A DETERMINATION
OF THE ABUNDANCES OF SODIUM
AND MAGNESIUM IN THE SOLAR
ATMOSPHERE FROM LINES NEAR
THE SERIES LIMITS

BY

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1. A theoretical investigation of the discontinuities at the edges of metal absorption continua in the solar spectrum has been carried out by UNSÖLD,[1] who demonstrated how the magnitude of a discontinuity could be determined from equivalent widths of the higher members of the corresponding series of absorption lines by an extrapolation process.

UNSÖLD in this way showed that the discontinuities of the absorption edges of the metals must be very small, indeed. This result, deduced from the observed strength of absorption lines, was in sharp contrast to the results obtained from the theoretical calculations of the coefficient of continuous absorption and its variation with wave-length, the latter leading to a discontinuity that was many times greater.

It is now known that the discrepancy was due to the fact that the effect of continuous absorption of the negative hydrogen ion had not been included in the theoretical calculations of the absorption coefficient. It is, in fact, assumed now that the greater part of the continuous absorption in the solar atmosphere is due to the negative hydrogen ion.[2, 3, 4, 5, 6, 7]

The aim of the present paper is to determine metal abundances from absorption edge discontinuities determined according to UNSÖLD's method. The ratio of the absorption coefficient of the metal at the edge in question to that of the negative hydrogen ion can be found from the magnitude of the discontinuity. The atomic continuous absorption coefficient of the metal and of the negative hydrogen ion being known from quantum-mechanical calculations, the abundance ratio of the metal in question and of hydrogen can be found.

The present investigation deals with two of the series considered by UNSÖLD, *viz.* the "sharp" series of neutral sodium

and the "Bergman" triplet series of neutral magnesium. The reasons for using these series were partly theoretical, because the absorption coefficients at the series limits were known, and partly practical, both of the series having lines, the total strength of which could be measured with reasonable certainty from the curves given in the Photometric Atlas of the Solar Spectrum.[8]

The atlas is very useful, because it makes it easy to find out if a line is disturbed to such a degree that it has to be omitted from the investigation. As we shall see later, few lines only could, in fact, be used.

2. The basic assumption in Unsöld's method is that we may treat the line absorption in the series as if it were continuous, that is, we may use the formulae for the intensity in the continuous spectrum when the absorption—expressed as an equivalent width—of each line is evenly distributed over the interval of wave-length "belonging to" that line in the series. This "continuous absorption" is a direct continuation of the true continuous absorption on the short wave-length side of the series limit, and its "coefficient of continuous absorption" may—in not too great a distance from the limit—be found by extrapolation from the known continuous absorption coefficient. The process is therefore equivalent to a displacement of the limit to the place in the spectrum of the line in question. When the correction of the line intensities to the case of absorption in an "optically thin layer" is small, i. e. when the intensity measured at the centre of the line is great, this method yields reliable results.

The atlas of the solar spectrum gives the spectral distribution of the radiation of the centre of the solar disc. We must then use the following formula giving the intensity for light of frequency ν in the direction perpendicular to the atmosphere:[9]

$$I_\nu (0, \theta) = B_\nu (T_e) \left[1 - \frac{1}{8} u_e \frac{1}{1 - e^{-u_c}} + \frac{1}{n_\nu} \cdot \frac{3}{16} \cdot u_e \cdot \frac{1}{1 - e^{-u_c}} \cos \theta \right], \quad (1)$$

where $\theta = 0$.

Here $B_\nu (T_e)$ is the black-body intensity at the effective temperature T_e, $u_e = \frac{h\nu}{kT_e}$ and n_ν is the ratio of the coefficient

of continuous absorption at the frequency ν to the opacity (the Rosseland-mean); h and k have their usual meanings.

This formula holds only if n_ν is constant through the atmosphere. We shall see later to what degree this assumption is justified in the cases in question.

On the short and the long wave-length side of the "limit" the intensity has the values $I_{\nu 1}$ and $I_{\nu 2}$, respectively, and the constant n_ν the values $n_{\nu 1}$ and $n_{\nu 2}$, respectively.

