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ON THE SYSTEMATIC CHANGES OF THE ECCENTRICITIES OF NEARLY PARABOLIC ORBITS

BY

ERIK SINDING

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It is well known that investigations of comets moving in nearly parabolic orbits have shown that the majority of the original orbits were certainly elliptical, and that no certain case of an originally hyperbolic orbit exists. The results of the investigations were based on calculations of planetary perturbations of the nearly parabolic orbits during the time of the passage of the comet from outer space through the inner parts of the solar system. The available material consists partly in approximate calculations by G. FAYET[1] for about 150 comets, partly in rigorous calculations carried out by a number of investigators according to the principles developed by E. STRÖMGREN[2].

In the great majority of cases the eccentricity of the original orbit is smaller than the osculating eccentricity in the part of the orbit where the comet is observed. This tendency is very marked, and clearly shown both by the rigorous and the approximate calculations. However, since FAYET's work can only be regarded as an approximation of statistical value, we shall here restrict ourselves to a consideration of the results of the rigorous calculations.

GEELMUYDEN-STRÖMGREN, *Lærebog i Astronomi*, 2. Udg., Oslo 1945, on p. 285 contains a list of 21 rigorous calculations of the eccentricities of original cometary orbits. From this list we have taken the data given below and added a column giving the value of $\Delta\left(\frac{1}{a}\right)$, i. e. original $\frac{1}{a}$ — osculating $\frac{1}{a}$ in the observable part of the orbit, a being the semi-major axis.

The osculating $\frac{1}{a}$ is the reciprocal of the value of a found by a definitive orbit determination. The original $\frac{1}{a}$-values given

[1] G. FAYET, *Recherches concernant les excentricités des comètes*, Paris 1906.
[2] E. STRÖMGREN, *Über den Ursprung der Kometen*, Copenhagen 1914.

were found through numerical calculations of the perturbations by Jupiter and Saturn (in some cases by other planets, too) which covered a period extending so far back in time that the further perturbations were negligible. The original $\frac{1}{a}$-values refer to the motion of the comet with regard to the centre of gravity of the sun, Jupiter, and Saturn.

No.	Comet	Osculating $\frac{1}{a}$	Original $\frac{1}{a}$	$\Delta\left(\frac{1}{a}\right)$
1...	1853 III	− 0.0008193	+ 0.0000829	+ 0.0009022
2...	1863 VI	− 0.0004949	+ 0.0000166	+ 0.0005115
3...	1882 II	+ 0.0118963	+ 0.0121488	+ 0.0002525
4...	1886 I	− 0.0006944	− 0.0000071	+ 0.0006873
5...	1886 II	− 0.0004770	+ 0.0003166	+ 0.0007936
6...	1886 IX	− 0.0005765	+ 0.0000630	+ 0.0006395
7...	1889 I	− 0.0006915	+ 0.000042	+ 0.0007335
8...	1890 II	− 0.0002151	+ 0.0000718	+ 0.0002869
9...	1897 I	− 0.0008722	+ 0.0000396	+ 0.0009118
10...	1898 VII	− 0.0006074	− 0.0000157	+ 0.0005917
11...	1902 III	+ 0.0000810	+ 0.0000054	− 0.0000756
12...	1904 I	− 0.0005040	+ 0.0002165	+ 0.0007205
13...	1905 VI	− 0.0001424	+ 0.0006210	+ 0.0007634
14...	1907 I	− 0.0004991	+ 0.0000252	+ 0.0005243
15...	1910 I	+ 0.0002143	(+ 0.0033021)	(+ 0.0030878)
16...	1914 V	− 0.0001465	+ 0.0000119	+ 0.0001584
17...	1922 II	− 0.0003806	+ 0.0000038	+ 0.0003844
18...	1925 I	− 0.0005665	+ 0.0000540	+ 0.0006205
19...	1925 VII	− 0.0002730	+ 0.0001150	+ 0.0003880
20...	1932 VI	− 0.0005948	+ 0.0000441	+ 0.0006389
21...	1936 I	− 0.000487	+ 0.000205	+ 0.000692

