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ON THE ORIGINAL ORBITS OF COMETS

1925 I, 1902 III AND 1897 I

BY

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This paper gives the results of calculations concerning the original orbits of comets 1925 I, 1902 III and 1897 I.

The first of these comets — comet 1925 I Orkisz — is one for which the basis of the calculation of the original orbit is excellent, the period of observation being more than one year, and the number of observations close to 600. The definitive orbit has been determined by ORKISZ, who took account of the perturbations by Mercury, Venus, the Earth, Mars, Jupiter and Saturn. The agreement between observation and theory is exceptionally good, the mean errors of the resulting elements are consequently quite small. While the orbit determined by ORKISZ is hyperbolic, the calculated original orbit is found to be elliptical.

For the second comet — comet 1902 III Perrine-Borrelly — the original orbit has already been investigated (cf. Publ. Copenhagen Observatory No. 19). The resulting original orbit was slightly hyperbolic, the mean error of $\frac{1}{a}$ being so large, however, compared with the numerical value of $\frac{1}{a}$, that the hyperbolicity was illusory. An investigation by Mr. RASMUSEN showed that the comet a few years before the passage through perihelion had been rather close to the planet Neptune. It was quite possible, therefore, that a calculation of the original orbit, when account was taken of the perturbations by Neptune also, might to an appreciable extent modify the result of the previous investigation. The calculation was made, and led to the result that the original orbit was slightly elliptical.

The third of the comets mentioned, namely comet 1897 I Perrine, has also been investigated previously (Publ. Cop. Obs. No. 19). The calculation of the definitive orbit (loc. cit. p. 52) gave an hyperbolic value of $\frac{1}{a}$. The calculation, with the aid of Encke's method, of the original orbit gave for 1891 Feb. 20.0 Berlin M. T. an elliptical orbit $\left(\frac{1}{a} = +0.0000367\right)$. A continuation of the calculation still further back in time with the aid of equation (21) in Publ. 19 gave for the beginning of the year 1883 a quite insignificant change of this value $\left(\frac{1}{a} = +0.0000368\right)$, but the numbers given on p. 54 in Publ. 19 show that this value would have been changed a little — probably by 10 to 20 units of the 7th decimal — if the calculation had been continued some years further. In order to have, for one particular comet, a complete check on the method of Publ. 19 just referred to we have carried out a calculation of the original orbit according to the method of direct integration of rectangular co-ordinates, going backwards from 1891 Feb. 20.0 as far as 1880 March 9.0. For this epoch the resulting value of $\frac{1}{a}$ agreed within 28 units of the 7th decimal with the value found in Publ. 19.

The results obtained for the three comets are given in more detail on the following pages.

1. Comet 1925 I Orkisz.

Comet 1925 I Orkisz was observed during a period of 402 days, the number of observations amounted to 598. The following definitive elements were calculated by L. Orkisz[1]. Perturbations by Mercury, Venus, the Earth, Mars, Jupiter and Saturn were taken into account in the calculation. The perturbations by Uranus and Neptune were negligible.

Epoch of osculation 1925 Oct. 25.0 Greenwich M.T.

$$T = 1925 \text{ April } 1.5111585 \text{ G.M.T.}$$

$$\left.\begin{aligned} \omega &= 36^\circ\ 11'\ 40''.1 \\ \Omega &= 318\quad 3\quad 41.5 \\ i &= 100\quad 0\quad 57.0 \end{aligned}\right\} 1925.0$$

$$q = 1.1095947 \pm 0.0000118$$

$$e = 1.0006286 \pm 0.0000344$$

Elements and equatorial constants reduced to the standard equinox 1950.0 are:—

Epoch of osculation 1925 Oct. 25.0 G.M.T.

