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ON THE PHOTO-DISINTEGRATION OF THE DEUTERON

BY

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The photo-disintegration of the deuteron and the neutron-proton capture are discussed from the point of view of the meson theory of nuclear forces in the form proposed by Møller and Rosenfeld. The calculations include all first order relativistic contributions. The general expression for the photoelectric cross-section turns out to be identical in form with the corresponding quantity in the "old" theory (assuming a spherical potential well), while the photo-magnetic cross-section contains an extra term due to the charged meson fields. The theory accounts in a satisfactory way for the magnitude of the cross-sections. The discrepancy with regard to the angular distribution of the ejected particles which exists in the old theory is removed; the satisfactory agreement with experiment here found appears to be due mainly to the extra term mentioned above. For large energies, the cross-sections decrease more rapidly ($\sim \nu^{-7/2}$) than in the old theory, while there is a marked difference in angular distribution in this energy region as compared with Bethe's "neutral" theory. The capture cross-section for thermal neutrons is proportional to v^{-1}; its value is in good agreement with experiment. In an Appendix, the reliability of some approximate expressions for the radial wave functions of the deuteron is discussed and the electric quadrupole transitions are given.

§ 1. Introduction.

The discovery, made by CHADWICK and GOLDHABER[1]), that the deuteron can be disintegrated by γ-rays of sufficient energy, provides most valuable information about the interaction of electromagnetic radiation with nuclear systems. This effect is closely connected with the capture process of neutrons by protons which especially plays a prominent rôle in slow neutron experiments. In the earliest treatments of photo-disintegration[2])[3]) as well as of proton-neutron capture, these effects were considered as photo-electric (*PE*) processes (interaction of the electric field of the incident wave with the nuclear system). The cross-sections thus obtained for the PE disintegration were in reasonable agreement with experiment, but there appeared to be a difference of several orders of magnitude between theoretical expectations and the measured values of the capture cross-section. This point was cleared up by the remark of FERMI[4]) that, besides the mentioned processes, one has also to take into account the photo-magnetic (*PM*) transitions due to the interaction of the magnetic field of the incident wave with the magnetic moments of the nuclear particles (cf. also BREIT and CONDON[5]); it was shown that the slow neutron capture is essentially of magnetic character and that the well-known $1/v$ law can be explained on this assumption. Thus, all experimental data known at the time could be accounted for by a theory based only on the assumption that the range of the nuclear forces is small compared to the wave-lengths involved.

More recent experiments by VON HALBAN[6]), however, seem to indicate a discrepancy with theoretical expectations on the angular distribution of the disintegration products. While for the *PM* effect (corresponding to a transition between the ^3S-state and the ^1S-state of the deuteron), this distribution is isotropic, the contribution of the *PE* effect (a $^3S \rightarrow {}^3P$ transition) per unit solid angle is proportional to $\sin^2\theta$, θ being the angle between the incident γ-ray and the ejected neutron (or proton). Therefore, from the expressions for the differential cross-section of both effects, which we shall denote by $d\Phi^{\mathrm{el}}(\theta)$ and $d\Phi^{\mathrm{magn}}$, we find for the ratio of the intensities at $\theta = 0$, $(\Phi_{\parallel})$ and $\theta = \pi/2$, $(\Phi_{\perp})$:

$$\frac{\Phi_{\parallel}}{\Phi_{\perp}} = \frac{d\Phi^{\mathrm{magn}}}{d\Phi^{\mathrm{magn}} + d\Phi^{\mathrm{el}}(\pi/2)}. \tag{1}$$

For ThC″ γ-rays this ratio was calculated to be 0.29, assuming the virtual ^1S-level of the deuteron to have an energy of about $10^5\, eV$; more detailed calculations of the *PE* cross-section which show that this cross-section had been underestimated [3,5]), lead to a value of 0.15. The same value has been obtained by RARITA and SCHWINGER[7]) on the assumption of a spherical well potential combined with a directional coupling. On the other hand, v. HALBAN's measurements give a value of about 5 %, in agreement with results obtained by CHADWICK, FEATHER and BRETSCHER[8]) on the distribution of the photo-protons. In order to explain this apparent inconsistency, a better insight into the nature of nuclear forces may be deemed necessary and it is therefore of interest to discuss these problems in accordance with our present notions on the interaction between nucleons.

It is the aim of this paper to treat the photo-effect as well as proton-neutron capture from the point of view of the theory of MØLLER and ROSENFELD[9]), according to which nuclear interaction is established by a specific mixture of vector and pseudoscalar meson fields, including charged and neutral mesons in a symmetrical way*. §§ 2—4 are devoted to the *PE* and *PM* effects, while the neutron capture is discussed in § 5.

* Recently, a discussion of the *PE* effect in the frame of the meson theory has been given by FRÖHLICH, HEITLER and KAHN[10]), assuming the interaction to be described by a field of the vector type. However, their treatment is clearly inconsistent with the general electromagnetic properties of nuclear systems; their results will therefore not be considered here.

The admixture of a D-state with the ^3S-state of the deuteron has practically no influence on the effects under consideration. The contribution of this D-state will therefore be neglected throughout and the ground state will thus be taken to be purely of the ^3S-type. Moreover, it may be noted here that the inclusion of the electric quadrupole transitions will neither influence essentially the value of the total cross-section, nor the quantity defined in (1), the angular distribution to which it gives rise being proportional to $\sin^2\theta\cos^2\theta$ ($S \to D$ transition). This effect is treated in an Appendix.

§ 2. a) The wave equation of the deuteron.

We begin with a survey of the properties of the deuteron wave functions, representing in a slightly different form results obtained by KEMMER[11]) in a paper on the neutron-proton interaction.

The two nucleons constituting the deuteron and all quantities which refer to them are labelled with the upper indices 1 and 2, respectively; $\vec{x}^{(1)}$ and $\vec{x}^{(2)}$, for instance, represent the spatial coordinates of the first and second particles. The deuteron is described by a 16-component wave function $\Psi_{E\mu}$ (μ stands for all those sets of values of the degeneracy parameters which belong to the same energy E). In the frame of reference in which the centre of gravity of the deuteron is at rest it satisfies the equation

$$\mathcal{H}_0 \Psi_{E\mu}(\vec{x}) = \left[\frac{\hbar c}{i}\vec{\alpha}\,\mathrm{grad} + \beta M c^2 + \mathcal{V}(r)\right]\Psi_{E\mu}(\vec{x}) = E\,\Psi_{E\mu}(\vec{x}) \qquad (2)$$

with

$$\vec{x} = \vec{x}^{(1)} - \vec{x}^{(2)},\quad r = \left|\vec{x}\right|,\quad \vec{\alpha} = \vec{\alpha}^{(1)} - \vec{\alpha}^{(2)},\quad \beta = \varrho_3^{(1)} + \varrho_3^{(2)},\quad M \cong M_N \cong M_P,$$

$$\mathcal{V}(r) = \left(\boldsymbol{\tau}^{(1)}\boldsymbol{\tau}^{(2)}\right)\left[g_1^2 + g_2^2\left(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)}\right)\right]\frac{e^{-\varkappa r}}{4\pi r},\quad \varkappa = \frac{M_m c}{\hbar},$$

M_m is the meson rest mass. The eigenvalue $+1$ (-1) of τ_3 denotes neutron (proton) states. According to KEMMER, the non-trivial proper solutions of (2) can be classified as follows:

Type Ia:	corresponds in non-relativistic approximation with	triplet state with $l = j \pm 1$,
Type Ib:		triplet state with $l = j$,
Type IIb:		singlet state.

