a higher temperature because of a higher degree of ionization.

Although the actual figures given by Jeans in the following quotation will not be the right ones if the theory given in this paper should be correct, we still think that this quotation will give us an adequate ending for this paper:

"The contrast between the slowness of cosmogonic events and the rapidity with which events on our earth move leads to some interesting reflections. Let us suppose that civilisation on earth is 10000 years old. If each planetary system in the universe contains 10 planets, and life and civilisation appear in due course on each, the civilisations appear at an average rate of one per 500 million years. It follows that we should probably have to visit 50000 galaxies before finding a civilisation as young as our own. And as we have only studied cosmogony for some 200 years, we should have to search through about 25 million galaxies, if they exist, before encountering cosmogonists as primitive as ourselves. We may well be the most ignorant cosmogonists in the whole of space."

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København, Universitetets Institut for teoretisk Fysik.
Purdue University, Department of Physics.

Observational data[1].

Earth's mass: $5.975 \cdot 10^{27}$ g.
Moon's mass: $7.35 \cdot 10^{25}$ g.
Sun's mass: $1.992 \cdot 10^{33}$ g.
Sun's mean radius: $6.965 \cdot 10^{10}$ cm.
Sun's rotational period (at the equator): 24.65 days.

Table I.
Elements of the planetary system.

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
Mean dist. from sun in 10^{12} cm.	5.8	10.8	15.0	22.8	77.9	143	287	450	591
Sidereal period	88 d	225 d	365 d	687 d	12 y	29 y	84 y	165 y	249 y
Eccentricity ..	0.206	0.007	0.017	0.093	0.048	0.056	0.047	0.009	0.247
Inclination of orbital plane to ecliptic.......	7°0′	3°24′	0°	1°51′	1°18′	2°29′	0°46′	1°47′	17°19′
Mass in earth's mass as unit..	0.05	0.8	1	0.1	318	95	15	17	0.9
Density in g cm^{-3}	4.1	4.9	5.5	3.9	1.3	0.7	1.3	1.6	5.5
Rad. in 10^8 cm.	2.5	6.2	6.4	3.4	69.8	57.6	25.5	25.0	6.4
Number of satellites	..	..	1	2	11	9 + rings	4	1	..
Inclination of equator to orbital plane ...	..	..	23°	25°	3°	26°	98°	141°	..
Axial rotational period........	88 d	..	24 h	25 h	10 h	10 h	11 h	12 h[2]	..
	inner or terrestrial planets				outer or major planets				

[1] All data are taken from RUSSELL, DUGAN, STEWART (3).
[2] I am indebted to Prof. Lundmark for giving me his new data about Neptune's rotational period before publication.

Table II.
Jupiter's satellites.

	5	Io	Europa	Ganymede	Callisto	6	7	10	11	8	9
…an dist. from Jup. in …cm..............	1.8	4.2	6.7	11	19	115	118	118	225	235	237
…an dist. in planetary …ii...............	2.5	5,9	9.4	15	26	161	165	165	315	330	332
…lination of orbit to …'s equat. plane...	27′	1′	28′	11′	15′	181°	243°	82°	232°	208°	61°
…centricity.........	0.0028	0.0000	0.0003	0.0015	0.0075	0.16	0.21	0.08	0.21	0.38	0.27
…ss (moon = 1)	..	0.99	0.64	2.11	1.32	..	..	..	..	..	..
…sity in g cm^{-3}....	..	2.7	2.9	2.2	1.3	..	..	..	..	..	..

Table III.
Saturn's satellites.

	Mimas	Enceladus	Tethys	Dione	Rhea	Titan	Hyperion	Iapetus	Phoebe
…an dist. from Sat. in …cm..............	1.9	2.4	2.9	3.8	5.3	12	15	36	130
…an dist. from Sat. in …netary radii	3.11	3.99	4.94	6.33	8.84	20.5	24.8	59.7	217
…lination of orbit to …'s equat. plane	1°31′	1′	1°5′	0′	21′	18′	17′-56′	14°	149°
…centricity..........	0.0201	0.0044	0.0000	0.0022	0.0010	0.0289	0.1043	0.0283	0.166
…ss (moon = 1)......	0.0005	0.001	0.009	0.014	0.03	1.9	< 0.0006	0.019	..
…sity in g cm^{-3}.....	0.8?	1.3?	1.0?	1.5?	1.0?	3.6?	1.3?	1.2?	..

Table IV.
Uranus' satellites.

	Ariel	Umbriel	Titania	Oberon
Mean distance from Uranus in 10^{10} cm	1.9	2.7	4.4	5.9
Mean distance from Uranus in planetary radii	7.4	10	17	22
Inclination of orbit to Uranus' equatorial plane	0°	0°	0′	0′
Eccentricity	0.007	0.008	0.023	0.010

Table V.
Other satellites.

	Moon	Phobos	Deimos	Triton
Mean distance from primary in 10^{10} cm	3.8	0.9	2.4	3.5
Mean distance from primary in planetary radii	60	2.8	6.9	14
Inclination of orbit to primary's equatorial plane	~ 20°	1°	2°	20°
Eccentricity	0.055	0.021	0.003	0.000
Mass (moon = 1)	1	extremely small		1.8
Density in g cm^{-3}	3.34	..	..	2.8

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ON THE INTRODUCTION OF MEASURES IN INFINITE PRODUCT SETS

BY

ERIK SPARRE ANDERSEN

AND

BØRGE JESSEN

KØBENHAVN
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1948