$$T = 1925 \text{ April } 1.5111585 \text{ G.M.T.}$$

$$\left.\begin{aligned} \omega &= 36^\circ\ 11'\ 33''.1 \\ \Omega &= 318\quad 24\quad 36.9 \\ i &= 100\quad 1\quad 6.5 \end{aligned}\right\} 1950.0$$

$$q = 1.1095947 \pm 0.0000118$$

$$e = 1.0006286 \pm 0.0000344$$

$$\frac{1}{a} = -0.0005665 \pm 0.0000310$$

$$\left.\begin{aligned} P_x &= +0.5354076 & Q_x &= -0.5348396 \\ P_y &= -0.7933299 & Q_y &= -0.0529366 \\ P_z &= +0.2897696 & Q_z &= +0.8432938 \end{aligned}\right\} 1950.0$$

From these the following equatorial co-ordinates and

[1] L. Orkisz, Die definitive Bahn des Kometen 1925 I (Orkisz), Warschau 1931.

velocity components for the epoch of osculation 1925 Oct. 25.0 are found:—

$$x = -2.0703958 \qquad 10\frac{dx}{dt} = -0.1033548$$

$$y = +0.5481689 \qquad 10\frac{dy}{dt} = +0.0833989$$

$$z = +2.2554015 \qquad 10\frac{dz}{dt} = +0.0375121$$

Basing on these co-ordinates and velocity components the motion of the comet has been followed backwards through a period of 20 years with the aid of the method of direct integration of rectangular co-ordinates. The attractions by the sun and the 8 major planets were taken into account. The following table gives the computed co-ordinates referred to the equinox 1950.0.

Equatorial Co-ordinates. Equinox 1950.0.

G. M. T.	x	y	z
1925 Nov. 6.0	— 2.192989	+ 0.647853	+ 2.298875
Oct. 27.0	2.091026	0.564838	2.262860
— 17.0	1.987050	0.481280	2.224665
— 7.0	1.880942	0.397207	2.184080
Sept. 27.0	1.772574	0.312659	2.140870
— 17.0	1.661808	0.227690	2.094760
— 7.0	1.548498	0.142375	2.045440
Aug. 28.0	1.432485	+ 0.056816	1.992546
— 18.0	1.313606	— 0.028853	1.935660
— 8.0	1.191692	0.114449	1.874295
July 29.0	1.066571	0.199730	1.807879
— 19.0	0.938087	0.284371	1.735747
— 9.0	0.806102	0.367937	1.657120
June 29.0	0.670529	0.449846	1.571092
— 19.0	0.531367	0.529317	1.476617
— 9.0	0.388764	0.605313	1.372514
May 30.0	— 0.243099	— 0.676461	+ 1.257493

G. M. T.	x	y	z
1925 May 20.0	— 0.095109	— 0.740983	+ 1.130235
— 10.0	+ 0.053957	0.796652	0.989563
April 30.0	0.202182	0.840835	0.834714
— 20.0	0.346906	0.870708	0.665745
— 10.0	0.484857	0.883698	0.483933
March 31.0	0.612586	0.878103	0.291979
— 21.0	0.727152	0.853631	+ 0.093741
— 11.0	0.826734	0.811533	— 0.106480
— 1.0	0.910877	0.754226	0.304786
Febr. 19.0	0.980280	0.684658	0.498235
— 9.0	1.036360	0.605736	0.684978
Jan. 30.0	1.080822	0.519982	0.864098
— 20.0	1.115368	0.429425	1.035335
— 10.0	1.141547	0.335620	1.198842
1924 Dec. 30.5	1.160700	0.239730	1.354995
— 20.5	1.173948	0.142604	1.504278
— 10.5	1.182225	— 0.044862	1.647205
Nov. 30.5	1.186295	+ 0.053048	1.784286
— 20.5	1.186791	0.150808	1.916001
— 10.5	1.184233	0.248185	2.042793
Oct. 31.5	1.179048	0.345017	2.165065
— 21.5	1.171598	0.441190	2.283180
— 11.5	1.162180	0.536626	2.397464
— 1.5	1.151045	0.631272	2.508213
Sept. 11.5	1.124444	0.818083	2.720121
Aug. 22.5	1.093146	1.001504	2.920710
— 2.5	1.058145	1.181560	3.111435
July 13.5	1.020189	1.358337	3.293495
June 23.5	0.979850	1.531956	3.467880
— 3.5	0.937572	1.702558	3.635414
May 14.5	0.893704	1.870285	3.796796
April 24.5	0.848526	2.035277	3.952616
— 4.5	0.802261	2.197673	4.103383
March 15.5	+ 0.755093	+ 2.357601	— 4.249534