We now introduce the normalized spin wave functions

$$^3\chi_0 = \frac{1}{\sqrt{2}}\left(\delta_{\sigma_3^{(1)}1}\,\delta_{\sigma_3^{(2)}-1} + \delta_{\sigma_3^{(1)}-1}\,\delta_{\sigma_3^{(2)}1}\right),$$

$$^3\chi_1 = \delta_{\sigma_3^{(1)}1}\,\delta_{\sigma_3^{(2)}1}, \qquad {}^3\chi_{-1} = \delta_{\sigma_3^{(1)}-1}\,\delta_{\sigma_3^{(2)}-1},$$

$$^1\chi_0 = \frac{1}{\sqrt{2}}\left(\delta_{\sigma_3^{(1)}1}\,\delta_{\sigma_3^{(2)}-1} - \delta_{\sigma_3^{(1)}-1}\,\delta_{\sigma_3^{(2)}1}\right),$$

and similarly the "isotopic spin wave functions" ζ and the "ϱ-wave functions" ξ:

$$^3\zeta_1 = \delta_{\tau_3^{(1)},1}\,\delta_{\tau_3^{(2)},1}, \ \textit{etc.};$$

$$^3\xi_1 = \delta_{\varrho_3^{(1)}1}\,\delta_{\varrho_3^{(2)}1}, \ \textit{etc.}$$

Then, to the first order in the velocities and apart from isotopic spin dependence ($\overset{(0)}{\Psi}$ is the velocity-independent part of Ψ which will be called "large component", $\overset{(1)}{\Psi}$ which is of the order of v/c is the "small component"),

$$\Psi = \overset{(0)}{\Psi} + \overset{(1)}{\Psi}$$

with

$$\left.\begin{array}{ll} \text{Type Ia, Ib:} & \overset{(0)}{\Psi} = {}^3\xi_1\left[{}^3\chi_1 Z^1 + {}^3\chi_0 Z^0 + {}^3\chi_{-1} Z^{-1}\right], \\ \text{Type IIb:} & \overset{(0)}{\Psi} = {}^3\xi_1\,{}^1\chi_0 Z^0, \end{array}\right\} \text{(3a)}$$

and

$$\left.\begin{array}{ll} \text{Type Ia, Ib:} & \overset{(1)}{\Psi} = {}^1\xi_0\left[{}^3\chi_1 z^1 + {}^3\chi_0 z_s^0 + {}^3\chi_{-1} z^{-1}\right] + {}^3\xi_0\,{}^1\chi_0 z_a^0, \\ \text{Type IIb:} & \overset{(1)}{\Psi} = {}^3\xi_0\left[{}^3\chi_1 z^1 + {}^3\chi_0 z_s^0 + {}^3\chi_{-1} z^{-1}\right]. \end{array}\right\} \text{(3b)}$$

Z and z only depend on the relative spatial coordinates; introducing polar variables ($x = r \sin \vartheta \cos \varphi$, $y = r \sin \vartheta \sin \varphi$, $z = r \cos \vartheta$), they may be expressed in the following way*

* The spherical harmonics are defined as in *loc. cit.*[9]), equ. (115); we also use the same normalization prescriptions as stated there.

Type I a $l = j-1$

$$\left\{\begin{array}{ll} Z^1 = \sqrt{(j+m-1)(j+m)} & Y_{j-1}^{(m-1)} \\ Z^0 = -\sqrt{2(j+m)(j-m)} & Y_{j-1}^{(m)} \\ Z^{-1} = \sqrt{(j-m-1)(j-m)} & Y_{j-1}^{(m+1)} \end{array}\right\} \cdot \frac{1}{\sqrt{2j(2j-1)}} \cdot \frac{R_I(j-1)}{r}$$

Type I a $l = j+1$

$$\left\{\begin{array}{ll} Z^1 = \sqrt{(j-m+1)(j-m+2)} & Y_{j+1}^{(m-1)} \\ Z^0 = \sqrt{2(j+m+1)(j-m+1)} & Y_{j+1}^{(m)} \\ Z^{-1} = \sqrt{(j+m+1)(j+m+2)} & Y_{j+1}^{(m+1)} \end{array}\right\} \cdot \frac{1}{\sqrt{2(j+1)(2j+3)}} \cdot \frac{R_I(j+1)}{r}$$

Type I b

$$\left\{\begin{array}{ll} Z^1 = -\sqrt{(j+m)(j-m+1)} & Y_{j}^{(m-1)} \\ Z^0 = -m\sqrt{2} & Y_{j}^{(m)} \\ Z^{-1} = -\sqrt{(j+m+1)(j-m)} & Y_{j}^{(m+1)} \end{array}\right\} \cdot \frac{1}{\sqrt{2j(j+1)}} \cdot \frac{R_I(j)}{r}$$

Type II b

$$Z^0 = \qquad - Y_j^{(m)} \cdot \frac{R_{II}(j)}{r}$$

Type I a $l = j-1$

$$\left\{\begin{array}{ll} z^1 = \sqrt{2(j+m)(j-m+1)} & Y_{j}^{(m-1)} \\ z_s^0 = 2m & Y_{j}^{(m)} \\ z^{-1} = -\sqrt{2(j+m+1)(j-m)} & Y_{j}^{(m+1)} \\ z_a^0 = 2j & Y_{j}^{(m)} \end{array}\right\} \cdot \frac{1}{2\sqrt{2j}} \cdot \frac{A^1(j-1)}{r}$$

Type I a $l = j+1$

$$\left\{\begin{array}{ll} z^1 = \sqrt{2(j+m)(j-m+1)} & Y_{j}^{(m-1)} \\ z_s^0 = 2m & Y_{j}^{(m)} \\ z^{-1} = -\sqrt{2(j+m+1)(j-m)} & Y_{j}^{(m+1)} \\ z_a^0 = -2(j+1) & Y_{j}^{(m)} \end{array}\right\} \cdot \frac{1}{2\sqrt{2(j+1)}} \cdot \frac{A^2(j+1)}{r}$$

Type* I b
II b

$$\left\{\begin{array}{ll} z^1 = \sqrt{2(j+m-1)(j+m)} & Y_{j-1}^{(m-1)} \\ z_s^0 = -2\sqrt{(j+m)(j-m)} & Y_{j-1}^{(m)} \\ z^{-1} = \sqrt{2(j-m-1)(j-m)} & Y_{j-1}^{(m+1)} \\ z_a^0 = 0 & \end{array}\right\} \cdot \begin{array}{l} \dfrac{-1}{2\sqrt{2j(j+1)(2j-1)}} \cdot \dfrac{C^1(j)}{r} \\ \dfrac{1}{2\sqrt{2(2j-1)}} \cdot \dfrac{H^1(j)}{r} \end{array}$$

$$+\left\{\begin{array}{ll} \sqrt{2(j-m+1)(j-m+2)} & Y_{j+1}^{(m-1)} \\ 2\sqrt{(j+m+1)(j-m+1)} & Y_{j+1}^{(m)} \\ \sqrt{2(j+m+1)(j+m+2)} & Y_{j+1}^{(m+1)} \end{array}\right\} \cdot \begin{array}{l} \dfrac{-1}{2\sqrt{2j(j+1)(2j+3)}} \cdot \dfrac{C^2(j)}{r} \\ \dfrac{1}{2\sqrt{2(2j+3)}} \cdot \dfrac{H^2(j)}{r} \end{array}$$

* The expressions between the braces are the same for both types. The upper expressions behind the braces refer to I b-states, the lower ones to II b-states.

The large radial functions $R_s(j)$ satisfy

$$\left\{\frac{\hbar^2}{M}\left(\frac{d^2}{dr^2}-\frac{j(j+1)}{r^2}\right)+\Gamma_{j,s}\frac{e^{-\varkappa r}}{r}+E\right\}R_s(j)=0, \tag{4}$$

where $s = \mathrm{I}$ or II and

$$\left.\begin{aligned} &\text{for type Ia, Ib:} \quad \Gamma_{j,I} = \left[1-2(-1)^{j+1}\right]\frac{g_1^2+g_2^2}{4\pi},\\ &\text{for type IIb:} \quad \Gamma_{j,II} = \left[1-2(-1)^{j}\right]\frac{g_1^2-3g_2^2}{4\pi}.\end{aligned}\right\} \tag{5}$$

Thus, for instance, $R_I(0)$ denotes the large radial wave function of the ground state. From (4) it follows that R_∞, the asymptotic solution for a radial function R other than the ground state function, is given by

$$\left.\begin{aligned} R_\infty &= \lambda\sqrt{\frac{2}{\pi}}\cos(kr+\varepsilon_j);\\ \varepsilon_j &= -\frac{\pi}{2}(j+1)+\delta_j;\\ k &= \sqrt{ME}/\hbar;\end{aligned}\right\} \tag{6}$$

$$\lambda = \sqrt{\frac{M}{2\hbar^2 k}} = \sqrt{\frac{1}{\hbar v}}. \tag{7}$$

The factor $\lambda\sqrt{2/\pi}$ normalizes R_∞ in the energy scale [12]); v is the nucleon velocity in the laboratory system. The phase constants δ_j are fixed by the exact solution of (4). With the exception of δ_0, they are very small if $(\hbar^2/ME)^{1/2} >> \varkappa^{-1}$. On account of (6), the asymptotic expression for the complete large wave function may generally be written as

$$\Psi_{E\mu}(r\to\infty) = B(E,\mu;\vartheta,\varphi)\cdot\frac{1}{r}\cos(kr+\varepsilon_j),\ (\Psi\text{ large}). \tag{8a}$$

Furthermore, the small radial wave functions are related with the large functions by

$$\left.\begin{aligned}
A^1(j-1) &= \left(\frac{d}{dr}-\frac{j}{r}\right)R_I(j-1)\\
A^2(j+1) &= \left(\frac{d}{dr}+\frac{j+1}{r}\right)R_I(j+1)\\
C^1(j) &= (j+1)\left(\frac{d}{dr}+\frac{j}{r}\right)R_I(j)\\
C^2(j) &= j\left(\frac{d}{dr}-\frac{j+1}{r}\right)R_I(j)\\
H^1(j) &= -\left(\frac{d}{dr}+\frac{j}{r}\right)R_{II}(j)\\
H^2(j) &= \left(\frac{d}{dr}-\frac{j+1}{r}\right)R_{II}(j)
\end{aligned}\right\} \cdot \frac{i\hbar}{Mc}\cdot\frac{1}{\sqrt{2j+1}} \tag{9}$$

and the asymptotic expression of the complete small wave function is

$$\Psi_{E\mu}(r\to\infty) = B(E,\mu;\vartheta,\varphi)\cdot\frac{i}{r}\sin(kr+\varepsilon_j),\ (\Psi \text{ small}). \tag{8 b}$$

b) Interaction with electromagnetic radiation.

