to $s = 0$ we find K^0, the dissociation constant of the anilinium ion at infinite dilution. From the variation with temperature (Fig. 3 and formulae 13 and 14) we find the heat of dissociation of the anilinium ion 7100 cal./mole.

In order to make the above extrapolation (2) we have measured cells of the following composition with the hydrogen electrode:

$$H_2 \mid \text{Solution X} \mid 3.5\ m.\ \text{KCl} \mid \text{Solution S} \mid H_2,$$

where X is a mixture of hydrochloric acid and sodium chloride solution, while S is 0.01011 n. HCl (Table 6). From the measurements we find $-\log \frac{f_{(X)}}{f_{(S)}}$, which we extrapolate to X = pure sodium chloride solution (Fig. 4 and 5).

In the last part of the paper we have discussed the definition of the single values of f. It has been shown from the measurements in this paper that the usual procedure of extrapolating to infinite dilution by means of Debye-Hückel's law for the real activity coefficients may give meaningless results.

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ON THE TRANSMUTATION OF ATOMIC NUCLEI BY IMPACT OF MATERIAL PARTICLES

I. GENERAL THEORETICAL REMARKS

BY

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PREFACE

As indicated by the title the present paper was intended to form the first part of a treatise consisting of three parts to appear in immediate succession. The second part was planned as a more detailed elaboration of the theory of nuclear collisions on the general lines discussed here while the third part should contain an analysis on such lines of the available experimental evidence about nuclear transmutations. The publication of the present paper which was in print in January 1937 was, however, postponed and the completion of the other parts of the treatise delayed due to a visit of the authors to American universities in order to attend a number of conferences where nuclear problems were discussed. In the meantime the subject has been in rapid development due to the publication of several important papers during the last few months. Moreover an admirable complete report of the present state of nuclear dynamics has been published by H. Bethe in "Reviews of Modern Physics", **9**, 69, (1937), in which are included detailed comments on some of the considerations here presented, based on verbal communications from the authors at a conference in Washington, February 1937. Under these circumstances the plan of publishing a more comprehensive treatise has been temporarily abandoned, and in order to bring the present paper up to date it has been completed by an addendum written in October 1937 and containing references and brief comments of the most important recent contributions to the subject.

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§ 1. Basic Ideas.

In a recent paper[1] it was pointed out that the extreme facility of energy exchanges between the densely packed particles in atomic nuclei plays a decisive role in determining the course of nuclear transmutations initiated by impact of material particles. In fact, the assumption underlying the usual treatment of such collisions, that the transmutation consists essentially in a direct transfer of energy from the incident particle to some other particle in the original nucleus leading to its expulsion, cannot be maintained. On the contrary, we must realize that every nuclear transmutation will involve an intermediate stage in which the energy is temporarily stored in some closely coupled motion of all the particles of the compound system formed by the nucleus and the incident particle. On account of the strong forces which come into play between any two material particles at the small distances in question, the coupling between the particles of this compound system is

[1] N. Bohr, Neutron capture and nuclear constitution, Nature **137**, 344 and 351, (1936), cited for brevity in the following as (A).

Added in proof. In a more recent article (Science **86**, 161, 1937) a brief account of the later developments of the views presented in the paper cited is given. A fuller account with more detailed references to the previous litterature on the subject is further contained in an address at the International Physical Congress in Paris, October 1937, which will soon appear in the congress communications.

in fact so intimate that its eventual disintegration — whether it consists in the release of an "elementary" particle like a proton or a neutron, or of a "complex" nuclear particle like a deuteron or an α-ray — must be considered as a separate event, independent of the first stage of the collision process. The final result of the collision may thus be said to depend on a free competition between all the various disintegration and radiation processes of the compound system consistent with the general conservation laws.

From this point of view the treatment of nuclear transmutations initiated by collisions will imply in the first place an examination of the balance between the separate processes involved in the formation and disintegration of the semistable intermediate system. Notwithstanding the suggestiveness of simple mechanical analogies (A, p. 351) the discussion of this problem obviously demands proper quantum theoretical considerations. In fact, not only are the possible energy states of the compound system generally restricted by the laws of quantum mechanics, but also the formation or disintegration of this system will often involve characteristic quantum mechanical effects of the kind well known from the successful explanations of the laws of radioactive decay due to Condon and Gurney, and especially to Gamow. Considerable modifications in the usual treatment of such problems, which rest upon the assumption that the incident particle within the nucleus to a first approximation moves in a fixed field of force, are necessitated, however, by the intimate coupling here assumed between the motion of the particles in nuclei. Still we shall see that the extreme thoroughness of this coupling actually introduces certain simplifications which permit one to draw a number of simple conclusions of a comprehensive character about nuclear reactions.

Results of great interest on the constitution of atomic nuclei have been obtained, as is well known, by treating such nuclei as quantum mechanical systems built up entirely of neutrons and protons. This offers not only an explanation of the fact revealed by the study of band spectra and of the hyperfine structure of series lines that the intrinsic spin of the nucleus of any isotope is an odd or even multiple of the unit $h/4\pi$, according as its mass number is odd or even respectively, but also allows a general understanding of the way in which the stability of nuclei, and hence the occurrence of isotopes and the values of their mass defects, vary with mass and charge number. In this connection it may be especially noted that the important information about the forces between nuclear particles at close distances obtained in this way by Heisenberg and his collaborators rests essentially upon an estimate of the average kinetic energy of these particles in the normal state of nuclei. Since protons as well as neutrons obey the Pauli exclusion principle, this kinetic energy will in fact be nearly independent of the conditions of motion assumed for the nuclear particles and, as regards its order of magnitude, will always be comparable with what would be obtained if each particle was assumed to move in a separate cell within the nucleus.

Any closer examination of the constitution of atomic nuclei based on the usual procedure in which, like the extra-nuclear electrons in atoms, each particle is assumed in the first approximation to move independently in a conservative field of force cannot, however, on account of the far more intimate coupling between nuclear particles, be expected to yield results which may be directly compared with the actual properties of nuclei. Notwithstanding the promising attempts at a more rigorous treatment of the constitution of very light nuclei, we must for the moment be content to consider atomic nuclei as a state of matter of extreme density and electrisation, whose properties can only be explored by the analysis of the experimental evidence on nuclear reactions. Still, the circumstance that the excitation energy of the compound nucleus involved in ordinary experiments on nuclear transmutations is very small compared with the total energy necessary for the complete separation of all its constituent particles permits, as we shall see, a simple comparison to be made between many properties of nuclear matter and the properties of ordinary liquid and solid substances.

§ 2. Nuclear Level Distribution.

