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WIRELESS
ECHOES OF LONG DELAY

BY

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Introduction*.

With regard to the origin of the long delayed echoes of short radio waves — λ about 31 m — observed by JÖRGEN HALS, C. STÖRMER[1], BALTH. VAN DER POL[2] and others, a rather great difference of opinion prevails between the various authors who have treated the question. Some, as f. inst. C. STÖRMER[1], P. O. PEDERSEN[3] and K. W. WAGNER[4,7] assume the echoes to be caused by the radio waves being reflected from, or propagated along, swarms or bands of electrons out in space, while others, as f. inst. BALTH. VAN DER POL[2], E. V. APPLETON[5] and M. v. ARDENNE[6,8], assume the long delay of the echoes to be due to particular conditions existing along the path of the waves in — or bounded by — the ionized part of the earths atmosphere. As to the manner in which the propagation of the waves

* Notes in square brackets are added after the date — 8th Februar 1929 — when the paper was read before The Royal Danish Society of Sciences.

1 C. STÖRMER: "Nature", Vol. 122, p. 681, 1928; Vol. 123, p. 16, 1929. "C. R." Tome 187, p. 811, 1928; "E. N. T." Bd. 5, p. 483, 1928. ["L'Onde Electrique", 7, p. 531—532, 1928.]

2 BALTH. VAN DER POL: "Nature" Vol. 122, p. 878, 1928. ["L'Onde Electrique", 7, p. 534—537, 1928.]

3 P. O. PEDERSEN: "Radiofoniens Aarbog" 1929, p. 9—25 (Copenhagen, Nov. 1928).

4 K. W. WAGNER: "E. N. T.", Bd. 5, p. 483, 1928.

5 E. V. APPLETON: "Nature", Vol. 122, p. 879, 1928.

6 M. v. ARDENNE: "Populær Radio", p. 345, Copenhagen, Decbr. 1928.

7 [DESLANDRES: "L'Onde Electrique", 7, p. 532—533, 1928.]

8 [H. S. JELSTRUP: "L'Onde Electrique", 7, p. 538—540, 1928.]

should then occur in order to show such great delays — up to about 30 seconds as were at the time observed — these authors have, however, very different opinions. M. v. Ardenne[1] thus assumes that the waves simply travel round the earth a sufficient numbers of times — some hundreds — in the Kennelly layer, the attenuation of which he assumes to be so small for short waves that in spite of the great distance travelled they will arrive at the receiver with sufficient strength. Appleton[2] points out that the length of the time of delay depends upon the group-velocity of the waves in the medium in question and if this velocity is small, then the length of path will also be correspondingly smaller. Appleton[2] further points out, however, that the waves will be too strongly attenuated in the ionized part of the atmosphere — at least at heights less than about 600 km. But he suggests another possibility, namely, that the lower boundary of the ionized layer of the upper atmosphere acts as a sharply defined reflecting "shell"[3]. B. v. d. Pol[4] also suggests the long echo-times to be due to very small group-velocities at places where the electrons are so densely crowded that the refractive index for the waves in question approaches zero. But he hardly pays the necessary attention to the attenuation of the waves under such circumstances.

We shall later discuss these various possibilities, but since a detailed treatment of the very complicated conditions of the path is extremely difficult, and since it is hardly possible to give an account of all the geometrically possible tracks of the waves, and, finally, since we know

[1] l. c.

[2] l. c.

[3] [Jelstrup, l. c., holds a similar view of the phenomenon.]

[4] l. c.

so very little about the atmosphere above 150 km we will try to find some criteria of general validity to throw light upon the matter.

1. The Attenuation of Electro-magnetic Waves in a homogeneous ionized Atmosphere.

Plane electro-magnetic waves propagated in a homogeneous medium are attenuated at the rate

$$e^{-\gamma_0 x + j\omega\left(t - \frac{x}{v}\right)} = e^{-\gamma_0 x + j\omega\left(t - n \cdot \frac{x}{c}\right)} \tag{1}$$

where γ_0 is the attenuation constant and

$$v = \frac{c}{n} \tag{2}$$

is the phase-velocity of the waves; $c = 3 \cdot 10^{10}$ cm sec^{-1} being the velocity of the waves in empty space and n the refractive index of the medium for waves of the frequency $f = \frac{\omega}{2\pi}$.