G. M. T.			x	y	z
1924	Febr.	24.5	+ 0.707173	+ 2.515184	— 4.391450
	—	4.5	0.658624	2.670537	4.529465
	Jan.	15.5	0.609552	2.823768	4.663873
1923	Dec.	26.5	0.560043	2.974975	4.794933
	—	6.5	0.510170	3.124253	4.922876
	Nov.	16.5	0.459997	3.271687	5.047909
	Oct.	27.5	0.409578	3.417359	5.170218
	Sept.	17.5	0.308174	3.703714	5.407314
	Aug.	8.5	0.206256	3.983863	5.635323
	June	29.5	0.104045	4.258283	5.855207
	May	20.5	+ 0.001710	4.527396	6.067770
	April	10.5	— 0.100619	4.791571	6.273697
	March	1.5	0.202843	5.051139	6.473576
	Jan.	20.5	0.304884	5.306395	6.667911
1922	Dec.	11.5	0.406681	5.557601	6.857145
	Nov.	1.5	0.508188	5.804996	7.041663
	Sept.	22.5	0.609367	6.048795	7.221806
	Aug.	13.5	0.710191	6.289192	7.397877
	July	4.5	0.810638	6.526363	7.570145
	May	25.5	0.910690	6.760472	7.738854
	March	6.5	1.109565	7.220082	8.066436
1921	Dec.	16.5	1.306753	7.669074	8.382132
	Sept.	27.5	1.502227	8.108340	8.687186
	July	9.5	1.695985	8.538655	8.982644
	April	20.5	1.888041	8.960687	9.269396
	Jan.	30.5	2.078419	9.375022	9.548201
1920	Nov.	11.5	2.267151	9.782178	9.819717
	Aug.	23.5	2.454275	10.182612	10.084516
	June	4.5	2.639829	10.576731	10.343102
	March	16.5	2.823855	10.964903	10.595920
1919	Dec.	27.5	3.006397	11.347456	10.843362
	Oct.	8.5	3.187497	11.724689	11.085760
	July	20.5	3.367199	12.096874	11.323495
	May	1.5	— 3.545547	+ 12.464256	— 11.556787

G. M. T.	x	y	z
1919 Febr. 10.5	— 3.722582	+ 12.827064	— 11.785914
1918 Nov. 22.5	3.898347	13.185505	12.011110
June 15.5	4.246233	13.890041	12.450540
Jan. 6.5	4.589528	14.579251	12.876570
1917 July 30.5	4.928539	15.254336	13.290454
Febr. 20.5	5.263553	15.916361	13.693255
1916 Sept. 13.5	5.594831	16.566276	14.085883
April 6.5	5.922609	17.204939	14.469118
1915 Oct. 29.5	6.247085	17.833128	14.843639
May 22.5	6.568432	18.451548	15.210037
1914 Dec. 13.5	6.886785	19.060838	15.568837
July 6.5	7.202252	19.661572	15.920506
Jan. 27.5	7.514918	20.254268	16.265462
1913 Aug. 20.5	7.824848	20.839385	16.604093
March 13.5	8.132096	21.417332	16.936747
1912 Oct. 4.5	8.436706	21.988472	17.263749
April 27.5	8.738719	22.553126	17.585400
1911 Nov. 19.5	9.038177	23.111580	17.901976
June 12.5	9.335120	23.664087	18.213742
Jan. 3.5	9.629592	24.210873	18.520937
1910 July 27.5	9.921638	24.752126	18.823772
Febr. 17.5	10.211321	25.288072	19.122514
1909 Sept. 10.5	10.498686	25.818833	19.417306
April 3.5	10.783802	26.344570	19.708356
1908 Oct. 25.5	11.066735	26.865421	19.995836
May 18.5	11.347564	27.381513	20.279911
1907 Dec. 10.5	11.626370	27.892965	20.560733
July 3.5	11.903245	28.399892	20.838445
Jan. 24.5	12.178288	28.902407	21.113176
1906 Aug. 17.5	12.451602	29.400627	21.385044
March 10.5	12.723297	29.894671	21.654158
1905 Oct. 1.5	12.993479	30.384669	21.920609
April 24.5	13.262255	30.870759	22.184480
1904 Nov. 15.5	— 13.529719	+ 31.353090	— 22.445840