As shown in (A), the distribution of energy levels of excited nuclei exhibits a striking difference from what would be expected if such excitations, as ordinarily assumed, were due to an abnormally high energy state of a single nuclear particle. Thus the experimental evidence concerning the capture of fast and slow neutrons by heavy nuclei with emission of radiation shows that the distance between the energy levels of such nuclei decreases rapidly with increasing excitation, with the result that the distribution of energy levels becomes practically continuous, even for excitation energies which — although sufficient for the escape of a neutron with great kinetic energy — are far too small to alter essentially the semistable character of the compound system. Even within the region of continuous distribution the mean life time of the compound system is probably more than a hundred thousand times as long as the time interval which a fast neutron would use in passing through a region

of nuclear dimensions. The typical features of the nuclear level distribution are easily understood, however, if we realize that the stationary states of a nucleus must correspond to some quantized collective type of motion of all its constituent particles. In fact the rapid approach of neighbouring nuclear levels with increasing energy resembles (comp. A p. 346) the characteristics of the multitude of the linear combinations which may be formed by a number of independent quantities (see Addendum I). The nuclear level distribution has therefore very much the same character as that of the quantum states of a solid body, well known from the theory of specific heats at low temperatures (see Addendum II).

This analogy suggests a more direct comparison between the excitation of a nucleus and the vibrations of elastic substances, a comparison which is much simplified by the circumstance that apart from the very lightest nuclei the density of matter and energy is practically the same in all nuclei. Denoting by N the total number of protons and neutrons in such a nucleus, the volume will in fact be approximately given by

$$V = N\delta^3, \tag{1}$$

where δ is about $3 \cdot 10^{-13}$ and may be taken as the diameter of the cell occupied by an individual nuclear particle. Further the average kinetic energy of each particle in such nuclei will be given approximately by the simple formula

$$K = \frac{h^2}{8\,\delta^2 \mu}, \tag{2}$$

where h is Planck's constant, and μ the nearly equal mass of a proton or a neutron. This gives for K approximately

20 M. e. V. and, since from measurements of mass defects the average binding energy of a neutron or a proton in a nucleus is found to be approximately 10 M. e. V., the average potential energy loss per nuclear particle becomes nearly 30 M. e. V. Just as δ may be considered as a characteristic unit of length in nuclear problems, a suitable unit of time in such problems is given by the interval τ, which an elementary particle of kinetic energy K would use in covering the distance δ. This time interval which is approximately given by

$$\tau = 2\frac{\mu\delta^2}{h}, \tag{3}$$

is of the order of magnitude of 10^{-22} sec.

The circumstance that the excitation energy of heavier nuclei is always very small compared with the total kinetic energy NK in the normal state of the nucleus invites us now to compare the nuclear excitations with the oscillations in volume and shape of a sphere under the influence of an elasticity ε or surface tension ω given by expressions of the type of

$$\varepsilon = C_\varepsilon K\delta^{-3}; \quad \omega = C_\omega K\delta^{-2}, \tag{4}$$

where the dimensionless factors C_ε and C_ω must be expected to be approximately constant for all but the lightest nuclei. Thus the frequences ν_ε and ν_ω of the oscillations of the simplest character of a sphere of volume V and density σ are given by the familiar formulas

$$\nu_\varepsilon \backsimeq \varepsilon^{1/2} V^{-\frac{1}{3}}\sigma^{-\frac{1}{2}}; \quad \nu_\omega \backsimeq \omega^{1/2} V^{-\frac{1}{2}}\sigma^{-\frac{1}{2}}, \tag{5}$$

which are easily tested by dimensional considerations. Putting $\sigma = \mu\delta^{-3}$ we obtain from (5) by means of (1), (2), and (4)

for the energy differences between successive quantum states of the nucleus corresponding to such oscillations

$$\Delta_\varepsilon E = h\nu_\varepsilon \backsimeq \sqrt{8C_\varepsilon}\, N^{-\frac{1}{3}}K; \quad \Delta_\omega E = h\nu_\omega \backsimeq \sqrt{8C_\omega}\, N^{-\frac{1}{2}}K. \tag{6}$$

Due to the difficulty of estimating the numerical values of the constants C_ε and C_ω the main interest of these formulas is the variation of the energy differences with N. Thus the fact that the average energy differences between the lower excited states of nuclei varies decidedly faster than $N^{-\frac{1}{3}}$ and even somewhat more rapidly than $N^{-\frac{1}{2}}$ shows that, at any rate for heavier nuclei, simple elastic vibrations corresponding to $\Delta_\varepsilon E$ are not responsible for the lowest excited states and can only be expected to be present for higher excitations. The circumstance, however, that $\Delta_\omega E$ corresponds more closely to the way the average distance of the lower levels decreases with N, suggests a more direct comparison of surface oscillations with the fundamental modes of nuclear excitation responsible for the main features of the level distribution. Still the fact that the proper surface energy of nuclei estimated from the mass defect curves[1] give, when introduced in (4) and (6), values for $\Delta_\omega E$ of more than a million volts even for heavy nuclei, where the average level distance is certainly not more than a few hundred thousand volts, shows the great difficulty involved in such a comparison (see Addendum III).

Obviously any such simple considerations can at most serve as a first orientation as regards the possible origin of nuclear excitations. In a closer discussion of this problem more detailed considerations regarding the specific character of the interactions between the individual nuclear particles on the stability as well as of the exci-

[1] cf. C. F. v. Weizsäcker, Die Atomkerne, Leipzig 1937.

tation mechanism of nuclei are needed. This is in fact not only indicated by the well known periodicities in the mass defect curve but also by the marked difference observed between the distances of the ground level and the excited levels for nuclei with even and odd mass and charge numbers. These effects must obviously be ascribed to the different degrees of saturation of the forces between pairs of nuclear particles obtainable in such nuclei on account of the restrictions implied by the Pauli principle in a more rigorous quantum mechanical treatment of the many body systems concerned. Due to the close coupling of the motion of the nuclear particles it would seem difficult, however, at the moment to discern to what extent conclusions concerning the exchange character or the spin dependence of the specific nuclear forces are reliable, when based on considerations of nuclear models with weak coupling between the particles.

In particular any attempt of accounting for the spin values by attributing orbital momenta to the individual nuclear particles seems quite unjustifiable. We must in fact assume that any orbital momentum is shared by all the constituent particles of the nucleus in a way which resembles that of the rotation of a solid body. Denoting by J the moment of inertia, we obtain

$$\Delta_r E = \frac{h^2}{8\pi^2 J} \backsimeq N^{-\frac{5}{3}} K \qquad (7)$$

as an estimate of the energy differences between the lowest quantum states of such rotations. For heavy nuclei (7) gives values small compared with the average level distance and may therefore possibly explain the fine structure observed for many energy levels of such nuclei. Part of

this fine structure and perhaps many other of the characteristic features of the structure of the lower level distribution may, however, be attributed to the orientations of the intrinsic spins of the nuclear particles relative to each other and to the resulting angular momentum of the nuclear motions (see Addendum IV).