We now examine the effect of an irradiation of the deuteron with a monochromatic polarized γ-ray beam. In this case the deuteron wave equation is

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[\mathscr{H}_0 + (\Omega\, e^{-i\nu t} + \text{conj.})\right]\Psi,$$

where[13])

$$\Omega = -\left\{\vec{P}\,\vec{\mathscr{E}} + \vec{M}\,\vec{\mathscr{H}} + (Q\ \text{grad})\,\vec{\mathscr{E}}\right\}. \tag{10}$$

$\vec{\mathscr{E}}$ and $\vec{\mathscr{H}}$ are the electric and magnetic fields taken at the centre of gravity of the system. $\vec{P}$, $\vec{M}$ and Q are the operators of electric dipole moment, magnetic dipole moment and electric quadrupole moment, respectively. For a general nuclear system, explicit expressions of these quantities in an arbitrary frame of reference have been given in a previous paper[14]); for the deuteron, the indices (i) and (k) occurring there take the "values" (1) or (2). As in the present case $\vec{P}$, $\vec{M}$ and Q of course refer to the system in which the centre of gravity is at rest, we have furthermore to replace $\vec{x}^{(1)}$ by $\vec{x}/2$ and $\vec{x}^{(2)}$ by $-\vec{x}/2$. Consequently,

$$\vec{P} = -\frac{e}{4}\left(\tau_3^{(1)}-\tau_3^{(2)}\right)\vec{x} - \frac{e}{8\pi\hbar c}\cdot\frac{g_1 g_2}{\varkappa}\left(\boldsymbol{\tau}^{(1)}\wedge\boldsymbol{\tau}^{(2)}\right)_3\left(\vec{\sigma}^{(1)}+\vec{\sigma}^{(2)}\right)\wedge\vec{x}_0\cdot e^{-\varkappa r}, \tag{11}$$

$$\left.\begin{aligned}\vec{M} = \frac{e}{2c}\sum_{i=1,2}\frac{1-\tau_3^{(i)}}{2}\varrho_3^{(i)}\vec{x}^{(i)}\wedge\vec{v}^{(i)} + \mu_0\sum_{i=1,2}\frac{1-\tau_3^{(i)}}{2}\varrho_3^{(i)}\vec{\sigma}^{(i)} \\ +\left(\frac{g_2}{\varkappa}\right)^2\cdot\frac{1}{4\pi\hbar c}\cdot\frac{e}{4}\sum_{i,k=1,2}\left(\boldsymbol{\tau}^{(i)}\wedge\boldsymbol{\tau}^{(k)}\right)_3\left[\left(\vec{\sigma}^{(i)}\wedge\vec{\sigma}^{(k)}\right)\left(1-2\varkappa r^{(ik)}\right)\right. \\ \left.+\left\{\left(\vec{\sigma}^{(i)}\wedge\vec{\sigma}^{(k)}\right)\vec{x}_0^{(ik)}\right\}\vec{x}_0^{(ik)}\left(1+\varkappa r^{(ik)}\right)\right]\cdot\frac{e^{-\varkappa r^{(ik)}}}{r^{(ik)}},\end{aligned}\right\}\quad(12)$$

$$Q^{mn} = \frac{e}{16}\left\{2-\left(\tau_3^{(1)}+\tau_3^{(2)}\right)\right\}x^m x^n, \quad (13)$$

with

$$\vec{x}_0 = \frac{\vec{x}}{r}, \quad \vec{x}_0^{(ik)} = \frac{\vec{x}^{(i)}-\vec{x}^{(k)}}{r^{(ik)}}, \quad \vec{v}^{(i)} = \frac{\hbar}{Mi}\operatorname{grad}^{(i)}.$$

$\mu_0 = e\hbar/2Mc$ is the nuclear magneton. The first two terms of $\vec{M}$ denote the "orbital" and spin magnetic moment of the nucleons, respectively*. It should be noted that the contribution of the (static) meson fields to Q^{mn} vanishes in the centre of gravity system. The expression (10) for the interaction operator is sufficiently accurate if the γ-ray wave-length is large compared to the "radius" of the deuteron, a condition which is well fulfilled for the whole energy region of interest.

The differential cross-section for photo-disintegration by γ-rays with a fixed direction of polarization is in a general way given by**

$$d\Phi = \pi^2\bar{\varrho}\, v \sin\vartheta\, d\vartheta\, d\varphi, \quad (14)$$

where

$$\varrho = \left|\sum_{\mu} B(E,\mu;\vartheta,\varphi)\cdot(E,\mu\,|\,\Omega\,|\,o)\cdot e^{i\varepsilon_j}\right|^2 \quad (15)$$

and where $\bar{\varrho}$ is meant to be the average of ϱ over the magnetic substates of the deuteron ground state (which is indicated by 0 in the above formula). The axis of the polar coordinate system is supposed to be taken in the direction of polarization of the photons.

* Details of the separation of the nucleon magnetic moment into these two parts are given in *loc. cit.*[13]); a term which is proportional to the electromagnetic field and one which may be written as a time derivative have been omitted, both being irrelevant for present purposes. It should be noted that the sign of the constant g_2 adopted here is different from that in *loc. cit.*[13]).

** Cf. *loc. cit.*[15]) pp. 59—61. (14) is derived in a way quite similar to the treatment of the *PE* effect of the hydrogen atom by BETHE[16].

§ 3. Calculation of the cross-sections*.

In this paper, the photo-disintegration is treated up to the first order in the velocities, *i. e.* effects of the order v/c are taken into account. Therefore, those and only those matrix elements $(F|\Omega|0)$ will have to be considered which belong to one of the following three types

$$\int \overset{(0)}{\Psi}{}_F^{\dagger}\, \Omega\, \overset{(0)}{\Psi}_0, \quad \int \overset{(1)}{\Psi}{}_F^{\dagger}\, \Omega\, \overset{(0)}{\Psi}_0, \quad \int \overset{(0)}{\Psi}{}_F^{\dagger}\, \Omega\, \overset{(1)}{\Psi}_0,$$

the integral sign denoting integration over spatial coordinates and summation over spin coordinates as well.

a) The *PE* effect.

We shall now consider the transitions due to $\Omega_{\mathrm{el}} = -\vec{\mathcal{E}}\,\vec{P}$. Taking the x-axis as the direction of propagation of the photon beam, and the z-axis as the direction of its electric vector,

$$\Omega_{\mathrm{el}} = -E P_z.$$

The amplitude E of the fields of the wave is chosen such as to normalize the radiation to one polarized photon per sec. per cm^2 (Heaviside units are used throughout):

$$|E|^2 = \frac{\hbar\nu}{2c}. \tag{16}$$

Let us first consider

$$\Omega_{\mathrm{el}}^{\mathrm{nucl}} = -E P_z^{\mathrm{nucl}} = \frac{eE}{4}\left(\boldsymbol{\tau}_3^{(1)} - \boldsymbol{\tau}_3^{(2)}\right) z.$$

As the ground state is antisymmetric in the isotopic spins and

$$^3\zeta_0 \left(\boldsymbol{\tau}_3^{(1)} - \boldsymbol{\tau}_3^{(2)}\right) {}^1\zeta_0 = 2, \qquad {}^1\zeta_0 \left(\boldsymbol{\tau}_3^{(1)} - \boldsymbol{\tau}_3^{(2)}\right) {}^1\zeta_0 = 0,$$

the final state must be symmetric with respect to $\boldsymbol{\tau}_3^{(i)}$. Taking further into account the behaviour with regard to rotations and spatial reflections and the fact that the ground state is of the

* I should like to thank Prof. C. Møller for the communication of preliminary calculations on the photo-effect which have provided a valuable check of the calculations given here.