For 1905 Oct. 1.5 G.M.T., when the comet was at a distance of 40 astronomical units from the sun, i. e. outside the orbit of Neptune, the following co-ordinates and velocity components $\left(x,\ y,\ z,\ \frac{dx}{dt},\ \frac{dy}{dt},\ \frac{dz}{dt}\right)$, and reductions to the center of gravity of the sun and the 8 major planets $\left(\xi,\ \eta,\ \zeta,\ \frac{d\xi}{dt},\ \frac{d\eta}{dt},\ \frac{d\zeta}{dt}\right)$, and the resulting reduced co-ordinates and velocity components $\left(\bar{x},\ \bar{y},\ \bar{z},\ \frac{d\bar{x}}{dt},\ \frac{d\bar{y}}{dt},\ \frac{d\bar{z}}{dt}\right)$ were obtained:—

$$x = -12.993479 \qquad y = +30.384669 \qquad z = -21.920609$$
$$\xi = -\quad 4853 \qquad \eta = -\quad 3166 \qquad \zeta = -\quad 1179$$
$$\bar{x} = -12.998332 \qquad \bar{y} = +30.381503 \qquad \bar{z} = -21.921788$$

$$160\frac{dx}{dt} = +0.2694623 \qquad 160\frac{dy}{dt} = -0.4880198 \qquad 160\frac{dz}{dt} = +0.2651485$$
$$160\frac{d\xi}{dt} = +\quad 8819 \qquad 160\frac{d\eta}{dt} = -\quad 8235 \qquad 160\frac{d\zeta}{dt} = -\quad 3702$$
$$160\frac{d\bar{x}}{dt} = +0.2703442 \qquad 160\frac{d\bar{y}}{dt} = -0.4888433 \qquad 160\frac{d\bar{z}}{dt} = +0.2647783$$

From these we find:—

$$\bar{r} = \sqrt{\bar{x}^2 + \bar{y}^2 + \bar{z}^2} = 39.65548,$$

while the square of the velocity is:—

$$\bar{V}^2_{160} = \left(160\,\frac{d\bar{x}}{dt}\right)^2 + \left(160\,\frac{d\bar{y}}{dt}\right)^2 + \left(160\,\frac{d\bar{z}}{dt}\right)^2 = 0.3821613.$$

The equation of conservation of energy is:—

$$\bar{V}^2_{160} = w^2 k^2 (1 + \Sigma m)\left(\frac{2}{\bar{r}} - \frac{1}{\bar{a}}\right)$$

or

$$\frac{1}{\bar{a}} = \frac{2}{\bar{r}} - \frac{\bar{V}^2}{w^2 k^2 (1 + \Sigma m)}.$$

Inserting the values of $\bar{r}$ and $\overline{V}^2$ given above, $w^2 k^2 = 7.5753525$ and $1 + \Sigma m = 1.00134197$ the following value of $\frac{1}{a}$ for 1905 Oct. 1.5 G.M.T. is found:—

$$\frac{1}{a} = +0.0000540.$$

Thus the hyperbolic value of $\frac{1}{a}$ from the calculation of the definitive orbit has been changed into an elliptic value.