§ 3. Radiative Properties of Nuclei.

As first revealed from the study of so-called internal conversion of γ-rays, the radiation emitted from excited nuclei will often show polarity properties differing essentially from that of an excited atom containing an electron in an abnormally high quantum state. While in the atomic case the intense radiations are always of dipole type, nuclear radiations corresponding to poles of higher order are found to be relatively intense. It is true that this is just what should be expected if nuclei could be considered as composed entirely of constituents like α-particles, all having the same charge and the same mass, because in that case the electric center would always coincide with the mass center and exclude the appearance of any dipole moment[1]. In the more general case, however, where nuclei must be considered to be built up of protons and neutrons, the appearance of dipole moments must — quite independent of the character of the forces between the particles — obviously be expected to appear, if the coupling is assumed to be so small that the state of the nucleus can be described by attributing well defined quantum states to each particle.

If, on the contrary, the coupling between the motions of the individual particles is assumed to be so intimate that we have only to do with collectively quantized states of the

[1] Comp. N. Bohr, Journ. Chem. Soc. p. 381 (1932).

whole nucleus, the situation will obviously be very different. In fact, unless the excitation is so high that the relative position of neighbouring particles is essentially affected, the radiative properties of the nucleus must be expected to show a close resemblance with that of a rotating or oscillating body with practically uniform electrisation and, due to the approximate coincidence of the charge and the mass center, dipole moments will under such conditions be absent or at any rate much suppressed. Such a comparison also makes possible a quantitative estimate of the probabilities of the radiative processes responsible for neutron capture. In fact, for an oscillation of the nuclear matter with frequency ν and relative amplitude α the quadrupole radiation emitted per unit time will be approximately

$$R \backsimeq (2\pi\nu)^6 \frac{E^2}{c^5} \alpha^2 d^4, \tag{8}$$

where $E = Ze$ is the total electric charge, and $d = \delta N^{1/3}$ the diameter of the nucleus. Further we have for a low quantum state

$$h\nu \backsimeq (2\pi\nu)^2 \alpha^2 d^2 M, \tag{9}$$

where $M = N\mu$ is the total mass of the nucleus. Eliminating α from (8) and (9) we get for the probability of a radiative transition in unit time

$$\Gamma_r = \frac{R}{h\nu} \backsimeq \tau^{-1} (2\pi\nu)^4 \frac{e^2}{hc} \frac{Z^2 \delta^4}{N^{1/3} c^4}. \tag{10}$$

Now the life time of the excited nuclear states formed by slow neutron impact on heavy nuclei corresponds to a value of Γ_r about $\tau^{-1} 10^{-7}$ and this agrees with (10), if $h\nu$ for the most probable radiative transition is of the order of a million volts, as would seem consistent with general experimental evidence.

Formula (10) holds of course only in the case of a transition actually accompanied by a quadrupole radiation. For nuclear excitations corresponding to radial pulsations or to simple rotations even the quadrupole moment will, however, disappear and radiative transitions will become still more improbable[1]. As regards the question of radiative transitions between any two levels of an excited nucleus, it must also be noted that the various possible types of oscillations can generally not be expected to be independent of each other. In fact an estimate by means of (9) of the amplitudes of these oscillations shows that even for heavy nuclei such amplitudes will be small compared with nuclear dimensions only for the lowest quantum states. In general there will therefore probably be a close coupling between the elastic vibrations of the different types, which may explain the observation of the frequent appearance of comparatively hard radiation from excited nuclei corresponding to transitions between distant nuclear levels[2]. In this connection it may be hoped that further experiments on the radiation emitted from excited nuclei as well as on nuclear disintegrations produced by γ-rays will help to clear up the question of the mechanisms of the excitation of nuclei (see Addendum V).

[1] C. F. v. WEIZSÄCKER, Naturwiss. **24**, 813, (1936) has recently suggested that the appearance of so-called isomeries among the artificial radioactive elements may be explained by the extremely small probabilities which radiative transitions with a change of angular momentum of several times $h/2\pi$ would have on any nuclear model. In this connection it may be of interest to call attention to the possibility that the uniformity of the electrisation of the densely packed nuclear matter may also make the probabilities of radiative transitions as well as of internal conversion processes between certain other pairs of nuclear states extremely small.

[2] Comp. esp. S. KIKUCHI, K. HUSIMI and H. AOKI, Nature **137**, 992, (1936).

§ 4. Escape of Neutrons from Excited Nuclei.

As already mentioned in § 1, the disintegration of the compound system involved in nuclear transmutations must be considered as an event depending only on the state of this system and not on the way in which it is formed. Such disintegrations demand in fact a so to speak fortuitous concentration on the individual particle released of an essential part of the energy temporarily stored in intrinsic motions of the nuclear matter. These characteristic features of nuclear dynamics appear especially clearly in the case of the disintegration of the compound system which results in neutron escape. In fact, in the case of release of charged particles the electric repulsion extending beyond the range of the proper nuclear forces may under certain circumstances have a considerable influence on the probability of the disintegration, and this essentially quantum mechanical effect cannot always, as we shall see in § 6 be unambiguously seperated from the kinematical conditions for the liberation of a particle from the nuclear matter. Even in the case of neutron collisions classical mechanical considerations cannot be unambiguously applied to the motion of the neutrons outside the nucleus, unless the de Broglie wave length

$$\lambda = \frac{h}{\mu v} \tag{11}$$

is shorter than or at any rate comparable with nuclear dimensions. Strictly we cannot speak of a definite establishment of interaction between a free neutron and some particle within the nucleus, unless λ is comparable with δ. The formation of a semistable compound system, which under such conditions will in almost every case result from contact

between the incident neutron and the surface of the nucleus, closely resembles in fact the adhesion of a vapour molecule to the surface of a liquid or solid body. Conversely the disintegration of the compound system with neutron release exhibits a suggestive analogy to the evaporation of such substances at low temperatures.