type I a with $l = 0$, $j = 1$, the following states are found to combine with the ground state (behind each state the spectroscopic symbol corresponding to the non-relativistic approximation is indicated):

$$\begin{aligned} &\mathrm{I\,a}, \; l = 1, \; j = 2 \; (^3P_2^-), \\ &\mathrm{I\,a}, \; l = 1, \; j = 0 \; (^3P_0^-), \\ &\mathrm{I\,b}, \; l = j = 1 \quad (^3P_1^-), \end{aligned}$$

while the familiar selection rule $\Delta m = 0$ holds. From (3) it is easily seen that, to the first order in the velocities, we have for all these transitions

$$\left(F \middle| \Omega_{\mathrm{el}}^{\mathrm{nucl}} \middle| 0\right) = \int \overset{(0)}{\Psi}{}_F^{\dagger} \, \Omega_{\mathrm{el}}^{\mathrm{nucl}} \, \overset{(0)}{\Psi}_0 . \tag{17}$$

The matrix elements are readily found to be

$$\begin{aligned} &\left(\mathrm{I\,a},\, l = 1,\, j = 2 \middle| \Omega_{\mathrm{el}}^{\mathrm{nucl}} \middle| 0\right) = \frac{eE}{2} J \cdot \begin{cases} \sqrt{6}/6 & 1 \to 1 \\ \sqrt{2}/3 & 0 \to 0 \\ \sqrt{6}/6 & -1 \to -1 \end{cases} \\ &\left(\mathrm{I\,a},\, l = 1,\, j = 0 \middle| \Omega_{\mathrm{el}}^{\mathrm{nucl}} \middle| 0\right) = -\frac{eE}{6} J \qquad\qquad 0 \to 0 \\ &\left(\mathrm{I\,b},\, j = 1 \middle| \Omega_{\mathrm{el}}^{\mathrm{nucl}} \middle| 0\right) = -\frac{eE}{2} J \cdot \begin{cases} \sqrt{6}/6 & 1 \to 1 \\ -\sqrt{6}/6 . & -1 \to -1 \end{cases} \end{aligned}$$

Behind each expression the corresponding magnetic transition has been indicated, while

$$J = \int_0^{\infty} dr \, R_I(0) \, R_I(1) \, r .$$

$\Omega_{\mathrm{el}}^{\mathrm{exch}}$, the second part of Ω_{el} which, according to (11), is given by

$$\Omega_{\mathrm{el}}^{\mathrm{exch}} = -E P_z^{\mathrm{exch}} = \frac{eE}{8\pi\hbar c} \cdot \frac{g_1 g_2}{\varkappa} \cdot \left(\boldsymbol{\tau}^{(1)} \wedge \boldsymbol{\tau}^{(2)}\right)_3 \left\{\left(\vec{\sigma}^{(1)} + \vec{\sigma}^{(2)}\right) \wedge \vec{x}_0\right\}_z e^{-\varkappa r}$$

does, in our approximation, not contribute to the *PE* effect. This will be shown in the Appendix.

With the help of (14) and (15), the differential cross-section now can directly be obtained. As the final states are all *P*-states having the same radial wave function, the factor $\exp(i\varepsilon_j)$ in (15) may be omitted. Using (7) and (2) and neglecting those terms in B that are proportional to $(v/c)^2$ we then get expressions which prove to be the same for the three possible magnetic states, so that they directly give the average value $\bar{\varrho}$. Denoting by θ, ψ the direction of the ejected neutron with respect to the direction of propagation of the incoming γ-ray beam, we obtain, after averaging over all directions of polarization (which gives a factor 1/2),

$$d\Phi^{\mathrm{el}}(\theta) = \frac{e^2 \nu}{32\, c} |J|^2 \cdot \sin^2\theta \cdot \sin\theta\, d\theta\, d\psi. \tag{18 a}$$

The total cross-section is

$$\Phi^{\mathrm{el}} = \frac{\pi e^2 \nu}{12\, c} |J|^2. \tag{18 b}$$

This result is identical in form with that obtained in the Bethe-Peierls theory for a spherical well potential. Deviations from this simple formula are at most to be expected in the second order with respect to the velocities.

b) The *PM* effect.

Again beginning with the nucleon terms we have, noting that the first term of (12) gives no contribution,

$$\Omega^{\mathrm{nucl}}_{\mathrm{magn}} = E\mu_0 \sum_i \frac{1-\tau_3^{(i)}}{2} \varrho_3^{(i)} \sigma_y^{(i)},$$

as the magnetic vector stands in the $-y$-direction. For transitions to states antisymmetric in the isotopic spins, this operator becomes $E\mu_0\left(\varrho_3^{(1)}\sigma_y^{(1)} + \varrho_3^{(2)}\sigma_y^{(2)}\right)/2$, while

$$\Omega^{\mathrm{nucl}}_{\mathrm{magn}} = -\frac{E\mu_0}{2}\left(\varrho_3^{(1)}\sigma_y^{(1)} - \varrho_3^{(2)}\sigma_y^{(3)}\right)$$

if the final state is symmetric in these coordinates. It is easily seen that the matrix elements vanish in the former case; as to the latter, the only combining state is IIb, $j = 0$ (^1S) with $\Delta m = \pm 1$. For $1 \rightarrow 0$ as well as for $-1 \rightarrow 0$, one finds

$$\left(\mathrm{II\,b},\ j = 0 \left| \Omega^{\mathrm{nucl}}_{\mathrm{magn}} \right| 0\right) = -\frac{i\mu_0 E\sqrt{2}}{2} \int_0^\infty dr\, R_I(0)\, R_{II}(0).$$

The extra magnetic moments of proton and neutron due to their proper (static) meson fields form part of the rest of (12), *viz.* those terms for which $i = k$*. They give of course an infinite contribution and can only be managed by using a cut-off prescription. The corresponding interaction is given by

$$\left.\begin{aligned}\Omega_{\text{magn}}^{\text{extra}} &= -\left(\frac{g_2}{\varkappa}\right)^2 \frac{1}{4\pi\hbar c}\cdot eE \lim_{\varrho=0}\sum_i \tau_3^{(i)} \Big\{\vec{\sigma}^{(i)}(1-2\varkappa\varrho) \\ &\quad + \left(\vec{\sigma}^{(i)}\vec{\varrho}_0\right)\vec{\varrho}_0(1+\varkappa\varrho)\Big\}_y \cdot \frac{e^{-\varkappa\varrho}}{\varrho}\end{aligned}\right\} \tag{19}$$

with $\varrho = |\vec{\varrho}|$ and $\vec{\varrho}_0 = \vec{\varrho}/\varrho$. Replacing $\left(\vec{\sigma}^{(i)}\vec{\varrho}_0\right)\varrho_{0y}$ by its directional average $\sigma_y^{(i)}/3$, as would seem appropriate if the nuclear point sources are considered as the limiting case of a spherically symmetrical distribution, and cutting off by putting

$$\frac{2}{3}\frac{g_2^2}{4\pi\hbar c}\frac{M}{M_m}\lim_{\varrho=0}\left(\frac{4}{\varkappa\varrho}-5\right)e^{-\varkappa\varrho} \tag{20 a}$$

equal to a finite quantity μ, gives

$$\Omega_{\text{magn}}^{\text{extra}} = -\mu_0\,\mu\,E\sum_i \tau_3^{(i)}\sigma_y^{(i)}.$$

μ is the "extra magnetic moment" in units μ_0. Experimentally, a slight dissymmetry between extra proton and neutron moment is found, the discussion of which, however, falls outside the scope of the present considerations[13]). Formally, we may account for it by writing instead of the last expression

$$\Omega_{\text{magn}}^{\text{extra}} = \mu_0 E\sum_i\left\{(\mu_P-1)\frac{1-\tau_3^{(i)}}{2} + \mu_N\frac{1+\tau_3^{(i)}}{2}\right\}\sigma_y^{(i)},$$

thus using the empirical value $\mu_P\,(\mu_N)$ of the magnetic proton (neutron) moment instead of μ. The introduction of a term proportional to $\left(\sigma_y^{(1)}+\sigma_y^{(2)}\right)$ which is involved in this change on account of $\mu_P+\mu_N-1 \neq 0$ is irrelevant, since it does not give rise to any transitions. The only allowed state is again the ^1S-state in which case the operator becomes

* Other terms of higher order which also contribute to these extra magnetic moments have to be discarded according to the prescription given in *loc. cit.*[13]).

$$-\frac{1}{2}\mu_0\left(\mu_P-\mu_N-1\right)E\left(\sigma_y^{(1)}-\sigma_y^{(2)}\right).$$

Consequently,

$$\left(\text{II b},\ j=0\left|\Omega_{\text{magn}}^{\text{extra}}\right|0\right)=-\frac{i\sqrt{2}}{2}\mu_0\left(\mu_P-\mu_N-1\right)E\int_0^\infty dr\,R_I(0)R_{II}(0), \tag{20b}$$

and the total contribution of the nucleons is

$$\left(\text{II b},\ j=0\left|\Omega_{\text{magn}}^{\text{nucl}}+\Omega_{\text{magn}}^{\text{extra}}\right|0\right)=-\frac{i\sqrt{2}}{2}\mu_0\left(\mu_P-\mu_N\right)E\int_0^\infty dr\,R_I(0)R_{II}(0). \tag{21}$$

