DET KGL. DANSKE VIDENSKABERNES SELSKAB
MATEMATISK-FYSISKE MEDDELELSER, BIND XXIV, NR. 5

THE SHORT-PERIODIC COMETS AND THE HYPOTHESIS OF THEIR CAPTURE BY THE MAJOR PLANETS

BY

ELIS STRÖMGREN †

1742

KØBENHAVN
I KOMMISSION HOS EJNAR MUNKSGAARD
1947

Printed in Denmark
Bianco Lunos Bogtrykkeri

A survey of the motions in our solar system shows that space around our sun is filled with bodies, large and small: the major planets and their satellites, asteroids, and comets; and filled with orbits. Objects that are situated in the outer parts of the system are invisible to us even if they are intrinsically bright; but if they move in very eccentric orbits, so that their perihelion distances are smaller than a certain limit, then they may come within a distance small enough to make them observable. Millions of planets and comets are certainly present in our solar system, but the great majority are never observed from the earth.

Most of the several hundred comets for which orbits have been determined move in very eccentric orbits, very close to parabolic motion. In the case of a few comets, for which slightly hyperbolic motion has been derived, it has been found that this hyperbolicity can be explained as a consequence of the perturbations by the major planets during the motion of the comet towards the inner parts of the solar system. We may maintain that there is not a single comet among those observed for which motion from the outside into our solar system has been established[1].

A small fraction of the comets observed until now move in relatively small ellipses round the sun, and the relation between these comets and the great majority of comets moving in very eccentric orbits has been made the subject of a number of investigations by various authors.

By the investigations of the orbit of the so-called Lexell's comet (comet 1770 I) attention was drawn to the possibility of great changes in the character of the motion of a comet if a near approach to one of the major planets took place. Later it became

---

[1] Cf. e. g. Publications of the Copenhagen Observatory Nos. 19, 44, 98, 105, 112, 114.

possible to establish by accurate calculations many cases of large changes of cometary orbits caused by one of the major planets (Jupiter). For an example see Publ. Copenhagen Observatory No. 106: H. Q. Rasmusen's calculation of the motion of comet Schwassmann-Wachmann 2 (1929 I). With the increasing knowledge of cometary orbits it became evident that groups of cometary orbits exist which apparently to a certain degree are connected with the major planets: a large group with aphelion distances about equal to the semi-major axis of the orbit of Jupiter, a small group with aphelion distances about equal to the semi-major axis of the orbit of Saturn, and two small groups apparently connected in a similar way with the orbits of Uranus and Neptune. It was supposed that the comets had come from distant regions of the solar system and had been captured by one of the major planets. Later investigations have shown that such an explanation is uncertain in the case of the three groups of comets apparently connected with the planets Saturn, Uranus, and Neptune, while the existence of a group of comets that have a direct mechanical connection with Jupiter may be regarded as an established fact.

How is it possible to establish whether a planet, e. g. Jupiter, has by its attraction captured a comet coming from a great distance, and changed its orbit into an ellipse of relatively small dimensions? The most exact and conclusive method would be that of carrying out for every single comet a calculation of the perturbations as far back in time as the moment when the great change of the orbit due to the close approach between planet and comet took place. The amount of work involved in carrying out this program would, however, be prohibitive, not only because the number of comets to be treated would, nowadays, be very large, but especially because it is not possible *a priori* to see how many revolutions would have to be calculated before the required situation of capture were reached and—still worse—how many times in the past a comet has had near approaches to the planet from the capture up to our time. It has, therefore, been necessary to be content with investigations of a more statistical nature.

In the course of time the problem has been attacked by many different investigators. We shall here mention the important researches made by G. Fayet and H. N. Russell.

Fayet in the Bulletin Astronomique (1911) investigated the situation of the minimum distances of the short-periodic comets known at the time from the orbit of Jupiter. Making use of amongst others the so-called Tisserand criterion, he found that a number of comets form a group of common origin. Russell in a paper in The Astronomical Journal (1920) studied the minimum distances of the short-periodic comets then known from the orbits of the major planets, and came to the conclusion that Jupiter in this problem plays an absolutely dominating role among the planets. Saturn does not seem to be of much importance. The importance of Uranus seemed to be small, and that of Neptune negligible.