Then
$$\Delta I_\nu = I_{\nu 2} - I_{\nu 1} = B_\nu (T_e) \cdot \left[\frac{1}{n_{\nu 2}} - \frac{1}{n_{\nu 1}} \right] \cdot \frac{3}{16} \cdot \frac{u_e}{1 - e^{-u_e}} \quad (2)$$

and

$$\frac{\Delta I_\nu}{I_{\nu 2}} = \frac{\left[\frac{1}{n_{\nu 2}} - \frac{1}{n_{\nu 1}} \right] \cdot \frac{3}{16} \cdot \frac{u_e}{1 - e^{-u_c}}}{1 - \frac{1}{8} \frac{u_e}{1 - e^{-u_c}} + \frac{1}{n_{\nu 2}} \cdot \frac{3}{16} \cdot \frac{u_e}{1 - e^{-u_c}}} = \frac{\frac{1}{n_{\nu 2}} - \frac{1}{n_{\nu 1}}}{\frac{16}{3} \cdot \frac{1 - e^{-u_e}}{u_e} - \frac{2}{3} + \frac{1}{n_{\nu 2}}}. \quad (3)$$

When small quantities are neglected, we get:

$$\frac{\Delta I_\nu}{I_\nu} = \frac{-\frac{\Delta n_\nu}{n_\nu^2}}{\frac{16}{3} \cdot \frac{1 - e^{-u_e}}{u_e} - \frac{2}{3} + \frac{1}{n_\nu}}, \quad (4)$$

where $I_\nu \infty I_{\nu 1} \infty I_{\nu 2}$, $n_\nu \infty n_{\nu 1} \infty n_{\nu 2}$ and $\Delta n_\nu = n_{\nu 2} - n_{\nu 1}$.

As to the solar atmosphere, the continuous spectrum is very nearly that of "grey" matter, n_ν being for all frequencies equal to 1.

When we put n_ν equal to 1 in the above equation, we get:

$$-\Delta n_\nu = \frac{\Delta I_\nu}{I_\nu} \cdot \frac{1}{3} \cdot \left[16 \cdot \frac{1 - e^{-u_c}}{u_e} + 1 \right]. \quad (5)$$

The negative sign in front of Δn_ν means that I_ν takes its greatest value when n_ν takes the smallest and vice versa.

The negative hydrogen ions are believed to produce practically the whole of the total absorption on the long wave-length side of the Balmer limit, where its coefficient of absorption is near its maximum value.[10]

The constant value of the continuous absorption coefficient is therefore put equal to this maximum value ($2.6 \cdot 10^{-17} \, \text{cm}^2$).

Δn_ν is the ratio of the metal absorption to the total absorption at the frequency ν. When a part x of the total absorption is due to negative hydrogen ions, the ratio of metal absorption to negative hydrogen ion absorption is $\dfrac{\Delta n_\nu}{x}$.

When the coefficient of continuous absorption per metal atom in the state in question is $\varkappa_{\nu m}$ and per negative hydrogen ion $\varkappa_{\nu H^-}$, the ratio of the number of metal atoms in this state to the number of negative hydrogen ions is

$$\frac{n_m}{n_{H^-}} = \frac{\Delta n_\nu}{x} \cdot \frac{\varkappa_{\nu H^-}}{\varkappa_{\nu m}}; \tag{6}$$

$\varkappa_{\nu H^-}$ is equal to x times its maximum value; $\varkappa_\nu$ is proportional to the oscillator strength per interval of energy $\dfrac{df}{dE}$;

$$\varkappa_\nu = 8.06 \cdot 10^{-18} \cdot \frac{df}{dE} \tag{7}$$

when E is measured in Rydberg units.[11]

When $\left(\dfrac{df}{dE}\right)_m$ denotes this quantity for the metal, we get:

$$\frac{n_m}{n_{H^-}} = \Delta n_\nu \cdot \frac{3.2}{\left(\dfrac{df}{dE}\right)_m}, \tag{8}$$

where x has disappeared.

As we shall see later, the ratio $\dfrac{n_m}{n_{H^-}}$ varies little for neutral sodium and magnesium. The situation will be similar for other neutral metals.