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2. Comet 1902 III Perrine-Borrelly.

The calculation of the definitive elements was based on 1182 observations distributed over a period of 212 days (cf. Publ. Cop. Obs. No. 19, p. 57):—

Epoch of osculation = T.

$$\begin{aligned}
T &= 1902 \text{ Nov. } 23.89393 \text{ M.T. Berlin} \\
\left.\begin{aligned}
\omega &= 152^\circ\ 57'\ 25''.3 \\
\Omega &= \ \ 49\ \ \ 19\ \ \ 10\ .3 \\
i &= 156\ \ \ 20\ \ \ 50\ .7
\end{aligned}\right\} & \ 1900.0 \\
\log q &= 9.603225 \pm 0.000001 \\
e &= 0.9999675 \pm 0.0000074 \\
\frac{1}{a} &= +0.0000810 \pm 0.0000184
\end{aligned}$$

The calculation of original elements published in Publ. No. 19 was based on these elements. Perturbations from Jupiter and Saturn were taken into account. The elements referred to the center of gravity were derived for 1896 Oct. 31.0 M.T. Berlin, and $\frac{1}{a}$ was found to be -0.0000204. The original value of $\frac{1}{a}$ was found to be $\frac{1}{a} = -0.0000168$.

This is the only case of a cometary orbit which has changed in hyperbolic direction in going back to the original orbit.

A closer investigation showed that in 1890 the comet passed Neptune at a distance of 2 astronomical units, to the North. The perturbations by Neptune therefore must have been appreciable. In order to get a more accurate value of the original $\frac{1}{a}$ the perturbed co-ordinates of the comet were computed for the period 1896—1884, account being taken of the perturbations from Jupiter, Saturn, and Neptune. The result was that $\frac{1}{a}$ in 1884 had the value $+0.0000054$. The main steps of the calculation are given below.

With the aid of the data given in Publ. No. 19 the following ecliptical co-ordinates and velocity components for 1896 Oct. 31.0 M.T. Berlin were obtained (1900.0). The rectangular co-ordinates of Jupiter and Saturn were taken from Publ. Cop. Obs. No. 19 (cf. footnote 2 on page 15 in this paper), the co-ordinates of Neptune were calculated from the polar co-ordinates in Berliner Jahrbuch.

$$x = +9.534042 \qquad y = +15.572743 \qquad z = -1.277228$$

$$160\frac{dx}{dt} = -0.3627868 \quad 160\frac{dy}{dt} = -0.8281268 \quad 160\frac{dz}{dt} = +0.1161004$$

With these values perturbed ecliptical co-ordinates were computed with the aid of numerical integration back to 1884.

Ecliptical Co-ordinates. Equinox 1900.0.