This analogy has been emphasized by Frenkel in a recent paper[1] in which he has derived, by a comparison with the well-known evaporation formula, an expression for the probability of neutron escape from an excited nucleus which, in our notation, can be written

$$\Gamma_n = N^{2/3} \tau^{-1} e^{-\frac{W}{kT}}, \tag{12}$$

where W is the work necessary for the liberation of a neutron from the nuclear matter, T the effective temperature and k Boltzmann's factor. This temperature energy of the nucleus Frenkel estimates by assuming that the excitation energy is distributed according to Planck's formula over a multitude of oscillators equal in number with the intrinsic degrees of freedom of a system consisting of N particles. If U is the total excitation energy of the nucleus, this gives

$$U = \sum_i \frac{h\nu_i}{e^{h\nu_i/kT} - 1}, \tag{13}$$

where the summation is extended over all the oscillators. Assuming further that the frequencies of these oscillators are all comparable with the lowest frequencies of the radiation emitted from excited nuclei, he obtains for the

[1] J. Frenkel, Sow. Phys. 9, 533, (1936).

compound system formed by the collision between a neutron and a heavy nucleus values for kT of a few hundred thousand electron volts. Introduced in (12), this gives values for Γ_n considerably smaller than the probability of neutron escape estimated from experiments. Since W is about 10 M. e. V. the formula is, however, very sensitive to the estimate of T and a far better agreement with experimental values is actually obtained, if we take into account that the possible oscillations of the nuclear matter have very different frequencies varying from the values given by formulas like (7) up to values of the same order of magnitude as K/h.

Practically all the excitation energy of the compound system is therefore stored in a few oscillations of the nuclear matter of smallest frequencies and accordingly the temperature of the nucleus calculated by (13) will be several times as high as that estimated by Frenkel, and becomes quite sufficient to secure an approximate agreement with the observed disintegration probabilities in the cases where a reasonable accuracy of formula (12) can be expected. A quantitative comparison between ordinary evaporation and neutron escape from the compound system is in fact limited not only by the difficulty involved in an accurate estimate of the effective temperatures of this system but also by the circumstance that the excitation of the residual nucleus left after the escape of a neutron will generally be much smaller than that of the compound system, in contrast with usual evaporation phenomena where the change in the heat energy of the bodies concerned, during the escape of a single vapour molecule, is negligibly small. A formula like (12) can therefore only be expected to give approximately correct results when the average excitation of the residual nucleus, although always smaller than that of the compound system, is still of the same order of magnitude. (See Addendum VI).

In such cases a comparison between neutron escape from the compound system and ordinary evaporation offers, too, a simple explanation of the relative probabilities of different disintegration processes leading to different excited states of the residual nucleus. In fact, formula (12) gives primarily an estimate of the probability of those disintegration processes in which the energy of the escaping neutron is approximately the same as that of a gas molecule at the temperature concerned, and the relative probabilities of the escape of neutrons with higher velocities must be expected to be smaller in approximate conformity with Maxwell's velocity distribution of gas molecules. Actually such a comparison offers a simple explanation of the observation that in nuclear reactions resulting in neutron release the probability of a neutron leaving the nucleus with the total energy available is generally very small, if this energy is large compared with the temperature energy. (See Addendum VII).

Similar considerations are also in qualitative agreement with the observed great probability of energy transfer in collisions between nuclei and neutrons of kinetic energy greater than the energy difference between the normal and the lowest excited states of the nucleus. While this effect contrasts so strikingly with the usual ideas of nuclear collisions it is (compare (A), p. 347) nevertheless readily explained by the smaller demands on the concentration of the energy stored in the nuclear matter necessary for neutron escape in such disintegrations of the compound system as leave the residual nucleus in an excited state than in such as leave it in its normal state. In very violent collisions, where the energy of the compound system is comparable or even larger than K, we should further expect that several particles would leave this system in successive separate disintegration processes[1]. If such a disintegration process results in the escape of a neutron its most probable energy will be of the same order of magnitude as the temperature energy of the compound system, while, if a charged particle is released, its energy will be higher on account of the additional effect of the electrical repulsion beyond the nuclear surface, which in a case like this has only a minor influence on the liberation process itself. (See § 6).

§ 5. Slow Neutron Collisions.

In the case of collisions between nuclei and neutrons with such small kinetic energies that the de Broglie wave length (11) is very long compared with nuclear dimensions we cannot, as has been mentioned, speak in an unambiguous way about contact between the neutron and the nucleus. Accordingly every simple basis is evidently lost for an ordinary mechanical description of the formation of the compound system or its disintegration. This is also shown most strikingly by the remarkable phenomena of capture of slow neutrons for which effective cross sections have been found amounting to several thousand times of simple nuclear cross sections. In these highly selective phenomena we have obviously to do with a typical quantum mechanical resonance effect where, although the collision process can

[1] The escape of more than one neutron in nuclear collision has recently been observed in fast neutron collisions by F. Heyn, Nature **138**, 723, (1936).

still be separated in well defined stages, the probabilities of successive stages cannot be estimated independently of each other.

In the first attempts to explain the appearance of such resonance the neutron was supposed to move within the nucleus in a fixed field forming a so-called potential hole. On account of the great fall in potential, the kinetic energy of the neutron within the hole would in fact be so large that its wave length became smaller than the diameter of the hole, although the wave length outside was much larger. This great change in wave length therefore effects an almost complete reflection of the neutron wave from the inner walls of the hole, allowing a standing wave of considerable intensity to be built up for suitable energy values of the neutron. As a consequence of the existence of such a semistable state of motion of the neutron within the nucleus there will appear for these energy values both an abnormally large scattering effect corresponding to the reemission of the neutron from this state and a considerable probability of capture of the neutron resulting from a radiative transition to a lower energy state within the potential hole. Although this picture in a very instructive way illuminates essential features of the resonance effect, it was soon found quite insufficient to account for the details of the phenomena observed. In particular an estimate of the probability of radiative effects in such simple collision processes shows that the probability of scattering will always be greater than or comparable with the probability of capture in contrast with the experimental results, according to which the often extraordinarily large capture probability of slow neutrons is in no case found to be accompanied by an excessively high scattering effect.

To overcome this difficulty, G. Breit and E. Wigner[1] have proposed a modification of the explanation of the resonance effects in slow neutron collisions according to which, in the intermediate state another nuclear particle through its interaction with the incident neutron is lifted from its normal state to a higher quantum state at the same time as the neutron becomes itself bound in some stationary state in the nuclear field with an energy too low to allow its immediate escape. On account of the small power of penetration of the incident neutron wave into a potential hole of nuclear dimensions even a relatively small probability of energy transfer from the neutron to another particle bound in the nucleus is in fact, as they showed, sufficient to reverse the balance between the scattering and the radiative processes in such collisions. Still, as was already pointed out in (A), the observed extraordinary sharpness of the resonance phenomena and their comparatively frequent occurrence demand a much longer life time of the intermediate system and a much closer distribution of its energy levels than any nuclear model with weak coupling between the individual particles can give.