The assumption of constant n_ν (and Δn_ν) through the atmosphere is therefore justified when x is near unity. The influence of the possible variation of the rest of the continuous absorption is of course smaller, the smaller $1-x$, and is wholly neglected in our calculations.

3. The coefficient of continuous absorption of the "sharp" and the "diffuse" series of neutral sodium is known from quan-

tum-mechanical calculation.[12] $\dfrac{df}{dE}$ for the two series is 0.13 and 0.82, respectively, at the (common) series limit and 0.08 and 0.31 at a distance of 0.05 Rydberg units on the short wavelength side of the limit.

The extrapolation of the $\dfrac{df}{dE}$-function on the long wavelength side of the limit could not be carried out with any certainty for the "diffuse" series, but was thought to be possible in the case of the "sharp" series, at least in not too great distance from the limit.

The first doublet of this series is placed in the infrared. The second (wave-lengths 6161 and 6154 Å) was considered to be placed too far from the limit. Of the lines visible in the solar spectrum only the lines of the third and fourth doublet remained. The wave-lengths of these lines are:[1]

third doublet: $\begin{cases} 5153.42 \text{ Å} \\ 5148.85 \text{ -} \end{cases}$

fourth doublet: $\begin{cases} 4751.83 \text{ -} \\ 4747.98 \text{ -} \end{cases}$

The line at 5153 Å could not be measured because it is strongly blended with other lines. The line at 5149 Å is placed close to another line of nearly the same intensity and shape. The curve in the atlas here has the form of a nearly symmetrical double figure. Because we are here nearly dealing with "absorption in an optically thin layer," the absorption may be taken to be equal to the sum of the absorptions from each of the lines. We may therefore take as the intensity of the sodium line its part of the "double" measured from the base of the figure, which is not necessarily thought to be placed at the top of the true continuous spectrum. Its equivalent width is found to be 0.009 Å.

In the fourth doublet the line at 4752 Å is placed in the wing of a stronger line and is therefore measured in a similar way. Its equivalent width was found to be 0.009 Å.

The second (weak) component of this doublet (at 4748 Å) is seen only as an irregularity of a stronger neighbouring line and its intensity therefore is not measurable.

The equivalent widths of the two measured lines are about one seventh of the (double) Doppler widths of the lines. Hence, the correction to absorption in "optically thin" layer is about 10 per cent. and the corrected intensities are taken to be 0.010 Å for both of the lines.

The line at 5149 Å is the weaker component of the doublet, and hence the sum of the corrected intensities of the doublet components should be 0.030 Å. The line at 4752 Å is the stronger doublet-component, and hence the total corrected intensity of this doublet is put equal to 0.015 Å.

The "interval per line" is found in the following way:

The wave number $\tilde{\nu}$ of a line in a series is given by $\tilde{\nu} = -\dfrac{R}{n_e^2} + \tilde{\nu}_0$, where R is the Rydberg wave number, n_e the "effective" quantum number of the upper state, and $\tilde{\nu}_0$ the wave number of the lower state common to the lines in the series.

Hence

$$\frac{d\tilde{\nu}}{dn_e} = \frac{2R}{n_e^3}; \tag{9}$$

$$\frac{d\lambda}{dn_e} = \frac{d\lambda}{d\tilde{\nu}} \cdot \frac{d\tilde{\nu}}{dn_e} = -\frac{1}{\tilde{\nu}^2} \cdot \frac{2R}{n_e^3} = -2R \cdot \frac{\lambda^2}{n_e^3}. \tag{10}$$

The difference between n_e and the term number is nearly constant for the lines in a series; therefore dn_e is put equal to one and $2R \cdot \dfrac{\lambda^2}{n_e^3}$ used as a measure of the "interval of wave-length" per line. For the two doublets considered, n_e takes the values 4.65 and 5.65 and hence the two intervals are 582 and 275 Å, respectively, and $\dfrac{\Delta I}{I}$ for the two lines is 0.000052 and 0.000055.