M. T. Berlin	x	y	z
1897 April 9.0	+ 9.165243	+ 14.734619	— 1.160325
1896 Oct. 31.0	+ 9.534042	+ 15.572743	— 1.277228

M. T. Berlin			x	y	z
1896	May	24.0	+ 9.891244	+ 16.391468	— 1.392556
1895	Dec.	16.0	10.238010	17.192434	1.506394
	July	9.0	10.575299	17.977041	1.618820
	Jan.	30.0	10.903911	18.746513	1.729906
1894	Aug.	23.0	11.224512	19.501899	1.839715
	March	16.0	11.537680	20.244163	1.948310
1893	Oct.	7.0	11.843901	20.974154	2.055745
	April	30.0	12.143602	21.692660	2.162071
1892	Nov.	21.0	12.437163	22.400399	2.267334
	June	14.0	12.724928	23.098039	2.371578
	Jan.	6.0	13.007220	23.786203	2.474844
1891	July	30.0	13.284345	24.465460	2.577165
	Febr.	20.0	13.556600	25.136340	2.678575
1890	Sept.	13.0	13.824268	25.799320	2.779096
	April	6.0	14.087620	26.454824	2.878743
1889	Oct.	28.0	14.346920	27.103218	2.977526
	May	21.0	14.602431	27.744814	3.075475
1888	Dec.	12.0	14.854401	28.379898	3.172635
	July	5.0	15.103056	29.008735	3.269049
	Jan.	27.0	15.348606	29.631564	3.364752
1887	Aug.	20.0	15.591232	30.248594	3.459774
	March	13.0	15.831102	30.860018	3.554139
1886	Oct.	4.0	16.068360	31.465999	3.647869
	April	27.0	16.303138	32.066689	3.740983
1885	Nov.	18.0	16.535547	32.662222	3.833501
	June	11.0	16.765682	33.252722	3.925438
	Jan.	2.0	16.993621	33.838302	4.016811
1884	July	26.0	17.219422	34.419070	4.107635
	Febr.	17.0	+ 17.443128	+ 34.995127	— 4.197925

From this table we get for 1884 July 26.0 M.T. Berlin: —

$x = +17.219422$	$y = +34.419070$	$z = -4.107635$
$\xi = + \quad 1987$	$\eta = - \quad 7258$	$\zeta = + \quad 47$
$\bar{x} = +17.221409$	$\bar{y} = +34.411812$	$\bar{z} = -4.107588$

$$160\,\frac{dx}{dt} = -0.224748 \qquad 160\,\frac{dy}{dt} = -0.578396 \qquad 160\,\frac{dz}{dt} = +0.090554$$

$$160\,\frac{d\xi}{dt} = +\quad 1111 \qquad 160\,\frac{d\eta}{dt} = +\quad 679 \qquad 160\,\frac{d\zeta}{dt} = -\quad 31$$

$$160\,\frac{d\bar{x}}{dt} = -0.223637 \qquad 160\,\frac{d\bar{y}}{dt} = -0.577717 \qquad 160\,\frac{d\bar{z}}{dt} = +0.090523$$

$$\frac{1}{\bar{a}} = 0.0516808 - \frac{0.391965}{7.575353 \times 1.0012923} = +0.0000054,$$

i. e. a slightly elliptic value.

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3. Comet 1897 I Perrine.

The reasons for carrying out the investigations of this comet here presented were of a nature of principle. The method used in Publ. 19 in the calculation of original orbits of comets which, in the inner parts of the solar system, that is in the neighbourhood of perihelion, had proved to move in hyperbolic orbits, was largely based on the use of Encke's method of special perturbations, this method being used to follow the comet for a certain number of years back in time. The calculation of exact perturbations according to this method, however, involves a very great amount of labour. As a consequence, the calculations according to Encke's method were not carried as far back in time as would have been desirable from certain points of view. On the other hand equation (33) in Publ. No. 19 provided a means of determining an upper limit to that part of the perturbations which had not been allowed for in the calculations. This equation, however, gives a value which is far greater than the actual uncertainty of the results, because its derivation is based on the assumption of the most unfavourable conditions in every respect. Actually, therefore, there was no doubt that the accuracy of the calculation of the original $\frac{1}{a}$ was far greater than found according to equation (33) [cf. the remark on page 19].

Calculations of perturbations by Encke's method through a considerably longer period than that adopted in Publ. No. 19 would have involved a considerable increase in the amount of labour. It therefore meant a great help in this problem, when the method of direct integration of rectangular co-ordinates was gradually introduced. When this method is used in connection with the tables published by Comrie[1] and those published by the authors[2-4] the saving of labour is very considerable.

After this method had been adopted we have been able to increase considerably the period through which the comet was followed backward in time. It was also desirable to carry out comparison, in one or more special cases, between the results obtained earlier and results derived from a calculation by the method of direct integration of rectangular co-ordinates used over a considerably longer period. As an example we have chosen comet 1897 I.