The decisive progress in the treatment of the resonance problems by Breit and Wigner consists, however, in the establishment of general formulas for the variation of the cross sections of neutron scattering and capture in the resonance region, which are of great value for the analysis of the experimental evidence. Denoting by Γ_n and Γ_r the probabilities of neutron disintegration and of radiative transitions of the compound system respectively these cross section formulas can be written

[1] Breit and Wigner, Phys. Rev. 49, 519, (1936).

$$\sigma_{sc} = \frac{\lambda^2}{4\pi} \frac{\Gamma_n^2}{(E-E_0)^2 h^{-2} + \frac{1}{4}(\Gamma_n+\Gamma_r)^2} \tag{14}$$

and

$$\sigma_r = \frac{\lambda^2}{4\pi} \frac{\Gamma_n \Gamma_r}{(E-E_0)^2 h^{-2} + \frac{1}{4}(\Gamma_n+\Gamma_r)^2}, \tag{15}$$

where λ and E are the wave length and kinetic energy of the incident neutron respectively, and E_0 is the energy value to be ascribed to the semistable stationary state of the compound system.

The remarkable resemblance of (14) and (15) with well known optical dispersion formulas is most suggestive and illustrates in particular the difficulties of separating simply in resonance collisions the probability of the formation of the compound system from the probabilities of the competing disintegration and radiation processes of this system. While the ratio between the latter probabilities as always alone determines the relative yields of scattering and capture, we see from the dependence of the absolute values of these yields on Γ_n and Γ_r how these probabilities also influence the degree of resonance obtainable and thereby the probability of the formation of the compound system.

As regards the discussion of the experimental evidence by means of (14) and (15), it is especially important that it in principle is possible from measurements of the breadth of the resonance region

$$\beta = h(\Gamma_n + \Gamma_r) \tag{16}$$

and of the maximum capture cross section

$$\sigma_r^{\max} = \frac{\lambda^2}{\pi} \frac{\Gamma_n \Gamma_r}{(\Gamma_n+\Gamma_r)^2} \tag{17}$$

to determine Γ_n as well as Γ_r. The closer analysis of the phenomena shows that for heavier elements Γ_r is of the order of

10^{14} sec^{-1}, and that the ratio of Γ_r and Γ_n for neutrons of temperature velocity is about 10^3. While Γ_r over a considerable energy region must be expected to vary only slowly with energy, Γ_n will, as follows from quite simple quantum mechanical arguments, be directly proportional to the velocity of the incident neutron in the energy region, where the neutron wave length is large compared with nuclear dimensions, because in such a case the balance between the processes will depend only on the probability of the presence of a neutron close to the nucleus[1].. We shall therefore expect that Γ_n and Γ_r will be of the same order of magnitude for neutron energies of about 10^5 volts. For still higher energies Γ_n must be expected to increase still more rapidly and soon become much greater than Γ_r in conformity with the experimental evidence regarding fast neutron collisions[2].

In the formulas (14) and (15) it is presumed that only one semistable state of the compound system is responsible for the anomalous variation of the cross sections for capture

[1] As pointed out by O. R. Frisch and G. Placzek, Nature **137**, 357, (1936), and by P. Weekes, M. Livingstone and H. Bethe, Phys. Rev. **49**, 471 (1936), such simple arguments offer a direct method of gauging small neutron velocities. In fact the cross section for nuclear disintegrations initiated by slow neutron impacts and resulting in the release of fast α-rays will over a large energy region with high approximation be inversely proportional to the neutron velocity, since in such a case the life time of the compound system will be very small and all typical resonance effects will disappear as also shown by formula (15), if β given by (16) is very large compared with the energy of the incident neutrons in the whole region concerned.

[2] In a recent paper by H. Bethe and G. Placzek, Phys. Rev. **51**, 450, (1937), a detailed discussion of the experimental evidence regarding slow neutron collisions is given. In this paper formulas of somewhat more general type than (14) and (15) are developed, in which an explicit account is taken of the influence on the resonance phenomena of the spin properties of the nuclei concerned.

and scattering. Just as in the case of optical dispersion it is possible, however, to account for the combined effects of several resonance levels, if only the breadth of each level is small compared with the distance between neighbouring levels. In case the compound system in the energy region concerned has a continuous level distribution such an analysis cannot be unambiguously performed, but — if in this region the wave length of the incident neutron is still large compared with nuclear dimensions — the cross section for scattering and capture will be given by the simple expression (17), if the Γ's are identified with the slowly varying probabilities of disintegration and radiation of the compound system. In fact, in contrast to the case of collisions with fast neutrons, the cross sections will in this region be determined by a balance between the processes of formation and disintegration of the compound system which quite resembles that in complete resonance. (Addendum VIII).

§ 6. Release of Charged Particles from Nuclei.

As is well known from the quantum mechanical explanation of α-ray disintegration of radioactive nuclei, a charged particle may escape from a nucleus even if its potential energy in the region just outside the proper nuclear surface would be larger than its kinetic energy at great distances. In fact, a most instructive explanation of the characteristic relation between the energy with which α-rays are expelled from radioactive nuclei and the average life time of such nuclei has been obtained by comparing these disintegrations with the escape of a particle through a fixed potential barrier around the nucleus formed by the combined action of the attraction between the nuclear particles at small distances and their electrostatic repulsion beyond the range of these forces. As well known from Gamow's theory, we get in this way for the probability of disintegration per unit time

$$\Gamma_\alpha \cong \tau^{-1} e^{-\frac{4\pi}{h}\int_a^b \sqrt{2m(P(r)-E)}\,dr}, \qquad (18)$$

where m and E are the mass of the particle and the energy with which it is expelled, $P(r)$ is the potential of the particle at the distance r from the center of the nucleus, a is the inner radius of this barrier, and b the classical distance of closest approach.

Formula (18) has in particular been used as a basis for estimates from the known disintegration constants of the radii of radioactive nuclei. The recognition of the decisive influence of energy exchanges between the individual nuclear particles on the probability of the release of uncharged particles from the compound system formed by nuclear collisions raises, however, the question to what extent such estimates are reliable. In fact we have to consider that the α-particle before its expulsion does in no way move freely in a fixed potential hole but that its escape from the nucleus must rather be considered as composed of two more or less sharply separated steps, of which the first consists in the release of the α-particle from the nuclear matter, and the second in its penetration as a free particle through a potential barrier. Comparing the first step of this process with the escape of fast neutrons from highly excited nuclei, Bethe[1] has in a recent paper concluded that the penetrability of the α-particle barrier must be many times larger than hitherto assumed and has thus arrived at values for nuclear radii which are considerably larger than those

[1] H. Bethe, Phys. Rev. **50**, 977 (1936).

ordinarily adopted and which would require a radical change in all estimates of the effect of the extranuclear electric forces in charged particle reactions.