Inserting the numerical values of the constants, we find

$$u_e = \frac{h\nu}{kT_e} = \frac{h}{k} \cdot \frac{c}{\lambda} \cdot \frac{1}{T_e} = \frac{1.439}{\lambda \cdot T_e}. \tag{11}$$

With $T_e = 5740°$ and $\lambda = 5151$ and 4750 Å we get $u_e = 4.87$ and 5.28. Hence, $1 - e^{-u_e}$ is practically equal to 1.

For Δn_ν numerically we find in the two cases 0.000074 and 0.000074. The $\dfrac{df}{dE}$-function is linearly extrapolated. As its numerical value is known in two points only, we have no indication for the extrapolation in any other way than this, the simplest one.

The E-values corresponding to the two wave-lengths above are 0.177 and 0.192, respectively. The limit is placed at $E = 0.223$. We then extrapolate the $\dfrac{df}{dE}$-values (p. 7) and find for the places of the two lines $\dfrac{df}{dE} = 0.17_5$ and 0.16.

We find from formula (8) $\dfrac{n_{Na3^2P}}{n_{H^-}} = 0.0013_5$ and 0.0014_8, where n_{Na3^2P} is the number of sodium atoms in the $3p$-state (the upper state of the resonance lines of sodium). As a mean value we shall adopt 0.0014.

From Saha's equation we get:[13]

$$\left.\begin{array}{l} \log\left(\dfrac{n_H}{n_{H^-}}\right) = -0.70 \cdot \theta + \dfrac{5}{2}\log T - 0.48 + \log\left(\dfrac{2 \cdot 2}{1}\right) - \log P_e \\[3mm] \text{and } \log\left(\dfrac{n_{Na^+}}{n_{Na3^2P}}\right) = -3.02 \cdot \theta + \dfrac{5}{2}\log T - 0.48 + \log\left(\dfrac{2 \cdot 1}{6}\right) - \log P_e, \end{array}\right\} \tag{12}$$

the ionization potential of the $3p$-state of neutral sodium being 3.02 volt and its statistical weight 6. θ is equal to $\dfrac{5040°}{T}$.

From these equations we get:

$$\log\left(\frac{n_H}{n_{Na^+}} \cdot \frac{n_{Na3^2P}}{n_{H^-}}\right) = 2.32 \cdot \theta + \log 12.$$

In the solar atmosphere practically all the hydrogen atoms are neutral and in the ground state and practically all the sodium atoms in the ground state of the Na^+ ion.

We see that the electron pressure has disappeared from the equation and that the variation of $\dfrac{n_{Na3^2P}}{n_{H^-}}$ with the temperature is small. For θ we use as a representative value $\theta_e = \dfrac{5040°}{T_e} = 0.878$. Hence we get:

$$\log\left(\frac{n_H}{n_{Na^+}} \cdot \frac{n_{Na3^2P}}{n_{H^-}}\right) = 3.12.$$

When inserting

$$\log \left(\frac{n_{Na\,3\,^2P}}{n_{H^-}} \right) = 0.15 - 3,$$

we get:

$$\log \left(\frac{n_H}{n_{Na^+}} \right) = 6.0.$$

This is in good agreement with the value (6.1) found by B. Strömgren.[14]

4. The magnesium series dealt with—the "Bergman" triplet series—has single lines, the multiplet structure being wholly negligible in comparison with the width of the lines in the solar spectrum. The lines of the corresponding singlet series are weaker and are more disturbed by other lines, because its series limit is placed at a shorter wave-length (6547 Å) than the limit of the triplet series (7289 Å).

The lines—especially the higher members of the series—are very much broadened by damping. Unless the lines are placed in regions practically free from disturbing influences of other lines, the wings cannot be traced to so great distances from the centres of the lines that the total intensity can be found with reasonable certainty. This is probably the reason for the seemingly irregular run of the intensities (cf. Unsöld[1]).