It appears from the table that for 20 out of 21 comets the orbits have shifted in the elliptical direction, when going back in time, i. e. $\Delta\left(\frac{1}{a}\right)$ is positive, while there is a slight shift in the hyperbolic direction for one comet. The order of magnitude of $\Delta\left(\frac{1}{a}\right)$ is $+0.0005$. Comet 1910 I is apparently an exception. An examination of the data given for this comet,[1] however, shows that the investigation contains an error in the derivation of the

[1] K. Lous, *Die ursprüngliche Bahn des Kometen 1910 I* (A. N. 220, 16 Kiel 1924).

elements for 1904, January 24. The correct value of $\frac{1}{a}$ is $+0.0006921$. Hence, $\Delta\left(\frac{1}{a}\right) = +0.0004778$, a result which is in good agreement with the other values of $\Delta\left(\frac{1}{a}\right)$.

The mean value of the values of $\Delta\left(\frac{1}{a}\right)$ now becomes $\Delta\left(\frac{1}{a}\right)_m = +0.000552$. Examination of the distribution of the $\Delta\left(\frac{1}{a}\right)$ around the mean value shows that this may be characterized as random, as the material is small (cf. Fig. 1).

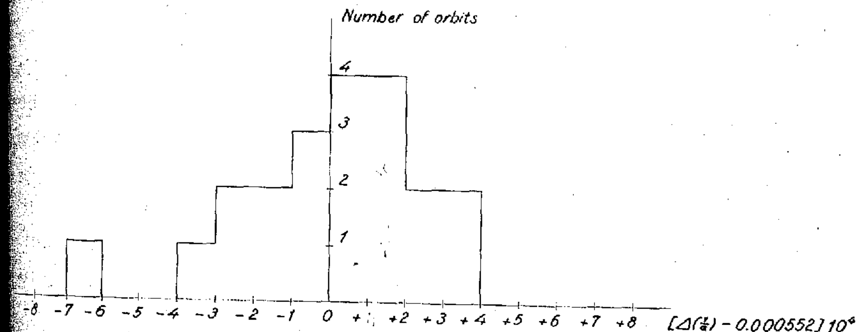

Fig. 1.

From the distribution it is found that the standard error of $\Delta\left(\frac{1}{a}\right)_m$ is ± 0.000055, thus

$$\Delta\left(\frac{1}{a}\right)_m = +0.000552 \pm 0.000055.$$

The prediction for an individual comet is

$$\Delta\left(\frac{1}{a}\right) = +0.00055 \pm 0.00025.$$

In what follows we shall attempt to show how the systematic change in the eccentricity may be understood on the basis of certain simplifying assumptions.

We consider a simplified system consisting of the sun, Jupiter and the comet. We assume that Jupiter is moving in a circular orbit with the sun at the centre. In this case a Jacobian integral is valid for the system.

The integral in question has been derived by H. SEELIGER.[1] SEELIGER considered the motion relative to the sun (not relative to the centre of gravity of the sun and Jupiter). Putting $k = 1$ (which corresponds to the adoption of a unit of time equal to about 58 days), we find

$$V^2 = 2\gamma + \frac{2}{r} + \frac{2\mu}{\varrho} + 2n'\sqrt{p}\cos i - \frac{\mu}{r'^3}(r^2 - \varrho^2). \qquad (1)$$

Here V is the velocity of the comet relative to the sun, r the distance of the comet from the sun, ϱ the distance between the comet and Jupiter, while p and i are the parameter and the inclination, respectively, of the comet's orbit (strictly speaking the inclination is relative to Jupiter's orbital plane). Finally μ is the Jupiter mass, r' Jupiter's distance from the sun, and n' the mean motion of the planet. The quantity 2γ is a constant.