As regards such an argumentation it must be remembered, however, that while the outer slope of the barrier is determined entirely by the electric repulsion between the nuclear particles at large distances, its inner rise is essentially due to the peculiar nuclear forces at small distances. The disintegration of the imaginary nucleus, which would remain after the complete elimination of the barrier, would therefore not be opposed by the nuclear forces in the same way as the escape of neutral particles from real nuclei, and the difference between two such processes will obviously be the larger the more the top of the potential barrier is raised over the energy of the escaping particle. In the particular case of radioactive nuclei in their normal state, where the height of the α-ray barrier is of the same order of magnitude as K, the instability of the nuclear system which remains after the elimination of this barrier would thus seem to be so large that the probability of disintegration of the nucleus would be practically determined by the barrier effect alone. Notwithstanding the ambiguity inherent in all estimates of nuclear radii without a closer discrimination between various possible types of nuclear reactions, it would therefore seem that estimates of the radii of radioactive nuclei by means of formulas of the type of (18) can hardly be changed greatly by taking the many body aspects of the problem into account. (See Addendum IX.)

Compared with α-ray decay of radioactive nuclei in their normal state the relative influence of the repulsive forces and the energy exchange between the individual nuclear particles on the disintegration probabilities is completely reversed in the highly excited compound nuclei formed by collisions where, as already remarked in § 4, the direct effect of the repulsive forces will often simply be a subsequent acceleration of the charged particles evaporating from the nuclear matter. This effect is especially clearly shown in the much studied nuclear transmutations initiated by α-ray impact on light nuclei and resulting in the expulsion of high speed protons. Resembling the circumstances of neutron escape from excited nuclei it is found that, as soon as the energy is large enough, it is more likely that the nucleus after the proton expulsion is left in an excited state than in its normal state. The only difference between the relative abundance of the various proton groups appearing in such transmutations and that of the corresponding neutron groups is, in fact, that due to the repulsion even the slowest protons have energies markedly higher than the temperature of the compound nucleus. Still, as regards the estimate of the absolute values of the disintegration probabilities by means of evaporation formulas of the type (12), it must be remembered that the latent heat of evaporation cannot be simply identified with the energy necessary to remove a proton in the normal state of the compound nucleus to infinite distance, but that the potential of the proton just outside the nuclear surface must be added to this energy.

§ 7. Collisions between Charged Particles and Nuclei.

In nuclear transmutations initiated by impacts of charged particles we can, if the energy of these particles is sufficiently large as in fast neutron collisions, consider the formation of the compound system as a direct consequence of a contact between the incident particle and the original nucleus. In case of charged particles, however, the energy

must of course be so large that even after the penetration through the electrostatic repulsive field around the nucleus the wave length of the incident particle is still small compared with nuclear dimensions. For impacts of high speed α-particles on lighter nuclei the approximate fulfilment of these conditions for a simple treatment of the formation of the compound system is proved by the fact that the total yield of the disintegration processes is nearly independent of the velocity of the incident particles. This is particularly clearly shown in certain cases where as well protons as neutrons can be released in comparable abundance as a result of the collision, and where it is found that the sum of the yield of protons and neutrons is remarkably constant over a large region of α-ray energy, even if their relative abundance may vary considerably within this region[1]. At the same time this observation shows most strikingly that in such collisions we have not to do with any direct coupling between the individual protons and neutrons expelled and the incident α-ray, but that the proton and neutron release represents competing disintegration processes of the compound system[2].

In α-ray impacts with smaller energies we meet with a more complicated situation, partly because the energy levels of the compound system are no longer continuously distributed but more or less sharply separated and partly because the establishment of contact between the incident particle and the original nucleus presents in itself a typical quantum mechanical problem. As regards the latter question it is

[1] See O. Haxel, Z. f. Phys. **93**, 400, (1935).

[2] Added in proof. This point has recently also been emphasized by W. D. Harkins, Proc. Nat. Acad. of Sci. **23**, 120, (1937), who, without entering more closely on the question of the mechanism of nuclear reactions, already several years ago has advocated the view that nuclear transmutations are always initiated by the formation of a compound system.

well known that Gamow's theory for the penetration through the potential barrier around the nucleus allows one, in many cases of nuclear disintegrations initiated by α-ray impact, to account satisfactorily for the variation of the output with increasing α-ray energy. It is, however, obvious that the remarkable maxima for certain α-ray energies observed in several nuclear disintegrations giving rise to the expulsion of high speed protons cannot be explained in the usual way by attributing such maxima to the presence of a semistable quantum state of the incident α-particle within the barrier, from which it may fall to some lower quantum state accompanied by the rise of the proton from its normal energy level within the nucleus to a level sufficiently high to allow its escape. In fact, no such explanation of the resonance effects, where in the first approximation the α-particle as well as the proton is supposed to move in a fixed nuclear field, can be reconciled with the large probability of proton emission by impact of faster α-particles, which may be assumed easily to penetrate into the interior of the nucleus. Indeed, as remarked incidentally by Mott[1] already some years ago, this fact implies a coupling between the α-particle and a proton, which would be far too close to permit a resonance to develop even for lower α-ray energies, where the penetration of the α-particle into the nucleus is assumed to be essentially influenced by the potential barrier, but where the excess energy is still sufficient to permit the proton to pass unhindered over the top of the barrier.

The resonance effect in question must clearly be attributed to a coincidence of the sum of the energies of the free α-particle and the original nucleus with that of a stationary state of the compound system corresponding to some quantized collective type of motion of all its constituent particles. The sharpness of these states and accordingly of the resonance effects will depend on the life time of the compound system, which will be determined by the sum of the probabilities of the various competing disintegration processes of this system. Apart from exceptional cases the release of protons will be far the most probable and will, as clearly shown by the velocity distribution of the emitted protons mentioned in § 6, depend on an evaporation like process of the nuclear matter which only indirectly will be influenced by the presence of the repulsive forces outside the nucleus. This is not only in conformity with the existence of resonance in the energy region, where a proton would be able to escape without difficulty from the potential barrier, but also explains the fact that the width of the resonance levels for not too fast α-rays varies only slowly with increasing α-ray energy, although the ease with which an α-particle passes through the potential barrier should increase very rapidly with increasing α-ray energy.