After the publication of Fayet's and Russell's results the material of cometary orbits has considerably increased.

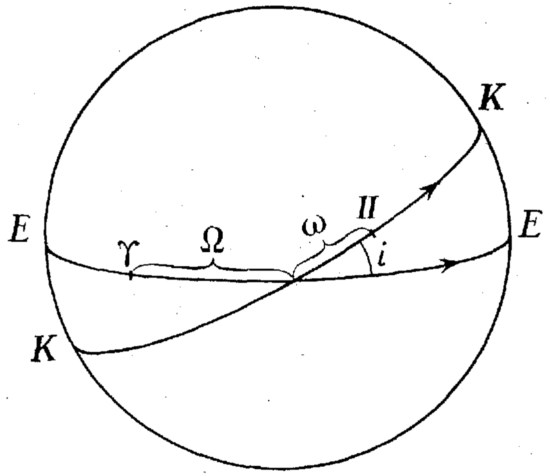
In Tables I and II we have compiled the material now available. Table I gives the orbits for all comets that have been observed in more than one apparition, while Table II contains the orbits of short-periodic comets that have not been rediscovered (the orbits of which are, however, on the whole to be considered as sufficiently reliable). In Table II we have, however, restricted the material to orbits with aphelion distances smaller than 35.0. The various columns in the tables give: the number, designation of the object, the perihelion time T, the longitude of the perihelion π, the longitude of the node Ω, the eccentricity e, the perihelion distance q, the inclination relative to the ecliptic i, the longitude of the perihelion reckoned from the node ω, the aphelion distance Q, and the period in years P. The table is arranged according to increasing period. In Table II we have not indicated the limit between the Jupiter group and the next group, as this limit is rather ill-defined. In our reasoning, as given below, we have identified the inclination relative to the ecliptic with the inclination relative to the orbit of the planet, an approximation that is of no importance to the reasoning. Similarly we have not reduced the elements to a common equinox. This is without any consequence whatever in our statistical discussion based on the elements i and ω. I am much indebted to Mr. K. A. Thernöe M.Sc. and to Mr. P. Naur for their valuable assistance in compiling the two tables.

It would undoubtedly be interesting to repeat the investigations of Fayet and Russell on the basis of the considerably greater material now available. We shall not, however, enter into this problem but restrict ourselves to a discussion from another point of view of the material contained in Tables I and II.

It has always been considered a fact that a small inclination (i) is favourable in as far as capture is concerned, and also that direct motion is more favourable than retrograde, as it makes it possible for the comet to move for a longer interval of time in the neighbourhood of the planet[1]. Statistical investigations of the orbits indeed indicate that small inclinations dominate, and that retrograde motion for short-periodic comets is very rare and does not occur at all in the Jupiter group. It is not, of course, excluded that a comet with large inclination (up to 180°) is captured in an exceptional case when the point of intersection with the ecliptic (the planetary orbit) so to speak accidentally falls *very* close to a point of the planetary orbit, but this is a question of extraordinary cases. There is another point that deserves more consideration.

It is a condition for referring a comet for instance to the Jupiter-group that the aphelion of the comet is at a distance from the sun about equal to the semi-major axis of the orbit of Jupiter (the range of the distances has proved to be rather wide, incidentally). This, however, is not sufficient. It is also required that the line of apsides of the comet lies in, or near the plane of the ecliptic (the planetary orbit). This condition is satisfied if the inclination is small, but it may also be satisfied if i is large, provided that the orbital element ω is equal to, or nearly equal to 0° or 180°.