The parameters γ_0 and n are determined by[1]

$$\gamma_0 = \frac{\omega}{c}\sqrt{\sqrt{\frac{\varepsilon^2}{4} + \left(2\pi c^2 \frac{\sigma}{\omega}\right)^2} - \frac{\varepsilon}{2}} \tag{3}$$

and

$$n = \sqrt{\sqrt{\frac{\varepsilon^2}{4} + \left(2\pi c^2 \frac{\sigma}{\omega}\right)^2} + \frac{\varepsilon}{2}} \tag{4}$$

where ε is the dielectric constant in E. S. U. of the medium and σ its conductibility in E. M. U.

From (3) and (4) follows that[2]

$$\gamma_0 n = 2\pi c \sigma. \tag{5}$$

[1] P. O. Pedersen: "Propagation of Radio Waves", p. 117. (G. E. C. Gad, Copenhagen 1927). Cited in the following as "P. R. W.".

[2] "P. R. W.", p. 118 (6 a).

A signal "carried" by a wave of frequency $\omega = 2\pi f$ travels with the velocity u determined by[1]

$$u = \frac{c}{n+\omega\dfrac{dn}{d\omega}} = \frac{nc}{n^2+\dfrac{1}{2}\omega\cdot\dfrac{d(n^2)}{d\omega}} = \frac{nc}{g};\; g = n^2+\frac{1}{2}\omega\cdot\frac{d(n^2)}{d\omega}. \quad (6)$$

The attenuation of a wave which has travelled the distance x is according to (1) determined by

$$e^{-\gamma_0 x} \quad (1')$$

and the attenuation of a signal having travelled t seconds is consequently

$$e^{-\gamma_0 u t} \quad (7)$$

since $x = u\cdot t$.

According to the equations (5) and (6) we have

$$\gamma_0 x = \gamma_0 u t = \frac{\gamma_0 n c}{n^2+\dfrac{1}{2}\omega\cdot\dfrac{d(n^2)}{d\omega}}\cdot t = \frac{2\pi c^2\sigma}{g}\cdot t = \frac{t}{T_0} \quad (8)$$

where

$$T_0 = \frac{g}{2\pi c^2\sigma} \quad (9)$$

is the time the signal-wave must travel to have its amplitude decreased in the ratio e^{-1}.

The above formulae are valid generally for homogeneous media in which the attenuation of the waves is not excessively high. If the medium is an ionized atmosphere with N ions (electrons) per cm^3 and these are on the average suffering ν collisions per second, then the dielectric constant of the medium and its conductibility are determined by[2]

[1] "P. R. W.", p. 174 (68).
[2] "P. R. W.", p. 121 (11).

$$\varepsilon = 1 - N \cdot 4\pi \frac{e^2}{m} \cdot \frac{1}{\omega^2 + \nu^2} \text{ [E. S. U.]}$$

and (10)

$$\sigma = N \cdot 4\pi \frac{e^2}{m c^2} \cdot \frac{\nu}{\omega^2 + \nu^2} \text{ [E. M. U.]}$$

and these formulae are valid in the cases in which we may neglect the influence of the magnetic field of the earth.

By inserting

$$k = N \cdot 2\pi \frac{e^2}{m} = \frac{1}{2} \varkappa \omega \quad \text{or} \quad \varkappa = N \cdot 4\pi \cdot \frac{e^2}{m\omega^2} \tag{11}$$

the equations (10) will be

$$\varepsilon = 1 - \frac{2k}{\omega^2 + \nu^2} = 1 - \varkappa \frac{\omega^2}{\omega^2 + \nu^2}$$

and (10′)

$$2\pi c^2 \frac{\sigma}{\omega} = \frac{\nu}{\omega} \cdot \frac{k}{\omega^2 + \nu^2} = \frac{\frac{1}{2} \varkappa \omega \nu}{\omega^2 + \nu^2}.$$

From equations (4) and (10) we then find[1]

$$g = n^2 + \frac{1}{2} \omega \cdot \frac{d(n^2)}{d\omega}$$

$$= \frac{1}{2} \left\{ 1 - \frac{\varkappa \omega^2 \nu^2}{(\omega^2 + \nu^2)^2} + \frac{\left[(1-\varkappa)\omega^2 + \nu^2\right]^2 - \left(1 - \frac{1}{2}\varkappa\right) \cdot \frac{\varkappa \omega^2 \nu^2}{(\omega^2 + \nu^2)^2}}{\sqrt{\left[(1-\varkappa)\omega^2 + \nu^2\right]^2 + \varkappa^2 \omega^2 \nu^2}} \right\}. \tag{12}$$