Of the lines given by Unsöld the following six are found in the atlas:

$\lambda\,(Å) = 8736.04,\ 8346.13,\ 8098.75,\ 7930.82,\ 7811.16,\ 7722.64.$[15]
Of these, the lines at 8346, 8099, 7931 Å are disturbed. The line at 7811 Å is blended with a sharper line placed at one of its wings. The other half of the line is free and hence the total intensity of the line may be found by doubling the equivalent width of this part. The line at 7723 Å is very broad and difficult to measure.

The equivalent widths of the lines at 8736 Å and 7811 Å are found to be 0.266 Å and 0.059 Å, respectively.

When reducing the values to the case of absorption in an "optically thin" layer, we use our knowledge of the intensity at the centres of the lines, which in this case—comparatively broad lines—may be taken from the profiles in the atlas. Hence, we need not know the value of the damping constants for the

lines, provided their profiles are determined by damping alone. This probably applies to the weaker of the two lines here considered. For the stronger, we calculate for comparison the reduction factor for the other ideal case, that of pure Doppler-broadening. The constants differ so little that we may use a mean value of the two factors.

In the calculation of these factors we have—for the sake of convenience—used the formula for the line profile in the integrated spectrum and put the value of $x_e = u_e\,(1 - e^{-u_e})^{-1}$ equal to 4. The errors introduced in this way are not serious in our, on the whole, comparatively rough calculations.

The formula used for r (the intensity in units of the intensity in the neighbouring continuous spectrum) hence is[16]

$$r = \frac{\lambda + \frac{2}{3}\sqrt{3\lambda}}{1 + \frac{2}{3}\sqrt{3\lambda}}, \qquad (13)$$

where $\lambda = \dfrac{1}{1 + \eta}$, and $\eta = \dfrac{l_\nu}{k_\nu}$, where l_ν and k_ν denote the coefficient of scattering in the line and the coefficient of continuous absorption at the place of the line, respectively. (We here neglect the absorption part of the line "absorption" coefficient.)

The central intensities of the two lines are 0.60 and 0.91, respectively. The corresponding values of λ are 0.33 and 0.817 and the values of η are 2.0 and 0.224, respectively.

In the damping and the Doppler case η is equal to

$$\frac{\eta_0}{1 + \left(\dfrac{\omega - \omega_0}{\delta}\right)^2} \quad \text{and} \quad \eta_0 \cdot e^{-\ln 2 \left(\frac{\omega - \omega_0}{\delta}\right)^2}, \qquad (14)$$

respectively, when η_0 is the value of η at the centre of the line.[17] $\dfrac{\omega - \omega_0}{\delta}$ is the distance from the centre of the line in units of δ, the distance for which η takes the value $\dfrac{\eta_0}{2}$.

With the aid of the formula (13) for r, we calculate the integral of $1 - r$ over the line and compare it with the value we should get, if η were vanishingly small.

We have

$$1 - r = \frac{1 - \lambda}{1 + \frac{2}{3}\sqrt{3\lambda}} = \frac{\frac{\eta}{1 + \eta}}{1 + \frac{2}{3}\sqrt{3\lambda}}; \quad (15)$$

when $\eta \to 0$, $\lambda \to 1$, and we get:

$$1 - r = \frac{\eta}{1 + \frac{2}{3}\sqrt{3}}. \quad (16)$$

Hence, the integral of η divided by $1 + \frac{2}{3}\sqrt{3}$ is the total intensity in the case of an "optically thin" layer, and this integral divided by the integral of the true $1 - r$ (both taken with the correct η_0) gives the required correction factor.

For the line at 8736 Å the correction factor in the case of pure damping broadening is 1.5 and for pure Doppler broadening 1.9. As a mean value we shall use 1.7.

For the line at 7811 Å the factor in the damping case is 1.2, which is the value adopted.

The corrected intensities then are 0.45 Å and 0.070 Å.

The effective quantum numbers of the upper terms of the two transitions are 7 and 11, respectively, and the corresponding wave-length intervals 488 Å and 101 Å.

Using the formula (5) we find for Δn_ν 0.00192 and 0.00135.