In a more detailed discussion of nuclear transmutation initiated by α-ray impacts it must further be taken into account that the wave length of the α-particle even in the resonance region is generally of the same order as nuclear dimensions and that therefore special attention must be paid to the possibilities of different values of its angular momentum relative to the nucleus and their influence on the absolute values of the effective cross sections for the disintegration process. This circumstance will in particular influence an estimate of the relative importance of the potential barrier and of energy exchange within the nucleus on the release probability of α-particles in the region concerned. It is in this connection also of interest to note that the

[1] N. F. Mott, Proc. Roy. Soc. **133**, 228, (1931).

phenomenon of so called anomalous scattering of α-rays in close nuclear collisions may not, as in the usual treatment, be entirely attributed to a deflection of the α-ray in a fixed field of force but may be essentially influenced by the possibility that an α-particle is temporarily taken up in the compound nucleus and subsequently emitted by a separate disintegration process.

In nuclear transmutations initiated by artificially accelerated protons the repulsive forces will on account of the comparatively small energy of the incident particle have a preponderent influence on the whole phenomenon. This is also shown by the great accuracy with which the relative variation of the output with proton energy, apart from cases of exceptional sharp resonance, is given by Gamow's theory. Simple calculations of the probability of penetration of the protons through the potential barrier can, however, not explain the often remarkably large differences between the absolute values of the output of transmutation processes by impact on different nuclei. These specific effects show in fact in a striking way the great extent to which the probability of the formation of the compound system in the proper quantum mechanical region may depend on the probabilities of disintegration processes of this system itself, which probabilities may again depend largely on the spin properties of the original nucleus and the disintegration products.[1]

In the particular case of highly selective capture of slow protons by certain light nuclei we meet, as regards the way in which the capture cross section depends on the probabilities of proton escape and of radiative transitions,

[1] Compare M. Goldhaber, Proc. Camb. Phil. Soc. **30**, 361 (1934); L. R. Hafstad, N. P. Heydenburg and M. A. Tuve, Phys. Rev. **50**, 504 (1936). (See also Addendum IV).

with an especially instructive analogy to slow neutron capture, at the same time as the two phenomena exhibit extreme differences in mechanical respects. In fact the cross section for proton capture and the breadth of the resonance region can obviously be expressed by general formulas of the same type as (15) and (16), but while the probability of neutron release Γ_n depends solely on energy exchanges within the nuclear matter the corresponding probability for proton escape Γ_p will also largely depend on the extra-nuclear repulsion. Still due to the high excitation of the compound system the situation is essentially different from α-ray disintegration of radioactive nuclei in their normal state, discussed in paragraph 6, and the influence on Γ_p of the mechanism of release of the proton from the nuclear matter will here be comparable with the barrier effect.

Essential new features are exhibited by transmutations initiated by deuteron collisions where the output over larger energy regions is often very much greater than estimated by the quantum mechanical probability of a material point with charge and mass like that of the deuteron in reaching the surface of the nucleus. As pointed out by Oppenheimer and Phillips[1] we must, however, here take into consideration that on account of the comparatively large size and small stability of the deuteron it may be disrupted during the collision with the result that the neutron is captured by the nucleus and the proton repelled by the extra nuclear field. For the smallest deuteron velocities this view seems actually to offer a satisfactory explanation of the experimental evidence. For larger deuteron velocities, where still the energy is too small to allow a sufficiently probable penetration of a charged mass point into the interior of the nucleus,

[1] J. R. Oppenheimer and M. Phillips, Phys. Rev. **48**, 500 (1935).

it is, however, necessary to assume that even a partial overlapping of the regions, to be ascribed to the motions of the elementary particles of which the nucleus and the deuteron respectively are composed, may result in a complete fusion of the two systems into a semistable compound nucleus.

On account of the weak binding energy of the deuteron the excitation of the compound nucleus will here be almost double as high as that which results from a neutron or proton impact. Still — except in the extreme case of mutual collisions between deuterons where the total energy approaches that of two free protons and two free neutrons too closely to allow an intermediate state of sufficient stability — the excitation energy of the compound system will be so small compared with the total binding energy of its particles that, like in other nuclear transmutations, the collisions can be separated into two well defined stages. In fact just the great variety of the disintegration processes of the compound system made possible by the high excitation in deuteron collisions offers many instructive examples of the competition responsible for the final result of nuclear reactions.

Addendum.

I. Under the simplifying assumption that each level represents a combination of a number of nearly equidistantly distributed quantities the density of nuclear levels for high excitation can be simply estimated by means of an asymptotic formula for the number of possible ways $p(n)$ any integer may be written as a sum of smaller positive integers which has been derived by G. H. HARDY and S. RAMANUJAN (Proc. London Math. Soc. (2) XLII, 75, 1918) and to which

our attention has recently be drawn. This formula can for large values of n be approximately written

$$p(n) = \frac{1}{4\sqrt{3}\,n}\, e^{\pi\sqrt{2/3\, n}}.$$

If now for the unity we assume an energy value of $2 \cdot 10^5$ e. V. corresponding approximately to the average distance between the lowest levels of heavier nuclei, we get for the number of combinations with which an excitation energy of $8 \cdot 10^6$ e. V. can be obtained $p(40) \cong 2 \cdot 10^4$, meaning an average level distance of about 10 e. V. which roughly corresponds to the densities of the level distribution estimated from slow neutron collisions.

II. A closer theoretical discussion of the characteristic features of the nuclear level distribution has been given by H. BETHE (Phys. Rev. **50**, 332, 1936, and Rev. mod. Phys. **9**, 69, 1937) who, on the basis of general theorems of statistical mechanics connecting the entropy of a thermodynamical system with the average energy, has estimated the density of energy levels of a highly excited nucleus for two different simplified models of nuclear excitation. In the first of these the coupling between the motion of the individual particles is entirely neglected for the sake of simplicity and the excitation energy is compared with that of a so called Fermi gas at low temperatures; in the second model the coupling is assumed to be close and the excitation energy is supposed to have its origin entirely in capillarity oscillations of the nuclear matter of the type discussed briefly in the text. Although none of these models can be assumed to reproduce the actual conditions in nuclei correctly, the calculations of BETHE offer instructive examples of the ways in which the typical character of the level scheme of nuclei follows from the assumption that the excitation energy is shared by the nuclear particles in a way corresponding to a thermal equilibrium.