Already Tisserand in his Traité de Mécanique Céleste (Tome IV, p. 206, 1896) has remarked that the short-periodic comets then known showed a certain regularity with regard to the orbital element ω. We have thought that an investigation based on all the material now available would be of a certain interest. A study of our two tables now leads to the following result. In the column i we have typographically indicated small values, and similarly we have in the column ω indicated values of ω that do not deviate from 0° or 180° by more than a small angle, in both cases up to 16° by bold letters, up to 22° by italics. We now see how the conditions i small and ω close to 0° or 180°, separately or together, characterize the situation. We do not now consider the Saturn-, Uranus-, and Neptune-groups—we shall treat them on another occasion—we restrict ourselves to the Jupiter group, and we see that we have here a powerful statistical confirmation of the capture theory, not only with regard to the comets that have been observed in more than one apparition, *but also regarding the comets that have not been rediscovered after the apparition of discovery.*



The Celestial Sphere from the Outside.

EE = the projection of the ecliptic on the celestial sphere. KK = the projection of a cometary orbit on the celestial sphere. γ = the vernal equinox. Ω = the longitude of the node. i = the inclination of the cometary orbit relative to the ecliptic. Π = the projection of the perihelion on the celestial sphere. ω = the arc from the ascending node to the perihelion.

---

[1] This remark seems to give the explanation of a fact that would otherwise seem difficult to understand: Why do all these comets of the Jupiter group obtain their aphelia in the neighbourhood of the planet's position at the time of capture, i. e. why have they there a motion at a right angle (approximately) to the radius vector from the sun? Could we not imagine that the motion of a comet could cross the orbit of the planet at a wide angle? Of course there are many cases where a comet crosses the planet's path at a wide angle, but then there will normally be no capture, because the planet and the comet then move in the neighbourhood of each other for a very short time only. Normally we get a capture only if the comet for a relatively long time moves near the planet. In other words: capturing requires normally a motion at a right angle (approximately) to the radius vector from the sun, and then there will be an aphelion (or a perihelion, cf. p. 9).

Table I: Short-Periodic Comets Observed in more than one Apparition.