$\frac{df}{dE}$ at the series limit is calculated from the corresponding hydrogen-value. For sodium it is known that a difference of nearly one between the true and the effective quantum number of the ground term of a series makes a difference of only 15 per cent. from the approximate correction factor

$$\left(\frac{\nu_H}{\nu}\right)^3 \cdot \left(\frac{n}{n_e}\right)^3, \quad (17)$$

where ν_H and ν are the frequencies of the hydrogen and the metal limits.[18] In our case $n - n_e$ is only 0.2, so that we simply use the correction factor given above.

$\frac{df}{dE}$ for the corresponding hydrogen limit is 2.2, being one half of the value of $f \cdot n^3$ for the high members of the series given by BETHE[19] (cf. our formula (9)). Hence, $\frac{df}{dE}$ for the magnesium limit is 1.84. We extrapolate to greater wave-lengths, using the approximate law found for hydrogen, i. e. $\frac{df}{dE}$ is proportional to λ^3. For the two lines we find $\frac{df}{dE}$ equal to 3.16 and 2.26 and $\frac{n_{Mg\,3^3D}}{n_{H^-}}$ equal to 0.00194 and 0.00191, respectively. (The good agreement between these two values is of course only accidental.) We use the value 0.0019 in our further calculations.

The equation corresponding to the sodium equation (12) is:

$$\log\left(\frac{n_{Mg^+}}{n_{Mg\,3^3D}}\right) = -1.69 \cdot \theta + \frac{5}{2}\log T - 0.48 + \log\left(\frac{2 \cdot 2}{15}\right) - \log P_e, \quad (18)$$

and we get:

$$\log\left(\frac{n_H}{n_{Mg^+}} \cdot \frac{n_{Mg\,3^3D}}{n_{H^-}}\right) = 0.99 \cdot \theta + \log 15.$$

As before, we put θ equal to $\theta_e = 0.878$, and we get:

$$\left(\log\frac{n_H}{n_{Mg^+}} \cdot \frac{n_{Mg\,3^3D}}{n_{H^-}}\right) = 2.05.$$

We thus find $\log\left(\frac{n_H}{n_{Mg^+}}\right) = 4.77$.

Magnesium is not so completely ionized in the solar atmosphere as is sodium. In the model atmosphere with $\log A = 3.8$[20] three per cent. of the magnesium atoms are neutral at $\theta = \theta_e$.

Hence we get 4.7_5 for $\log\left(\frac{n_H}{n_{Mg}}\right)$.

5. If we trust this hydrogen-magnesium ratio, we find that the relative proportions of the metals magnesium, calcium, sodium, and potassium are nearly equal to those found by GOLDSCHMIDT for these metals in meteorites.[21]

Normalizing GOLDSCHMIDT's relative proportions so that the sum of the abundances of the metals is equal to one, we get the values in the first column of the table below.

Fe	0.30	—	—
Si	0.33	—	—
Mg	0.30	0.000018	0.00006
Ca	0.020	0.0000016	0.00008
Na	0.015	0.0000009	0.00006
K	0.0023	0.00000020	0.00009

In the next column we write the abundances relative to hydrogen. For *Ca* and *K* we use the values found by B. Strömgren, for *Mg* the value found above, and for *Na* the mean value of B. Strömgren's and our values. (The *Mg* abundance found by B. Strömgren was determined by comparison of a *Mg* and a *Ca* line and is probably less accurate than the other abundances).

The third column gives the ratio of the abundances in the two first. The mean of the different values of this ratio is found to be $0,00007_2$. The logarithm of A—the number of hydrogen atoms per metal atom—hence being 4.1_5.

The value of $\log A$ (3.9) found by B. Strömgren in his investigations is a parameter of the model atmosphere which measures the number of free electrons per hydrogen atom. In view of the various sources of errors no great weight should be attached to this discrepancy. Nevertheless the following consideration may be of some interest. That part of the electrons—nearly 50 per cent.—which, if we trust the numbers given above, is delivered by elements other than the metals, may be due to the ionization of carbon. This element has an ionization potential intermediate between the potentials of hydrogen and the metals, and its abundance is possibly so high that its contribution to the electron pressure may be of the required order.

A method for the correction of the model atmosphere tables for different values of the carbon abundance is given by B. Strömgren.[22]

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