Finally, the terms from the second part of (12) with $i \neq k$ must be considered. The corresponding interaction, $\Omega_{\text{magn}}^{\text{exch}}$, is

$$\Omega_{\text{magn}}^{\text{exch}}=\frac{eE}{2}\frac{g_2^2}{4\pi\hbar c}\left(\boldsymbol{\tau}^{(1)}\wedge\boldsymbol{\tau}^{(2)}\right)_3\left[\left(\vec{\sigma}^{(1)}\wedge\vec{\sigma}^{(2)}\right)_y\left(\frac{1}{\varkappa^2 r}-\frac{2}{\varkappa}\right)\right.$$
$$\left.+\left\{\left(\vec{\sigma}^{(1)}\wedge\vec{\sigma}^{(2)}\right)\vec{x}_0\right\}x_{0y}\left(\frac{1}{\varkappa^2 r}+\frac{1}{\varkappa}\right)\right]e^{-\varkappa r}.$$

As ${}^3\zeta_0\left(\boldsymbol{\tau}^{(1)}\wedge\boldsymbol{\tau}^{(2)}\right)_3{}^1\zeta_0=-2i$, ${}^1\zeta_0\left(\boldsymbol{\tau}^{(1)}\wedge\boldsymbol{\tau}^{(2)}\right)_3{}^1\zeta_0=0$, the final states must be symmetric in the isotopic spins. The only states combining with the ground state turn out to be

Type II b, $j=0$, $({}^1S)$,
Type II b, $j=2$, $({}^1D)$.

The matrix elements are found to be again of the type (17). They are

$$\left.\begin{aligned}&\left.\begin{matrix}1\to 0\\-1\to 0\end{matrix}\right\}\left(\text{II b},\ j=0\left|\Omega_{\text{magn}}^{\text{exch}}\right|0\right)\\&=\frac{2i\sqrt{2}}{3}\frac{g_2^2}{4\pi\hbar c}\frac{M}{M_m}\mu_0 E\int_0^\infty dr\,R_I(0)\,R_{II}(0)\left(\frac{4}{\varkappa r}-5\right)e^{-\varkappa r},\end{aligned}\right\} \tag{22}$$

$$\left.\begin{aligned}&\left.\begin{matrix}1\to 0\\-1\to 0\end{matrix}\right\}\left(\text{II b},\ j=2\left|\Omega_{\text{magn}}^{\text{exch}}\right|0\right)\\&=-\frac{2i\sqrt{10}}{15}\frac{g_2^2}{4\pi\hbar c}\frac{M}{M_m}\mu_0 E\int_0^\infty dr\,R_I(0)\,R_{II}(2)\left(\frac{1}{\varkappa r}+1\right)e^{-\varkappa r}.\end{aligned}\right\} \tag{23}$$

The obvious similarity in form of (20a, b) and (22) becomes clear if one remembers that, for an $S\to S$ transition, $\left\{\left(\vec{\sigma}^{(1)}\wedge\vec{\sigma}^{(2)}\right)\vec{x}_0\right\}x_{0y}$ should be replaced by its directional average which is $\left(\vec{\sigma}^{(1)}\wedge\vec{\sigma}^{(2)}\right)_y/3$.

Having verified that the contribution of (23) is very small, we have ignored this transition.

The differential cross-section is computed from (21) and (22) in the same way as (18 a) was found; averaging over the initial magnetic substates gives a factor 2/3. The result is

$$d\Phi^{\mathrm{magn}} = \frac{1}{12}\mu_0^2 \frac{\nu}{c} |K|^2 \cdot \sin\theta \, d\theta \, d\psi, \tag{24 a}$$

with

$$K = (\mu_P - \mu_N)\int_0^\infty dr R_I(0) R_{II}(0) - \frac{4}{3}\frac{g_2^2}{4\pi\hbar c}\frac{M}{M_m}\int_0^\infty dr R_I(0) R_{II}(0)\left(\frac{4}{\varkappa r} - 5\right) e^{-\varkappa r}.$$

Therefore,

$$\Phi^{\mathrm{magn}} = \frac{\pi}{3}\mu_0^2 \frac{\nu}{c} |\mathrm{K}|^2. \tag{24 b}$$

§ 4. Numerical evaluations.

Approximate expressions for the radial wave function of the ground state and the ^1S-state have been obtained by HULTHÉN[17, 18], assuming the nuclear potential to be of the form as determined by (4) and (5). They are:

$$R_I(0) = \sqrt{\varrho\varkappa}\, e^{-\beta\varkappa r}\left\{(1 - e^{-\varkappa r}) - c_1 (1 - e^{-\varkappa r})^2\right\}, \quad \beta = \frac{M}{M_m}\left(\frac{E_0}{Mc^2}\right)^{1/2}, \tag{25}$$

$$R_{II}(0) = \lambda \sqrt{\frac{2}{\pi}} \cdot \frac{1}{1 - c_2}\left\{(1 - e^{-\varkappa r}) - c_2 (1 - e^{-\varkappa r})^2\right\} \sin(kr + \delta). \tag{26}$$

ϱ is a normalizing factor and is given by loc. cit.[17]) equ. (35). The constants c_1 and c_2 are determined by variational methods. For nucleon energies not much larger than zero, $c_2 = 0{,}349$ while, taking $E_0 = 2{,}16\ MeV$,

α	$\varkappa$	β	ϱ	c_1	$g_2^2/4\pi\hbar c$
200	$0{,}52 \cdot 10^{13}$ cm.	0,442	4,66	0,370	0,065
300	$0{,}78 \cdot 10^{13}$ cm.	0,294	2,48	0,365	0,095

α denoting the ratio between meson and electron mass. g_2 and the phase δ in (26) depend upon the energy E_0' of the virtual ^1S-level, for which we have taken $E_0' = 5 \cdot 10^4\ eV$. With $\gamma = (ME_0')^{1/2}/\hbar = 3{,}48 \cdot 10^{11}\ \mathrm{cm}^{-1}$,

$$\cos\delta = \frac{\gamma}{\sqrt{\gamma^2 + k^2}}, \quad \sin\delta = \frac{k}{\sqrt{\gamma^2 + k^2}}.$$

The P-function is, assuming as usual the interaction between proton and neutron to be negligible in this state,

$$R_I(1) = \lambda \sqrt{\frac{2}{\pi}} \left(-\cos kr + \frac{\sin kr}{kr}\right).$$

Inserting all this in (18 b) and (24 b) we get (it should be borne in mind that e is expressed in Heaviside units)

$$\Phi^{\mathrm{el}} = \frac{\varrho}{3} \cdot \frac{e^2}{\hbar c} \cdot \left(\frac{Mc}{\hbar}\right)^2 \cdot \frac{\hbar \nu}{Mc^2} \cdot \frac{\varkappa}{k} \cdot \frac{1}{k^4} \left| (1-c_1) P_0^2 + (2 c_1 - 1) P_1^2 - c_1 P_2^2 \right|^2 \tag{27}$$

$$\left.\begin{aligned} \Phi^{\mathrm{magn}} = \frac{\varrho}{12} \cdot \frac{1}{(1-c_2)^2} \cdot \frac{e^2}{\hbar c} \cdot \frac{\hbar \nu}{Mc^2} \cdot \frac{\varkappa}{k} \cdot \frac{1}{k^2} \cdot \frac{\gamma^2}{\gamma^2 + k^2} \Big| (\mu_P - \mu_N) F(X) \\ + \frac{16}{3} \cdot \frac{g_2^2}{4 \pi \hbar c} \cdot \frac{M}{M_m} \cdot F(Y) \Big|^2 \end{aligned}\right\} \tag{28}$$

where the following abbreviations have been used:

$$P_n = \left\{1 + (\beta + n)^2 \left(\frac{\varkappa}{k}\right)^2\right\}^{-1},$$

$$\begin{aligned} F(X) = (1 - c_1)(1 - c_2) X_0 - (4 c_1 c_2 - 3 c_1 - 3 c_2 + 2) X_1 \\ + (6 c_1 c_2 - 3 c_1 - 3 c_2 + 1) X_2 - (4 c_1 c_2 - c_1 - c_2) X_3 + c_1 c_2 X_4, \end{aligned}$$

with

$$X_n = \left\{1 + (\beta + n) \frac{\varkappa}{\gamma}\right\} P_n.$$

$F(Y)$ is understood to be obtained from $F(X)$ by replacing X_n by Y_n, where

$$Y_n = \frac{k}{\varkappa} \left[\operatorname{arc\,tg}\left\{(\beta + n) \frac{\varkappa}{k}\right\} + \frac{k}{2\gamma} \ln\left\{(\beta + n)^2 + \frac{k^2}{\varkappa^2}\right\}\right] + \frac{5}{4} X_{n+1}.$$