Further interesting contributions to this problem have been given by L. LANDAU (Sow. Phys. **11**, 556, 1937) and V. WEISSKOPF (Phys. Rev. **52**, 295, 1937) who, without introducing any special assumptions as regards the origin of nuclear excitation, have calculated the nuclear level density by thermodynamical methods, assuming that the mean value of the excitation energy for a heavy nucleus is proportional to the square of its absolute temperature. This condition, which also is fulfilled in the first of the two special cases discussed by BETHE, does actually mean that the fundamental modes of motion in nuclei have energy values which are nearly equidistant. It is therefore interesting to note that the formulas for the nuclear level density derived from thermodynamical analogies are — at any rate as regards the exponential dependence on the total excitation energy of the nucleus — practically identical with the expression for $p(n)$ in Addendum I, if we by the number n understand the measure of the total energy with the energy differences between the lowest states, as unit.

III. The question of the origin of nuclear excitation involves great difficulties due not only to the scarcity of our knowledge of the specific nuclear forces but also to the complications of the quantum mechanical problem concerned. The aim of the simple remarks in the text is therefore in the first line to discuss certain possibilities of a simplified semi-empirical treatment. While in this respect the existence of quasi elastic oscillations of nuclei suggests itself by a straightforward correspondence argument, it is,

however, very doubtful whether such an argument can be legitimately applied to an analogy of nuclear excitation with capillary oscillations. In fact the comparison with a non viscous fluid involved in this analogy can hardly be maintained in view of the close coupling between the motions of the individual nuclear particles. Besides such a comparison would — as kindly pointed out to us by Prof. PEIERLS at a recent discussion in Copenhagen — force us to consider other types of inner nuclear motions as well, which in particular would be inconsistent with the comparison mentioned in the text of the rotational motion within a nucleus and that of a rigid body.

IV. The problem of the interaction between the orbital momenta and spin vectors of the nuclear particles has often been discussed not only in connection with the spin values of nuclei but also in attempts of accounting for the remarkable selection rules for various nuclear transmutations. Usually these effects are ascribed to a loose coupling between the orbital momenta of the individual particles and their spin vectors like that in atoms. In a recent paper by F. KALCKAR, J. R. OPPENHEIMER and R. SERBER (Phys. Rev. **52**, 279, 1937) it is shown, however, that it seems possible to explain these rules merely by assuming that the total angular momentum and the resulting intrinsic spin of the nuclear particles are coupled sufficiently loosely to allow a well defined quantum mechanical specification of their relative orientations.

V. A treatment of the nuclear photoeffect consistent with the views on nuclear excitation and radiation here discussed is attempted in a recent paper by F. KALCKAR, J. R. OPPENHEIMER and R. SERBER, (Phys. Rev. **52**, 273, 1937). In particular it is shown how it is possible from the remarkable experiments by W. Bothe and W. Gentner with high energy γ-rays (Naturwiss., 25, 90, 126, 191, 1937), to estimate the probabilities of radiative transitions from excited states to the normal state of the nucleus. For nuclei of medium atomic weight and 17 M. e. V. excitation these probabilities are in certain cases found to be of the order $\tau^{-1} \cdot 10^{-9}$ sec^{-1}, i. e., about $^1/_{100}$ of the most probable radiation probabilities for such nuclei. This comparatively large probability of such distant transitions contrasts strikingly with what might at first sight be expected from a simple comparison (see L. Landau, Sow. Phys. **11,** 556, 1937) of the radiation from an excited nucleus and a black body with the temperature of about a million volt per degree of freedom (see § 4). Still it may be remarked that such a comparison involves difficulties due to the high degree of polarity of nuclear radiation and the close coupling between the various modes of excitation mentioned in the text. Moreover the apparently capricious way in which the yield of the nuclear photoeffects varies from element to element suggests that we have in transitions from these highly excited nuclear states to the normal state to do with some peculiar features of the radiative mechanism connected perhaps with the appearance of dipole moments.

VI. A closer examination of the conditions for the application of an evaporation formula of the usual type to nuclear disintegration problems is given by V. Weisskopf in a recent paper cited in Addendum II. On the basis of general methods of statistical mechanics a detailed discussion is given there not only of the limitation of simple thermodynamical analogies in nuclear problems due to

the comparatively few degrees of freedom of the system concerned, but also of the generalisations of the usual thermodynamical procedure required for the proper treatment of such systems.

VII. The energy distribution of neutrons escaping from highly excited nuclei has been especially closely studied in the case of the usual neutron source of Beryllium bombarded with α-rays. While here the distribution of the fast neutrons is found to agree closely with the theoretical expectations, an apparent deviation is exhibited by the relative abundance of neutrons with energies far below the estimated temperature of the compound nucleus. This apparent difficulty disappears, however, if we assume that the slow neutrons in question must, as first suggested by P. Auger (Journ. de Physique, **4,** 719, 1933), be ascribed to a more complex process, the first stage of which is the escape of an α-ray from the compound system leaving a beryllium nucleus in an excited state, while the second stage consists in the subsequent breaking up of this nucleus into two α-particles and a slow neutron. This view is further strongly supported by a recent experimental investigation by T. Bjerge (Proc. Roy. Soc., in print).

VIII. The question of the quantum mechanical resonance effects in case of continuous level distribution has recently been more closely discussed by F. Kalckar, J. R. Oppenheimer and R. Serber in the paper cited in Addendum V, on the nuclear photoeffect, which presents special features analogous to the problem of nuclear transmutations initiated by impact of slow particles. A more comprehensive quantum mechanical treatment of nuclear reactions will further be given in a paper by F. Kalckar to appear shortly and in which it will especially be attempted to develop general arguments resembling the correspondence treatment of atomic radiation problems.

IX. The question of the proper estimate of the nuclear radii to be derived from the analysis of α-ray disintegration of radioactive nuclei is further discussed by Bethe in his recent report on nuclear dynamics (Rev. of Mod. Phys., **9**, 69, 1937), where he makes extensive use of the enlarged values of such radii which he has proposed in the paper cited on page 26. In this connection Bethe also comments on the criticism of this procedure of estimating nuclear radii given in the text and presented at the Conference in Washington (see Preface). Meanwhile an important contribution to this problem has been given by Landau in his paper cited in Addendum II, where he has succeeded from very general arguments in deducing a comprehensive formula for the dependence of the probability of nuclear disintegrations under release of charged particles on the external repulsion as well as on the density of the level distribution of the nucleus in the energy region concerned. In the case of radioactive decay, where the levels are widely separated, Landau's formula leads to values for the nuclear radii which differ only little from those derived from ordinary potential barrier formulas but which are essentially different from those proposed by Bethe. The closer connection between Landau's treatment and the argumentation given in the text will be discussed in the forthcoming paper of Kalckar which was mentioned above.

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THE STABILITY OF CRYSTALLINE PEPSIN

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