The calculation of the definitive orbit for this comet had given the following result (Publ. No. 19, p. 52):—

[1] L. J. Comrie, Planetary co-ordinates for the years 1800—1940, referred to the equinox of 1950.0, London 1933, Naut. Alm. Office.

[2] E. Strömgren, Koordinaten der Planeten Jupiter und Saturn nebst den diesen zwei Planeten entstammenden Störungskomponenten der Sonne von 1871 Febr. 5 bis 1904 Dez. 9, alles auf 1900.0 bezogen. Copenhagen 1914, Publ. Cop. Obs. No. 19 p. 28—35.

[3] H. Q. Rasmusen, Äquatoreale Jupiter- und Saturn-Koordinaten für den Zeitraum 1800—1900 bezogen auf das Äquinoctium 1950.0, Kiel 1936, A. N. 6172 and Publ. Cop. Obs. No. 107.

[4] H. Q. Rasmusen, Hilfstafeln für die numerische Integration der rechtvinkligen Koordinaten eines Himmelskörpers, Kiel 1936, A. N. 6235—36 and Publ. Cop. Obs. No. 109.

Epoch of osculation 1897 Jan. 29.0 M.T. Berlin.

$$T = 1897 \text{ Feb. } 8.140779 \text{ M.T. Berlin}$$

$$\left.\begin{aligned} \omega &= 172^\circ\ 19'\ \ 2''.02 \\ \Omega &= \ \ 86\ \ \ 31\ \ \ \ 3\ .51 \\ i &= 146\ \ \ \ 8\ \ 14\ .49 \end{aligned}\right\} 1900.0$$

$$\log q = \ \ 0.0264443 \pm 0.0000034$$

$$e = \ \ 1.0009270 \pm 0.0000506$$

$$\frac{1}{a} = -0.0008722 \pm 0.0000476$$

i. e. an hyperbolic orbit.

The calculation of the perturbations from Jupiter and Saturn with the aid of Encke's method gave the following perturbed co-ordinates and velocity components for 1891 Feb. 20.0 M.T. Berlin (Ecliptic 1900.0):—

$$x = +6.173828 \qquad y = +16.003986 \qquad z = +3.483160$$

$$160\frac{dx}{dt} = -0.1444654 \quad 160\frac{dy}{dt} = -\ 0.9166608 \quad 160\frac{dz}{dt} = -0.0601508$$

From these starting-values the co-ordinates of the comet were calculated by direct integration as far back as the beginning of 1880:—

Ecliptical Co-ordinates. Equinox 1900.0.

M. T. Berlin	x	y	z
1892 Jan. 6.0	+ 5.865270	+ 14.122429	+ 3.352011
1891 July 30.0	6.024685	15.075696	3.420431
Febr. 20.0	6.173828	16.003986	3.483160
1890 Sept. 13.0	6.314013	16.909769	3.540958
April 6.0	6.446330	17.795127	3.594436
1889 Oct. 28.0	6.571695	18.661826	3.644097
May 21.0	+ 6.690886	+ 19.511375	+ 3.690359

M. T. Berlin			x	y	z
1888	Dec.	12.0	+ 6.804570	+ 20.345079	+ 3.733574
	July	5.0	6.913323	21.164063	3.774043
	Jan.	27.0	7.017637	21.969312	3.812020
1887	Aug.	20.0	7.117943	22.761686	3.847727
	March	13.0	7.214612	23.541945	3.881358
1886	Oct.	4.0	7.307966	24.310761	3.913080
	April	27.0	7.398283	25.068733	3.943043
1885	Nov.	18.0	7.485803	25.816395	3.971382
	June	11.0	7.570730	26.554232	3.998211
	Jan.	2.0	7.653240	27.282677	4.023638
1884	July	26.0	7.733475	28.002128	4.047758
	Febr.	17.0	7.811553	28.712950	4.070658
1883	Sept.	10.0	7.887569	29.415483	4.092415
	April	3.0	7.961595	30.110047	4.113100
1882	Oct.	25.0	8.033683	30.796944	4.132779
	May	18.0	8.103871	31.476476	4.151511
1881	Dec.	9.0	8.172188	32.148937	4.169352
	July	2.0	8.238655	32.814619	4.186351
	Jan.	23.0	8.303292	33.473814	4.202558
1880	Aug.	16.0	8.366113	34.126816	4.218021
	March	9.0	8.427134	34.773920	4.232788
1879	Oct.	1.0	+ 8.486372	+ 35.415422	+ 4.246908