The PE cross-section decreases with increasing $\varkappa$, as is shown by the following table:

$\hbar\nu$ in MeV	Φ^{el} in 10^{-28} cm² $\alpha = 200$	$\alpha = 300$
2,64	15	12
4	34	23
6,2	24	18
10	16	11
17	9	5,5

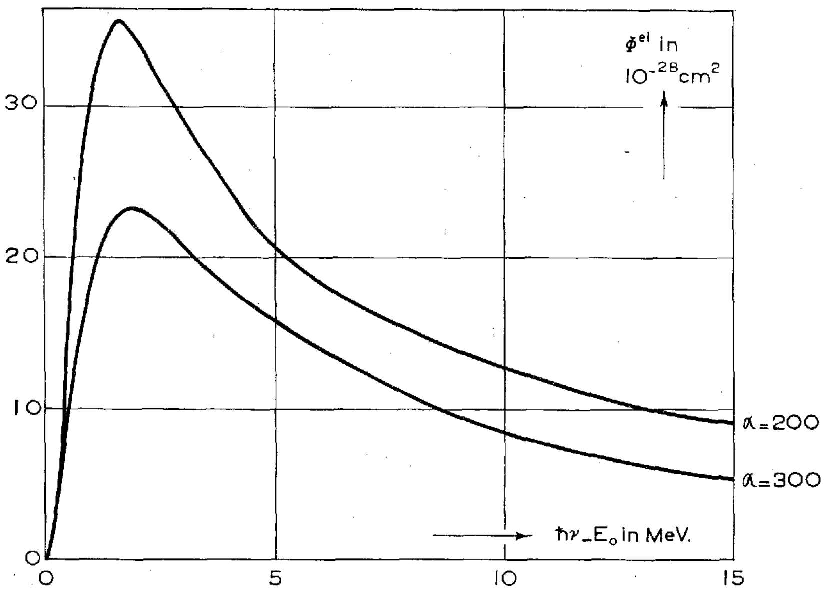
Φ^{el} in $10^{-28}cm^2$
30
20
10
0
0
5
10
15
$\alpha = 200$
$\alpha = 300$
$h\nu - E_0$ in MeV.

Fig. 1.

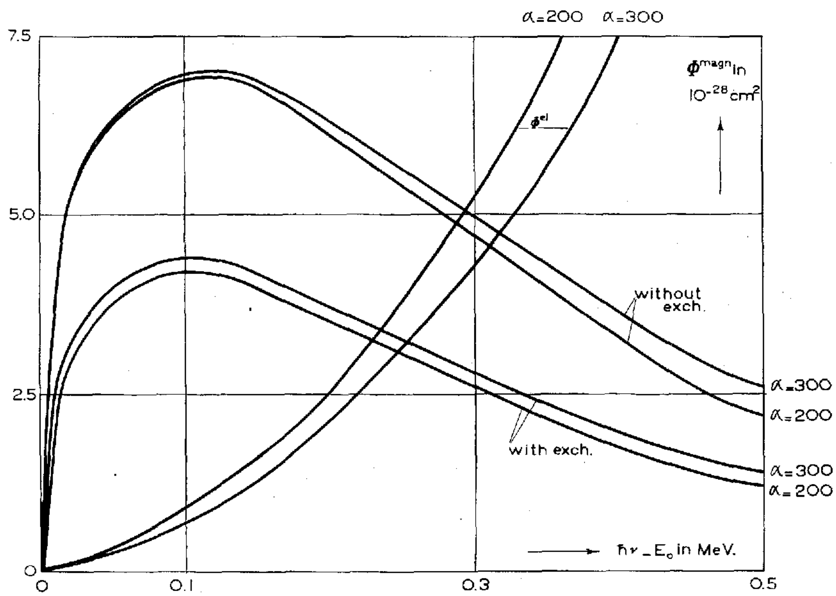
$\alpha = 200$ $\alpha = 300$
Φ^{magn} in $10^{-28}cm^2$
Φ^{el}
7.5
5.0
2.5
0
0
0.1
0.3
0.5
without exch.
with exch.
$\alpha = 300$
$\alpha = 200$
$\alpha = 300$
$\alpha = 200$
$h\nu - E_0$ in MeV.

Fig. 2.

In Fig. 1, Φ^{el} has been plotted against $\hbar\nu - E_0 = 2E$. From the considerations in the Appendix, Note 1, it will become clear that one should be careful in drawing quantitative conclusions from this graph.

The magnetic cross-section is given in Fig. 2 (curves marked "with exch."); we have taken $\mu_P = 2{,}78$, $\mu_N = -1{,}93$. In particular, one has for the ThC″ γ-rays, $\hbar\nu = 2{,}64\,MeV$:

$$\Phi^{\text{magn}} = \begin{array}{l} 1{,}3 \cdot 10^{-28} \text{ for } \alpha = 200, \\ 1{,}5 \cdot 10^{-28} \text{ for } \alpha = 300. \end{array}$$

It follows that for this energy

$$\frac{\Phi_{\parallel}}{\Phi_{\perp}} = \begin{array}{l} 0{,}055, \quad \alpha = 200, \\ 0{,}075, \quad \alpha = 300 \end{array}$$

which shows that the present theory gives a good account of the angular distribution. In order to understand better the origin of the difference between this and the "old" theory, we have also computed the value of $\Phi_{\parallel}/\Phi_{\perp}$ which would be obtained by omitting the term due to the meson fields in (28); for doing so, Φ^{magn}, too, would have the same general form as in the old theory (though it should be remembered that other expressions for the radial wave-functions are used). The curves of Fig. 2 marked "without exch." refer to Φ^{magn} as calculated by omitting $F(Y)$. We then find, for the ThC″ γ-rays, $\Phi^{\text{magn}} = 2{,}4 \cdot 10^{-28}$ for $\alpha = 200$, and $2{,}8 \cdot 10^{-28}$ for $\alpha = 300$, which would lead to $10\,\%$ ($\alpha = 200$) and $13\,\%$ ($\alpha = 300$) for $\Phi_{\parallel}/\Phi_{\perp}$. Thus, the present result is essentially due to the existence of meson exchange currents*.

One can, therefore, not expect to get satisfactory values for (1) by means of a "neutral" theory, as this does not exhibit exchange phenomena, notwithstanding the influence on the angular distribution of a strong directional coupling which may be inherent in such a theory, as is *e. g.* the case in the theory put forward by Bethe[21]). As a matter of fact, it can be seen from the form of the radial wave-functions of the states involved

* A discussion of the obtained values in connection with the magnitude of Φ^{el} is given in the Appendix, Note 1.

in the transitions due to this coupling that the angular distribution will not change appreciably for energies not much larger than E_0.

From Fig. 2 it appears that with increasing energy Φ^{magn} initially increases much more rapidly than Φ^{el}. Indeed, it can be seen from (27) and (28) that for $k << \kappa$, $\Phi^{\mathrm{el}} \backsim k^3$ and thus increases like $E^{3/2}$, while $\Phi^{\mathrm{magn}} \backsim k(\gamma^2+k^2)^{-1}$; from the latter result one infers, moreover, that Φ^{magn} attains its maximum at the "resonance value" $k \cong \gamma$, corresponding to $\hbar\nu - E_0 =$ 0,1 *MeV*, *i. e.* twice the value of the energy of the ^1S-level. The angular distribution just above the threshold should apparently be nearly isotropic.

In a recent paper*, MYERS and VAN ATTA[27]) report the results of photo-disintegration experiments in which X-rays are used with energies ranging from 0—0,25 *MeV* above the threshold; the major part of the intensity lies within 0,1 *MeV* of this limiting energy (for which they find $2{,}183 \pm 0{,}012$ *MeV*). The ratio of the intensities at 90° and 0° appears to be $1{,}15 \pm 0{,}10$. We should like to point out that no comparison with theoretical results is possible without a detailed knowledge of the X-ray spectrum employed: writing the intensity ratio under consideration as $1+x$, it is seen that, in the energy region $\hbar\nu - E_0 = 0 - 0{,}1$ *MeV*, x increases proportionally to E and, thus, varies rapidly for the energies concerned. For reference, we give the values of x at 0,1 *MeV*: 0,31 (0,19) for $\alpha = 200$ and 0,23 (0,15) for $\alpha = 300$ (the values in brackets are obtained if the contributions of the meson exchange currents are omitted).

