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AN INTEGRATING BEAM METER FOR HIGH TENSION WORK

BY

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When observing the output of nuclear disintegrating processes in high tension plants it is often desirable to know the amount of protons etc. which hit the target on the cathode. Data regarding this magnitude can be provided by measurements of current and time; it is difficult, however, to keep the current constant during the time of observation and, frequently, a simple instrument directly indicating the quantity of electricity is needed. In the following, it will be shown that a galvanometer of the moving coil reflecting type is applicable to this purpose, provided that certain conditions are fullfilled.

The equation of motion of the system in an ordinary galvanometer is

$$K \frac{d^2 \varphi}{dt^2} + n \frac{d \varphi}{dt} + D \varphi - \frac{1}{10} HsI = 0, \tag{1}$$

where φ is the angle of rotation, t the time, K the moment of inertia, n the damping coefficient, and D the torsional moment of the suspension. If the strength of the magnetic field is denoted by H and the total area of windings by s, the total flux through the coil will be $Hs\varphi$. When the system moves, an e. m. f. of $10^{-8} Hs \frac{d\varphi}{dt}$ volts is produced, and the induced current will accordingly be $10^{-8} \frac{Hs}{R} \frac{d\varphi}{dt}$ amperes; R being the total resistance (in ohms) of the circuit, $R = r_g + r_e$, where r_g is the resistance of the galvanometer itself, and r_e the resistance of the rest of the circuit. The retarding moment due to this induced current will be $\frac{1}{10} Hs \cdot 10^{-8} \frac{Hs}{R} \frac{d\varphi}{dt} = 10^{-9} \frac{(Hs)^2}{R} \frac{d\varphi}{dt} = n \frac{d\varphi}{dt}$. The current to be measured is denoted by I (amperes) and will produce an angular momentum $\frac{1}{10} HsI$.

In the present case, it was desired that the angular deflection of the galvanometer should be proportional to the quantity of electricity Q ($= \int I dt$) which, during a comparatively long time, e. g. 100 sec., passes through the instrument. The movement of the coil must therefore be extremely slow, and the term $K \frac{d^2\varphi}{dt^2}$ in (1) will be negligible, provided that K is small. The equation of motion will thus be

$$n \frac{d\varphi}{dt} + D\varphi - \frac{1}{10} HsI = 0 \qquad (2)$$

and, if the system moves from φ_0 to φ_1 during the time t,

$$(\varphi_1 - \varphi_0) = \frac{1}{10} \frac{Hs}{n} Q - \frac{D}{n} \int_0^t \varphi \, dt. \qquad (3)$$

The first term on the right side of this equation is the deflection proportional to Q, and the last term is the error which, for an arbitrarily varying, direct current, will not exceed $-\frac{D}{n}(\varphi_1 - \varphi_0) t$; if the current is almost constant, the error is only half this value. The relative error, which corresponds to the present application, will accordingly be

$$F = -\frac{D}{2n} \frac{(\varphi_1 - \varphi_0)}{(\varphi_1 - \varphi_0)} t \, 100\,\% = -50 \frac{D}{n} t\,\%. \qquad (4)$$

The claims, which should be fulfilled by such an instrument, are therefore: 1) a very small torsional moment D and, furthermore, a large damping coefficient $n \left(= 10^{-9} \frac{(Hs)^2}{R}\right)$, which means 2) a large value of Hs and 3) a small value of R.

Condition 1) means that the instrument must be of a similar type as the fluxmeter with its suspension of unspun silk. According to condition 2), the magnetic field H has to be very strong and, if possible, the coil should have a large cross section and many turns. As the moment of inertia of the coil is of minor importance the cross section may, within certain limits, be rather large and of the shape of a square. Many turns, however, will disaccord with claim 3), viz. low resistance. A closer examination reveals that, for a given mass of the coil, the number of turns is insignificant.

The external resistance r_e must be low, so as to get a large damping, and thus the instrument has to be shunted by a small resistance. Generally the galvanometer, when used to indicate the proton current (i) to the target, is coupled with a very large resistance. We have in this case (cf. the figure), $(i-I)\, r_e = I\, r_g$ or $i = \frac{r_g + r_e}{r_e} I = \frac{R}{r_e} I$, where r_e is the resistance

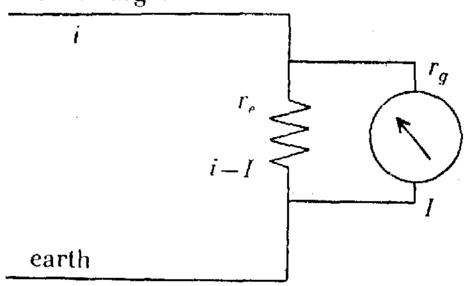

Fig. 1.

of the shunt. If q denotes the quantity of electricity from the target, we get

$$q = \int i dt = \frac{R}{r_e} \int I dt = \frac{R}{r_e} Q.$$

From (3) follows

$$(\varphi_1 - \varphi_0) = \frac{1}{10} \frac{Hs}{n} Q = 10^8 \frac{R}{Hs} Q = 10^8 \frac{r_e}{Hs} q, \qquad (5)$$

which means that the deflection is proportional to q and r_e, so that the sensitivity may easily be varied by changing the shunt. The error may be estimated from (4).