No.	Comet	T	π	Ω	e	q	i	ω	Q	P in years
1	Encke	1937 Dec. 27.2	159°6	334°7	0.846	0.3	**12°5**	**184°9**	4.1	3.3
2	Grigg-Skjellerup[1] ..	1942 May 23.2	211.8	215.4	0.704	0.9	*17.6*	**356.4**	4.9	4.9
3	Tempel 2[1]	1946 July 2.3	310.3	119.4	0.54	1.4	**12.4**	**190.9**	4.7	5.3
4	Neujmin 2 ...	1927 Jan. 16.2	161.4	327.7	0.565	1.3	**10.6**	**193.7**	4.8	5.4
5	Brorson 1	1879 April 1.0	116.2	101.3	0.810	0.6	29.4	**14.9**	5.6	5.5
6	Tempel 3-L. Swift	1908 Oct. 1.4	44.0	290.3	0.62	1.2	**5.4**	113.7	5.2	5.7
7	de Vico-E. Swift	1894 Oct. 12.7	345.4	48.8	0.57	1.4	**3.0**	296.6	5.1	5.9
8	Tempel 1.....	1879 May 7.6	238.3	78.8	0.463	1.8	**9.8**	*159.5*	4.8	6.0
9	Pons-Winnecke[1] ..	1945 July 10.6	264.5	94.4	0.654	1.2	*21.7*	**170.1**	5.6	6.1
10	Kopff[1]	1945 Aug. 11.3	284.6	253.0	0.556	1.5	**7.2**	31.5	5.3	6.2
11	Forbes	1929 June 26.0	285.0	25.5	0.56	1.5	**4.6**	259.5	5.3	6.4
12	Schwassmann-Wachmann 2	1942 Feb. 14.3	124.0	126.0	0.385	2.1	**3.7**	**358.0**	4.8	6.5
13	Perrine 1.....	1922 Dec. 25.7	49.6	242.3	0.66	1.2	**15.7**	**167.3**	5.8	6.6
14	Giacobini-Zinner[1]	1946 Sept. 18.5	8.1	196.3	0.717	1.0	30.7	**171.9**	6.0	6.6
15	Biela	1852 Sept. 24.2	109.2	245.9	0.756	0.9	**12.6**	223.3	6.2	6.6
16	d'Arrest......	1943 Sept. 23.8	318.0	143.6	0.611	1.4	*18.0*	**174.4**	5.7	6.7
17	Daniel	1943 Nov. 22.2	76.6	70.5	0.574	1.5	*19.9*	**6.1**	5.7	6.8
18	Finlay	1926 Aug. 7.2	5.9	45.3	0.70	1.1	**3.4**	320.6	6.2	6.9
19	Holmes	1906 Mar. 14.6	346.0	331.7	0.42	2.1	*20.8*	**14.3**	5.1	6.9
20	Borelly	1932 Aug. 26.3	69.6	77.1	0.617	1.4	30.5	**352.5**	5.8	6.9
21	Brooks 2[1]	1946 Aug. 25.8	13.3	177.7	0.484	1.9	**5.5**	**195.6**	5.4	7.0
22	Reinmuth	1935 May 1.4	133.8	125.0	0.504	1.9	**8.1**	**8.8**	5.7	7.2
23	Faye	1940 April 23.0	46.7	206.4	0.566	1.6	**10.6**	*200.3*	5.9	7.4
24	Whipple	1941 Jan. 13.3	19.0	188.8	0.349	2.5	**10.2**	**190.2**	5.2	7.5
25	Oterma 3[2]....	1942 Sept. 13.7	153.9	154.9	0.143	3.4	**4.0**	**359.0**	4.6	8.0
26	Schaumasse ..	1943 Nov. 4.5	137.7	86.7	0.705	1.2	**12.0**	51.0	6.8	8.2
27	Wolf 1.......	1942 June 7.6	5.3	204.3	0.405	2.4	27.3	*161.0*	5.8	8.3
28	Comas Solá ..	1944 April 11.6	104.6	65.7	0.576	1.8	**13.7**	38.9	6.6	8.5
29	Gale	1938 June 18.5	276.4	67.3	0.761	1.2	**11.7**	209.1	8.7	11.0
30	Tuttle 1	1939 Nov. 10.1	116.8	269.8	0.821	1.0	54.7	207.0	10.3	13.6
31	Schwassmann-Wachmann 1	1941 Sept. 1.2	323.4	323.0	0.142	**5.5**[3]	**9.4**	**0.4**	7.3	16.3
32	Neujmin 1 ...	1931 May 7.4	334.3	347.3	0.775	1.5	**15.2**	347.0	12.0	17.7
33	Pons-Forbes ..	1928 Nov. 5.0	86.0	250.1	0.93	0.7	28°9	**195.9**	18.0	27.9
34	Stephan-Oterma	1942 Dec. 18.9	76.7	78.6	0.858	1.6	*17.9*	**358.1**	17.9	37.8
35	Westphal.....	1913 Nov. 26.8	43.9	346.8	0.92	1.3	40.9	57.1	30.0	61.7
36	Brorson 2-Metcalf	1919 Oct. 17.4	80.3	310.8	0.97	0.5	*19.2*	129.5	33.2	69.1
37	Pons-Brooks ..	1884 Jan. 26.2	93.3	254.1	0.955	0.8	74.0	*199.2*	33.7	71.6
38	Olbers	1884 Oct. 9.0	149.8	84.5	0.931	1.2	44.6	65.3	33.6	72.7
39	Halley	1910 April 20.2	169.0	57.3	0.97	0.6	*162.2*	111.7	35.3	76.0
40	Herschel-Rigollet.....	1939 Aug. 9.5	24.4	355.1	0.974	0.7	64.2	29.3	57.2	150.7

[1] The list of short-periodic comets observed in more than one apparition was compiled in the autumn of 1944 for the "Festskrift" for Prof. N. E. NÖRLUND. Six of the comets in this list have been rediscovered at a later date. References for Comet Grigg-Skjellerup: U. A. I. Circ. 1080; for Comet Tempel 2: H. A. C. 745 and U. A. I. Circ. 1040; for Comet Pons-Winnecke: U. A. I. Circ. 998 and 1005; for Comet Kopff: U. A. I. Circ. 1019; for Comet Giacobini-Zinner: U. A. I. 755; for Comet Brooks 2: U. A. I. Circ. 1057.