Calculations on the magnetic effect have mostly been performed for vanishing range of the nuclear forces[5, 7]). For comparison, we shall give the numerical result in this limiting case on the present theory. It is easily seen that $F(Y) = 0$ for $\varkappa^{-1} = 0$. As $g_2^2\varkappa^{-1}$ practically does not depend on $\varkappa$[9]), the exchange effect therefore vanishes for zero range. Furthermore, only X_0 is now different from zero, this quantity being independent of $\varkappa$, while $\varrho\varkappa$ in the limit tends to a finite value. Φ^{magn} then turns out to be $3{,}2 \cdot 10^{-28}$, as compared with $3{,}3 \cdot 10^{-28}$ found by RARITA and SCHWINGER[7]). It is to be noted that the dependence

* I have not been able to see this article myself; an abstract of its contents has kindly been communicated to me by dr. L. HULTHÉN.

of Φ^{magn} on range is different from that of Φ^{el}. To this point we shall come back in § 5.

As to the absolute value of the photo-disintegration cross-section, there is reasonable agreement with the measured values, *viz.* $5 \cdot 10^{-28}$ (CHADWICK and GOLDHABER[1])) and $9 \cdot 10^{-28}\,\text{cm}^2$ (v. HALBAN[6])) for $\hbar\nu = 2{,}64$, and $11{,}6 \cdot 10^{-28}\,\text{cm}^2$ for $6{,}2\,MeV$ (ALLEN and SMITH[19])); cf. especially the Appendix, Note 1. The cross-section reaches a maximum at about $4\,MeV$ and then decreases rapidly. In fact, it is easily seen from (27) and (28) that for very large energies both Φ^{el} and Φ^{magn} decrease as $\nu^{-7/2}$, *i. e.* more rapidly than in the old theory ($\sim \nu^{-3/2}$).

Theories which, in contrast to the mixed theory, involve a strong directional coupling of the dipole interaction type give rise to an angular distribution of a quite different kind. This has been calculated by RARITA and SCHWINGER[20]) for the $Li + H$ γ-ray energy ($17{,}5\,MeV$) for which in all theories the magnetic effects are negligible. It is seen that the "symmetrical" theory gives a total cross-section of $3{,}8 \cdot 10^{-28}$ and an angular distribution such that $\Phi_{\parallel}/\Phi_{\perp} = 0{,}01$ while, in the "neutral" theory[21]) these quantities are $7{,}7 \cdot 10^{-28}$ and $0{,}27$, respectively. Here, the emission in the forward direction is due to electric transitions induced by the non-central forces which lead to a ${}^3D$ contribution to the ground state. While the total cross-sections are seen not to differ much from the value given here on mixed theory, the angular distributions cannot be directly compared with (18a), as in our approximation the non-central forces do not come into consideration. Indeed, it is an essential feature of the mixed theory, distinguishing it from all other current meson theories, that the tensor interaction responsible for the S-D coupling is of non-static nature. Thus, the matrix elements of the corresponding transitions are of higher order in v/c (which for $\hbar\nu = 17{,}5\,MeV$ is $\sim 0{,}1$) compared with those given here, so that their contribution, even for this energy, will be relatively small; therefore $\Phi_{\parallel}/\Phi_{\perp}$ would, according to the mixed theory, seem to be of the same order as in the symmetrical theory with directional coupling and would at any rate be much smaller than in the neutral theory.

In the foregoing, it has been tacitly assumed that the centre of gravity system of the deuteron may be identified with the actual system of measurement. In the high energy region a correction is necessary, however, as here the photon momentum may not be neglected.

§ 5. Capture of neutrons by protons.

Although the old theory could not account for the *PM* disintegration, the agreement of the theoretically found capture cross-section for thermal neutrons with experiment was satisfactory. As the latter process is entirely of magnetic origin in the energy region concerned and its probability is intimately connected with Φ^{magn}, it might be feared that the change of magnitude of Φ^{magn} as compared with the old theory would affect the capture cross-section in an unfavourable way. However, this is not the case, due to the circumstance that, for thermal energies, the influence of the exchange terms is considerably less than for the energies of interest in the discussion of the photo-effect.

The cross-section for this process can immediately be inferred from (27) and (28). We have in fact, calling the cross-sections for "electric" and "magnetic" capture Φ_c^{el} and Φ_c^{magn},

$$\Phi_c^{\mathrm{el}} = \frac{9}{2}\left(\frac{\nu}{kc}\right)^2 \Phi^{\mathrm{el}}, \qquad \Phi_c^{\mathrm{magn}} = \frac{3}{2}\left(\frac{\nu}{kc}\right)^2 \Phi^{\mathrm{magn}}.$$

We are especially interested in the behaviour of these expressions in the region of thermal neutron energies; in this case, $\gamma^2 + k^2$ may be replaced by γ^2. It is seen that for these energies $\Phi_c^{\mathrm{el}} \backsim k$ and, thus, may be neglected compared with Φ_c^{magn} which is $\backsim k^{-1}$. For very small k

$$\Phi_c^{\mathrm{magn}} = \frac{\varrho}{8} \cdot \frac{1}{(1-c_2)^2} \cdot \frac{e^2}{\hbar c} \cdot \left(\frac{\hbar\nu}{Mc^2}\right)^3 \cdot \left(\frac{Mc}{\hbar}\right)^2 \cdot \left(\frac{k}{\varkappa}\right)^3 \cdot \frac{1}{k^4} \cdot$$

$$\left| (\mu_P - \mu_N)\, F(\bar{X}) + \frac{16}{3} \cdot \frac{g_2^2}{4\pi\hbar c} \cdot \frac{M}{M_m}\, F(\bar{Y}) \right|^2$$

with

$$\bar{X}_n = \frac{1}{(\beta+n)^2}\left\{1 + (\beta+n)\frac{\varkappa}{\gamma}\right\}$$

$$\bar{Y}_n = \frac{\varkappa}{\gamma}\ln(\beta+n) + \frac{5}{4}\,\bar{X}_{n+1}.$$

(The term of $\bar{Y}_n$ containing an *arc tg* need not be written down, as these various terms cancel each other). For $\hbar\nu$ one may take E_0. The following numerical results have been obtained:

v in cm/sec.	Φ_c^{magn} in 10^{-24} cm²			
	$\alpha = 200$	$\alpha = 300$	"Old" th.	Exp.
$2{,}2 \cdot 10^5$	0,23 (0,39)	0,26 (0,39)	0,35	0,27 [22])
$2{,}5 \cdot 10^5$	0,20 (0,34)	0,23 (0,34)	0,31	0,31 [23]).

The values in brackets are obtained by omitting the exchange term. In the last column but one, the values according to the old theory are given; cf. *loc. cit.*[12]), equ. (95). The agreement with experiment is satisfactory.

We have considered in some detail the dependence of this effect on $\varkappa$. Just as for *PM* disintegration, it appears that for $\varkappa^{-1} = 0$ the contribution due to the exchange currents vanishes and that only $\bar{X}_0$ differs from zero. For $v = 2{,}2 \cdot 10^5$ cm/sec., $\Phi_c^{magn} = 0{,}39 \cdot 10^{-24}$ in this case. Further, by disregarding the exchange effect, one obtains the range dependence due to the form of the radial wave-functions and, thus, to the Yukawa potential employed in the present calculations. It then appears that, for small values of $\varkappa^{-1}$, Φ_c^{magn} is practically constant* and then decreases very slowly. As, for very small $\varkappa$, $\varrho \backsim \varkappa^{-3}$, it can be seen from the analytical expression of the quantity considered that for $\varkappa \rightarrow 0$ it tends to a finite value differing from zero.

* This has also been found for the case of a Morse potential[25]). However, in the present case, calculations up to $20 \cdot 10^{-13}$ cm. show a steady decrease of the capture cross-section, whereas in *loc. cit.* a sharp increase is found at $6 \cdot 10^{-13}$ cm.

Appendix.