Introducing the expression which defines the osculating a of the comet,

$$V^2 = \frac{2}{r} - \frac{1}{a},$$

we get

$$\frac{1}{a} = -2\gamma - \frac{2\mu}{\varrho} - 2n'\sqrt{p}\cos i + \frac{\mu}{r'^3}(r^2 - \varrho^2). \qquad (2)$$

Usually the Jacobian integral is written in terms of the motion relative to the centre of gravity of the system. In this case we get the following equation:

$$\overline{V}^2 = \frac{2}{r} + \frac{2\mu}{\varrho} + 2n'\left(\bar{x}\frac{d\bar{y}}{dt} - \bar{y}\frac{d\bar{x}}{dt}\right) - C. \qquad (3)$$

Here $\overline{V}$ is the velocity of the comet relative to the centre of gravity, r and ϱ as before denote the distances sun—comet and Jupiter—comet, while μ and n' are the mass and the mean motion of Jupiter. The quantities $\bar{x}$ and $\bar{y}$ are the co-ordinates of the comet relative to the centre of gravity, and C is a

[1] H. SEELIGER, *Notiz über einen Tisserand'schen Satz* (A. N. **124**, 209, Kiel 189

new constant. The latter constant is practically equal to -2γ, however, and for our purposes we may put $C = -2\gamma$.

If we consider the motion of the comet at a great distance from the sun, and using

$$\overline{V}^2 = (1+\mu)\left(\frac{2}{r} - \frac{1}{\bar{a}}\right),$$

where we have put r equal to ϱ equal to the distance of the comet from the centre of gravity, we find

$$\frac{1+\mu}{\bar{a}} = -2\gamma - 2n'\sqrt{\bar{p}}\cos\bar{i}\sqrt{1+\mu}. \qquad (4)$$

The quantities $\bar{a}$, $\bar{p}$ and $\bar{i}$ refer to the orbit relative to the centre of gravity.

From (4) we get

$$\frac{1}{\bar{a}} = -\frac{2\gamma}{1+\mu} - \frac{2n'}{\sqrt{1+\mu}}\sqrt{\bar{p}}\cos\bar{i} \qquad (5)$$

and finally, from (2) and (5),

$$\left.\begin{aligned} \Delta\left(\frac{1}{a}\right) = \frac{1}{\bar{a}} - \frac{1}{a} &= \frac{2\mu}{\varrho} - \frac{\mu}{r'^3}(r^2 - \varrho^2) + \\ &+ \frac{\mu}{1+\mu}2\gamma - \left(\frac{2n'}{\sqrt{1+\mu}}\sqrt{\bar{p}}\cos\bar{i} - 2n'\sqrt{p}\cos i\right). \end{aligned}\right\} \qquad (6)$$

Here ϱ and r denote the distances from the comet to Jupiter and the sun, respectively, at the epoch of osculation of the definitive orbit in question. If we wish to study the average value of $\Delta\left(\frac{1}{a}\right)$, it is only necessary to take account of the terms in (6) which are systematic in character. This leads to

$$\Delta\left(\frac{1}{a}\right)_m = \frac{2\mu}{\varrho} - \frac{\mu}{r'^3}(r^2 - \varrho^2). \qquad (7)$$

Introducing an average value of ϱ equal to $r' = 5.203$, and putting $r = 1$ and $\mu = \frac{1}{1047}$, we find

$$\Delta\left(\frac{1}{a}\right)_m = +0.000544,$$

in good agreement with the value obtained from the material of rigorous calculations.

These considerations show that the phenomenon that nearly parabolic orbits change systematically in the elliptical direction when epochs further and further back in time are considered, is connected with the fact that the sun and Jupiter act as one combined mass when the comets are at a great distance from the sun, and that the influence of Jupiter dominates over that of the other planets.

In conclusion it may be noted that the above considerations of course apply also to the case of the change of a nearly parabolic orbit throughout the time interval following the perihelion passage.

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ON SOME APPROXIMATIVE DIRICHLET-POLYNOMIALS IN THE THEORY OF THE ZETA-FUNCTION OF RIEMANN

BY

PAUL TURÁN

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