From these values the following co-ordinates and velocity components, and reductions to the center of gravity of the sun, Jupiter and Saturn, and the resulting reduced co-ordinates and velocity components were obtained:—

1880 March 9.0 M.T. Berlin.

$x = +8.427134$	$y = +34.773920$	$z = +4.232788$
$\xi = -\quad 7257$	$\eta = -\quad 441$	$\zeta = +\quad 221$
$\bar{x} = +8.419877$	$\bar{y} = +34.773479$	$\bar{z} = +4.233009$

$$160\,\frac{dx}{dt} = -0.060127 \qquad 160\,\frac{dy}{dt} = -0.644253 \qquad 160\,\frac{dz}{dt} = -0.014435$$

$$160\,\frac{d\xi}{dt} = +12 \qquad 160\,\frac{d\eta}{dt} = -1446 \qquad 160\,\frac{d\zeta}{dt} = +7$$

$$160\,\frac{d\bar{x}}{dt} = -0.060115 \qquad 160\,\frac{d\bar{y}}{dt} = -0.645699 \qquad 160\,\frac{d\bar{z}}{dt} = -0.014428$$

and from these values we find for the original $\frac{1}{a}$:—

$$\frac{1}{\bar{a}} = +0.0000396$$

i. e. a value, which agrees within 28 units in the 7th decimal with that given in Publ. No. 19 $\left(\frac{1}{\bar{a}} = +0.0000368\right)$.

The numerical calculations the results of which are published in this communication have been made by Mr. HANS Q. RASMUSEN. Mag. scient. B. SVANHOF has checked certain steps of the calculations. Economic support from the Carlsbergfond is gratefully acknowledged.

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Postscript: Recently from other sides two investigations made after the principles used in the Copenhagen papers have been published on the original orbits for two comets which in the neighbourhood of perihelion moved in hyperbolic orbits. One of these, concerning comet 1932 VI Geddes, has been carried out by G. VAN BIESBROECK (Publ. Yerkes Observatory VIII, 3). The other, concerning comet 1922 II Baade, has been made by A. GENNARO (Publ. Padua Observatory No. 54). In both cases an hyperbolic orbit was changed into a slightly elliptic orbit.

Dr. GENNARO in his paper makes an investigation for the comet 1922 II following the method outlined on p. 25 of Publ. Cop. Obs. No. 19: — «Es ist zu der Anwendung der Formel (33) zu bemerken, dass wir für $\left|\triangle\left(\frac{1}{a}\right)\right|$, d. h. für die Maximalgrenze der unberücksichtigten Störung in $\frac{1}{a}$, immer in jedem konkreten Fall einen bedeutend niedrigeren Wert feststellen können, ohne dass wir genötigt wären, die Störungsrechnung weiter rückwärts zu führen. Bei der Ableitung der Formel (33) haben wir für b und für $P_n^3\,(\cos\Phi)$ ihre theoretischen Maximalwerte benutzt. Es ist aber leicht, in jedem konkreten Fall, d. h. für jede Kometenbahn, je nach den Werten der Bahnelemente, spezielle Maximalwerte zu berechnen, die einen niedrigeren Betrag aufweisen».

As was to be expected, the value for the upper limit of the neglected perturbation in $\frac{1}{a}$ thus obtained by Dr. GENNARO for the comet investigated by him was considerably smaller than the value following from the general equation (33) in Publ. 19.

E. S.

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