Note 1. On the PE effect. This has been calculated using the operator $-\vec{\mathscr{E}}\vec{P}$. But as

$$\vec{\mathfrak{A}}\dot{\vec{P}} = \vec{\mathscr{E}}\vec{P} + \frac{d}{c\,dt}\left(\vec{\mathfrak{A}}\vec{P}\right)$$

and as the second term on the right has vanishing matrix elements for the transitions concerned, because of energy conservation, $-\vec{\mathfrak{A}}\dot{\vec{P}} = \bar{\Omega}_{\mathrm{el}}$ may be taken just as well. This will be done here; in the centre of gravity system[13])

$$\dot{\vec{P}} = \frac{e}{2}\sum_i \left(1-\tau_3^{(i)}\right)\vec{\alpha}^{(i)} + \frac{e}{4\pi\hbar c}\left(\boldsymbol{\tau}^{(1)}\wedge\boldsymbol{\tau}^{(2)}\right)_3\left\{g_1^2+g_2^2\left(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)}\right)\right\}\vec{x}_0\cdot e^{-\varkappa r}. \quad (29)$$

The first term gives

$$-\frac{Ae}{2}\left[\left(1-\tau_3^{(1)}\right)\alpha_z^{(1)} + \left(1-\tau_3^{(2)}\right)\alpha_z^{(2)}\right],$$

where $A = -icE\nu^{-1}$ is the amplitude of the vector potential. With the help of (9), the matrix elements are found to be

$$\left.\begin{aligned}
&\left(\mathrm{I\,a},\ l=1,\ j=2\,\middle|\,\bar{\Omega}^1_{\mathrm{el}}\,\middle|\,0\right) = -\frac{ieA\hbar}{Mc}J_1\cdot\begin{cases}\sqrt{6}/6 & 1\to 1\\ \sqrt{2}/3 & 0\to 0\\ \sqrt{6}/6 & -1\to -1\end{cases}\\
&\left(\mathrm{I\,a},\ l=1,\ j=0\,\middle|\,\bar{\Omega}^1_{\mathrm{el}}\,\middle|\,0\right) = \frac{ieA\hbar}{3\,Mc}J_1 \qquad\qquad 0\to 0\\
&\left(\mathrm{I\,b},\qquad\ j=1\,\middle|\,\bar{\Omega}^1_{\mathrm{el}}\,\middle|\,0\right) = \frac{ieA\hbar}{Mc}J_1\cdot\begin{cases}\sqrt{6}/6 & 1\to 1\\ -\sqrt{6}/6 & -1\to -1\end{cases}
\end{aligned}\right\}\quad (30)$$

with

$$J_1 = \int_0^\infty dr\,R_I(1)\left(\frac{d}{dr}-\frac{1}{r}\right)R_I(0).$$

The second term of (29) is treated in the same way; the matrix elements are obtained from (30) by replacing

$$\frac{\hbar}{Mc} J_1 \quad \text{by} \quad -\frac{g_1^2+g_2^2}{2\pi\hbar c} J_2,$$

where

$$J_2 = \int_0^\infty dr\, e^{-\varkappa r} R_I(1)\, R_I(0).$$

Writing down the equations for $R_I(0)$ and $R_I(1)$ in accordance with (4) and (5) and multiplying the first with $R_I(1)r$, the second with $R_I(0)r$, it is after subtraction and partial integration easy to see that

$$\frac{\nu}{2c} J = -\frac{\hbar}{Mc} J_1 + \frac{g_1^2+g_2^2}{2\pi\hbar c} J_2.$$

For J, cf. p. 13. Therefore,

$$\left(F\middle|\bar{\Omega}_{\text{el}}\middle|0\right) = -\left(F\middle|\vec{\mathcal{E}}\vec{P}_{\text{nucl}}\middle|0\right) \tag{31}$$

and the following expression for Φ^{el} is equivalent with (18 b):

$$\Phi^{\text{el}} = \frac{\pi e^2 c}{3\nu}\left|\frac{\hbar}{Mc} J_1 - \frac{g_1^2+g_2^2}{2\pi\hbar c} J_2\right|^2. \tag{32}$$

It can also be seen more directly that the matrix elements corresponding to P_{exch} vanish in our approximation; for these are all proportional to

$$\frac{\nu}{\varkappa c} \cdot A \cdot \frac{g_1 g_2}{\hbar c} J_2$$

and thus are obviously of higher order in the velocities than the matrix elements corresponding to the second term of (29).

As (18 b) and (32), of course, only would give identical numerical results if exact explicit expressions for the radial wave-functions are used, this provides a check as to the reliability of the approximate expressions for these functions proposed by various authors. Previously[15]), we had employed for $R_I(0)$ Wilson's result[25]) and it appeared that then the cross-section obtained from (32) for $\hbar\nu = 2{,}64\ MeV$ is 4,5 times the corresponding quantity obtained from (18 b), if $M_m = 0{,}1\ M$; it is thus quite impossible in this case to predict anything with regard to such a sensitive effect as the angular distribution. Using (25), this ratio becomes 0,7 for the same energy.

In order to understand better the origin of these discrepancies we have computed the ratio of the two expressions for Φ^{el} at the photo-electric threshold: $\hbar\nu = E_0$. Here, the Bessel-function representation for $R_I(1)$ is certainly accurate so that, in this limiting case, the remaining deviation should be entirely ascribed to the ground state function. The threshold ratio is found to be 1,5 for WILSON's function and 0,94 if (25) is used. HULTHÉN's expression is therefore a much better approximation. The larger deviations for energies greater than E_0 will be partly due to the inaccuracy of $R_I(1)$; in this connection, it should be remarked that the values of the matrix elements may be very sensitive even for small changes in the radial functions.

From the above it seems reasonable to assume that the values for Φ^{el} given in § 4 are too large for energies not much greater than E_0. The agreement of the theoretical values for the cross-section with experiment will therefore be better if more accurate approximations for the wave-functions are used, while the results with regard to $\Phi_{\|}/\Phi_{\perp}$ would remain satisfactory. In fact, it follows from the results stated for 2,64 MeV that this quantity would lie between 5,5 % and 8 % for $\alpha = 200$ and between 7,5 % and 10,5 % for $\alpha = 300$. As to the meson rest mass, one might infer from this result that $\alpha = 200$ is a more probable value than $\alpha = 300$.

Finally, it should be noted that (31) also holds in a pure vector or pure pseudoscalar meson theory, provided the dipole interaction potential (including cut-off) may be considered a perturbation and the contribution of the ^3D wave-function to the ground state may be disregarded.

Note 2. The electric quadrupole effect. In accordance with (10) and (13) and remembering the assumption on the direction of propagation and polarization of the γ-rays, the operator of the quadrupole transitions may be written as

$$\Omega_{\text{quadr}} = -\frac{i\nu eE}{16\,c}\left[2-\left(\tau_3^{(1)}+\tau_3^{(2)}\right)\right]xz.$$

The only allowed transitions are to D-states that are anti-symmetric in isotopic spin:

$$\begin{aligned}
&\mathrm{I\,a},\ l = 2,\ j = 3\ \ ({}^3D_3)\\
&\mathrm{I\,a},\ l = 2,\ j = 1\ \ ({}^3D_1)\\
&\mathrm{I\,b},\ l = j = 2\ \ \ \ ({}^3D_2).
\end{aligned}$$

There is therefore no interference with the electric or magnetic dipole transitions. For these states, $\Omega_{\text{quadr}} = -i\nu e E x z/8c$ and

$$(\mathrm{I\,a},\ l = 2,\ j = 3\,|\,\Omega_{\text{quadr}}\,|\,0) = -\frac{i\nu eE}{60\,c}\cdot G\cdot\begin{cases}\sqrt{5}/2 & 1\to 2\\ -\sqrt{6}/4 & 1\to 0\\ 1 & 0\to 1\\ -1 & 0\to -1\\ \sqrt{6}/4 & -1\to 0\\ -\sqrt{5}/2 & -1\to -2\end{cases}$$

$$(\mathrm{I\,a},\ l = 2,\ j = 1\,|\,\Omega_{\text{quadr}}\,|\,0) = \frac{i\nu eE}{80\,c}\cdot G\cdot\begin{cases}1 & 1\to 0\\ 1 & 0\to 1\\ -1 & 0\to -1\\ -1 & -1\to 0\end{cases}$$

$$(\mathrm{I\,b},\ l = j = 2\,|\,\Omega_{\text{quadr}}\,|\,0) = \frac{i\nu eE}{240\,c}\cdot G\cdot\begin{cases}\sqrt{10} & 1\to 2\\ -\sqrt{15} & 1\to 0\\ -\sqrt{5} & 0\to 1\\ -\sqrt{5} & 0\to -1\\ -\sqrt{15} & -1\to 0\\ \sqrt{10} & -1\to -2\end{cases}$$

with

$$G = \int_0^\infty dr\,R_I(2)\,R_I(0)\,r^2.$$

The contributions of the different magnetic substates of the ground state appear to be equal, as in the electric dipole case. Averaging over directions of polarization gives a factor 1/2. The result is

$$d\,\Phi_{\text{quadr}} = \frac{e^2}{512}\cdot\left(\frac{\nu}{c}\right)^3 |\,G\,|^2 \sin^2\theta\cos^2\theta\cdot\sin\theta\,d\theta\,d\psi,$$

$$\Phi_{\text{quadr}} = \frac{\pi e^2}{960}\left(\frac{\nu}{c}\right)^3 |\,G\,|^2.$$

This expression would also have been obtained in the old theory; the electric dipole and quadrupole cross-sections thus both have the same general form as in the old theory.

For $R_I(2)$ we have to take $\lambda (kr)^{1/2} J_{5/2}(kr)$. Estimations show Φ_{quadr} to be at most of the order of 10^{-29} cm^2; as this effect has, furthermore, no bearing on the quantity (1), these transitions are of no practical interest. The electric quadrupole capture is of course negligible, since a jump of l from 2 to 0 would be involved, which is highly improbable for small energies[26]).

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