[2] Comet Oterma 3 has been moved from Table II to Table I.

[3] Cf. p. 10.

Table II: Short-Periodic Comets Observed in one Apparition only (Aphelion distance < 35,0).

Comet	T	π	Ω	e	q	i	ω	Q	P in years	No.
1766 II	1766 April 28.2	252°0	71°6	0.834	0.4	**7°8**	**180°4**	4.5	3.9	1
1819 IV......	1819 Nov. 20.8	67.5	77.4	0.699	0.9	**9.1**	**350.1**	5.0	5.1	2
1678[1]	1678 Aug. 18.8	322.8	163.3	0.627	1.1	**2.9**	*159.5*	4.9	5.2	3
1930 VI......	1930 June 14.2	269.1	76.8	0.666	1.0	*17.3*	**192.3**	5.0	5.3	4
1884 II	1884 Aug. 17.0	306.1	5.1	0.584	1.3	**5.5**	301.0	4.9	5.4	5
1743 I	1743 Jan. 8.7	93.3	86.9	0.721	0.9	**1.9**	**6.4**	5.3	5.4	6
1941 e	1941 July 21.2	298.8	229.6	0.579	1.3	**3.2**	69.2	4.9	5.4	7
1770 I	1770 Aug. 14.0	356.3	132.0	0.786	0.7	**1.6**	224.3	5.6	5.5	8
1886 IV......	1886 June 7.2	230.3	53.5	0.579	1.3	**12.7**	**176.8**	5.0	5.6	9
1940 a[2]	1939 Oct. 3.5	70.4	137.6	0.448	1.7	**4.8**	292.8	5.1	5.6	10
1783.........	1783 Nov. 20.4	50.3	55.7	0.552	1.5	45.1	**354.6**	5.1	5.9	11
Giacobini[3]	1928 Mar. 26.8	182.0	196.8	0.71	1.0	**1.4**	**345.2**	5.9	6.4	12
1890 VII	1890 Oct. 27.0	58.4	45.1	0.471	1.8	**12.8**	**13.3**	5.1	6.4	13
1900 III	1900 Nov. 28.5	7.8	196.7	0.733	0.9	29.8	**171.1**	6.1	6.5	14
1858 III	1858 May 3.5	200.8	175.1	0.674	1.1	*19.5*	25.7	5.9	6.6	15
1892 V.......	1892 Dec. 11.0	16.3	206.4	0.594	1.4	31.3	**169.9**	5.6	6.6	16
1896 V.......	1896 Oct. 28.5	334.0	193.5	0.589	1.5	**11.4**	140.5	5.6	6.6	17
1918 III	1918 Sept. 30.7	37.4	117.9	0.468	1.9	**5.6**	259.5	5.2	6.7	18
1916 I	1928 Oct. 22.4	103.8	108.3	0.487	1.8	*20.7*	**355.5**	5.3	6.8	19
1895 II	1895 Aug. 21.3	338.1	170.3	0.652	1.3	**3.0**	**167.8**	6.2	7.2	20
1894 I	1894 Feb. 9.9	130.6	84.4	0.697	1.1	**5.5**	46.2	6.4	7.4	21
1925 I	1925 Jan. 24.0	84.6	260.5	0.374	2.4	23.1	**184.1**	5.3	7.5	22
1906 VI......	1906 Oct. 10.3	34.6	194.6	0.584	1.6	**14.6**	*200.0*	6.2	7.8	23
1936 IV......	1936 Oct. 3.4	1.5	164.2	0.650	1.5	**13.3**	*197.3*	6.9	8.5	24
1881 V.......	1881 Sept. 13.8	18.4	65.9	0.828	0.7	**6.9**	312.5	7.7	8.7	25
1889 VI......	1889 Nov. 30.1	40.2	330.4	0.685	1.4	**10.3**	69.8	7.2	8.9	26
1939 IV......	1939 Feb. 9.0	179.9	135.5	0.624	1.8	**11.1**	44.4	7.6	10.1	27
1929 III	1929 June 28.2	298.7	158.2	0.585	2.0	**3.7**	140.5	7.8	10.9	28
1846 VI......	1846 June 1.6	240.0	260.4	0.729	1.5	30.7	*339.6*	9.7	13.4	29
1944 c	1944 June 17.5	279.4	22.3	0.781	1.3	*18.6*	257.1	10.4	14.0	30
1585.........	1585 Oct. 8.5	9.9	38.0	0.826	1.1	**5.4**	331.9	11.4	15.5	31
1916 III	1916 June 14	319	224	0.93	0.5	103	95	12.4	16.4	32
1866 I	1866 Jan. 11.6	42.4	231.4	0.905	1.0	*162.7*	**171.0**	19.7	33.2	33
1863 V.......	1863 Dec. 27.1	59.3	305.0	0.946	0.8	63.6	114.3	27.8	53.2	34
1873 VII	1873 Dec. 2.5	85.9	250.0	0.949	0.7	29.2	**195.9**	28.1	54.8	35
1931 III	1931 June 10.8	150.4	191.6	0.935	1.0	42.5	318.8	30.6	62.9	36
1827 II	1827 Jan. 7.7	337.0	317.7	0.949	0.8	136.4	*19.3*	31.1	63.8	37
1883 II	1883 Dec. 25.6	41.9	264.3	0.981	0.3	114.7	137.6	31.9	64.6	38