Whether a given instrument is suited to the present purpose can be tested in the following manner. The sensitivity f is determined. As this is the current, which causes a deflection of the light spot of 1 mm on the scale at 1000 mm distance, it corresponds to a rotation of the coil of $(\varphi_1 - \varphi_0) = \frac{1}{2000}$. From equation (2) we find

$$D(\varphi_1 - \varphi_0) = \frac{1}{10} HsI \text{ or } D \frac{1}{2000} = \frac{1}{10} Hsf$$

and

$$f = \frac{1}{200} \frac{D}{Hs}. \tag{6}$$

The damping of the instrument may be observed, e. g. by determining the time $t_{\frac{1}{2}}$, in which a given deflection is reduced to half its value. With $I = 0$ and the shunt r_e it follows from (2) that

$$n = D \frac{t_{\frac{1}{2}}}{\ln 2} \text{ or } t_{\frac{1}{2}} = 0.694 \cdot 10^{-9} \frac{(Hs)^2}{DR}. \tag{7}$$

If R is measured, D and Hs may be found and introduced into the expressions

$$(\alpha_1 - \alpha_0) = 2.10^{10} \frac{r_e}{Hs} q \text{ and } F = -5 \cdot 10^{10} \frac{DR}{(Hs)^2} \cdot t\,\%_0 \tag{8}$$

derived from (5) and (4), respectively, and where α_0 and α_1 denote the positions of the light spot on the scale (in mm/m) before and after the passage of the quantity q coulombs.

An instrument, corresponding to the results of the present considerations, was constructed. The moving coil had 400 turns 0.10 mm wire, the cross section of the coil was 2×2 cm, $r_g = 100$ ohms (including the connecting ligaments). The suspension was 15 μ Vistra silk, 15 cm. long. The connecting ligaments were made of silver, 4 by 150 μ. The sensitivity was found to be $5.9 \cdot 10^{-10}$ amp/mm/m. The damping coefficient n was found from the time $t_{\frac{1}{2}} = 280$ sec. ($r_e = 0.9$ ohms). We obtained $n = 225$, $(Hs) = 4.7 \cdot 10^6$ and the torsional moment $D = 0.55$. The greater part of D is due to magnetic impurities in the copper coil, the present circumstances making it impossible to obtain unmagnetic material. Better material or a compensation of the induced magnetic moment would give a far smaller D (< 0.1). The error F amounted to 7.5 % during an observation period of 60 sec.; smaller D will result in a correspondingly smaller error.

Examples of some tests performed with this instrument:

1) Sensitivity proportional to the shunt r_e. $t = 60$ sec.

i	r_e	$\alpha_1 - \alpha_0$
6.0 μA	8.0 ohms	106.0 mm/m.
12.0 -	4.0 -	106.0 -
24.0 -	2.0 -	106 -
40.0 -	1.2 -	106 -
200 -	0.24 -	106 -

2) Deflection proportional to the quantity of electricity q, with corrections according to (4). Constant current $i = 24\,\mu A$, and different periods of observation t.

t	$\alpha_1 - \alpha_0$	$\alpha_1 - \alpha_0/t$	$\alpha_1 - \alpha_0/t$ corr.
10 sec.	9.3 mm/m.	0.93	0.94
20 -	18.9 -	0.95	0.97
40 -	36.6 -	0.92	0.96
60 -	53.4 -	0.89	0.96
120 -	100.0 -	0.83	0.96

3) Deflection proportional to q. Constant period of observation and different currents. (No correction needed). ($r_e = 4{,}0$ ohms $t = 60$ sec.

i	$\alpha_1 - \alpha_0$	$\alpha_1 - \alpha_0/i$
3.0 μA	26.7 mm/m.	8.9
6.0 -	53.3 -	8.9
12.0 -	105 -	8.8
24.0 -	209 -	8.7

It was the aim with this note to show that small amounts of electricity (0.1 — 10 milli-coulombs) due to proton currents (from a few to several hundred micro-amperes) conveniently can be measured with a reflecting galvanometer of the fluxmeter type, shunted with a low resistance.

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