[1] Possibly identical with No. 7 in Table I (cf. LEVERRIER, Astr. Nachrichten **26**, p. 382-384).

[2] This comet was overlooked in the list of 1944.

[3] Very uncertain elements.

We have assumed—as is indicated by the statistics—that the aphelion of the captured comet is, within certain limits, situated close to the orbit of the planet (cf. the footnote on p. 6). Why the aphelion? Is it not possible that the result of the capture were that the perihelion of the comet came to be situated close to the planetary orbit? Capture results because the comet at a certain time moves very close to the planet. When the planet has loosened its grip, the comet quickly glides into a practically unperturbed two-body motion relative to the sun.

If the comet through the capture has obtained a motion that makes it approach the sun, the aphelion of the new orbit will be in the neighbourhood of the point where the planet was situated in its orbit during the capture. Apparently this is the normal, and until recently it was the only case represented in our tables. The aphelion of the new cometary orbit is close to the planetary orbit, and its perihelion is situated on the other side of the sun. If, however, the comet through the capture has obtained an orbit in which it recedes from the sun, the perihelion of the new cometary orbit will be close to the planetary orbit, and the aphelion will be located on the opposite side of the sun. *A priori* this case would appear to be as probable as the other. A case of perihelion in the neighbourhood of the orbit of Jupiter now actually exists in comet Schwassmann-Wachmann 1 (No. 31 in Table I). The explanation of the fact that this case appears as an exception in the existing material instead of occurring quite often is probably the simple one that a comet having its perihelion close to the orbit of Jupiter during its entire motion will stay at a great distance from the earth, and therefore will be visible from the earth only in the case of abnormally great brightness. We must imagine that captured comets exist with all values of q from 0 up to the radius of the orbit of Jupiter (approximately) and such with values of Q from the value of the radius of the orbit of Jupiter (approximately) up to very high values, but the majority of them will never be discovered.

The problem of capture of comets contains many other questions of interest. Some of them we shall treat elsewhere.

Bibliography.

G. Fayet, Criterium de Tisserand pour les comètes à courte période (Bull. Astr. 1911).

H. N. Russell, On the Origin of Periodic Comets (The Astron. Journal 1920).

S. Oppenheim, Kometen (Enzyklopädie der Matematischen Wissenschaften, 1923).

A. Kopff, Die kurzperiodischen Kometen (Handbuch der Astrophysik Bd. IV, 1929).

Indleveret til Selskabet den 29. Marts 1947.
Færdig fra Trykkeriet den 15. December 1947.