For $(1-\varkappa)\omega^2 + \nu^2 \gg \varkappa \omega \nu$ or $\varkappa \ll \dfrac{\omega^2 + \nu^2}{\omega^2 + \omega \nu}$ we get

$$g \cong 1 - \frac{1}{2} \cdot \frac{\varkappa \omega^2 \nu^2}{(\omega^2 + \nu^2)^2} \cdot \frac{\left(2 - \frac{3}{2}\varkappa\right)\omega^2 + \left(2 - \frac{1}{2}\varkappa\right)\nu^2}{(1-\varkappa)\omega^2 + \nu^2}. \tag{13}$$

If further $\nu \ll \omega$, we have

[1] "P. R. W.", p. 172 (55).

$$g \cong 1 - \varkappa \frac{\nu^2}{\omega^2} \cdot \left(1 + \frac{\varkappa}{4(1-\varkappa)}\right) \tag{13'}$$

which for $\varkappa \ll 1$ reduces to

$$g \cong 1 - \varkappa \frac{\nu^2}{\omega^2}. \tag{13''}$$

In all cases where $\varkappa \ll \frac{\omega^2 + \nu^2}{\omega^2 + \omega \nu}$, $g = 1$ will consequently give a good approximation.

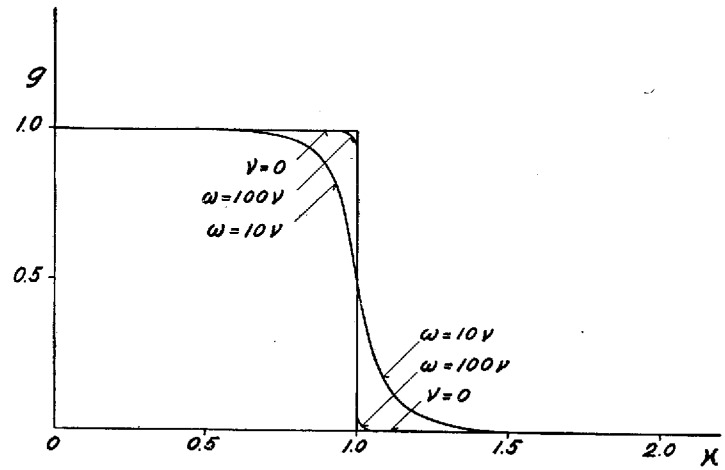

Fig. 1. The Factor g as a Function of $\varkappa$.

For $\varkappa = 2$ we have

$$g = \frac{\nu^4}{(\omega^2 + \nu^2)^2} \cong \frac{\nu^4}{\omega^4}. \tag{13'''}$$

Fig. 1 shows g as a function of $\varkappa$ and we see that for $\nu \leqq 10^{-2} \cdot \omega$ we get with good approximation $g = 1$ for $0 < \varkappa < 1$ while g is very nearly zero for $\varkappa > 1$. For $\varkappa = 1$ we get $g \cong \frac{1}{2}$.

From (6) it appears that for $g = 1$ the group-velocity is proportional to n. Therefore it will be of some interest to determine the smallest value which n can assume. By means of the equations (4), (10) and (11) we easily deduce that n will have minimum value for $\varkappa = 2$ and that

$$n_{\min} = \frac{\nu}{\sqrt{\omega^2 + \nu^2}} \cong \frac{\nu}{\omega} \tag{14}$$

while the corresponding value of the dielectric constant will be

$$\varepsilon = \frac{\nu^2 - \omega^2}{\nu^2 + \omega^2} \cong -1. \qquad (15)^1$$

Another point of particular interest is the one corresponding to $\varkappa = \frac{\omega^2 + \nu^2}{\omega^2} \cong 1$, here we get $n \cong \sqrt{\frac{\nu}{2\,\omega}}$ and $\varepsilon = 0$.

The density of electrons[2] corresponding to the various values of $\varkappa$ is according to (11) determined by

$$N = \frac{\varkappa\,\omega^2}{4\,\pi\,\frac{e^2}{m}} = 3.14 \cdot 10^{-10} \cdot \varkappa\,\omega^2 \text{ electrons per cm}^3. \tag{16}$$

The refractive index is thus independently of the air pressure minimum for

$$N = 6.29 \cdot 10^{-10} \cdot \omega^2 = 2\,N_0. \tag{17}$$

Assuming that $\nu \ll \omega$ we further get $\varepsilon \cong 0$ for $N = N_0 = 3.14 \cdot 10^{-10} \cdot \omega^2$.

[1] In the deduction of (14) no account is taken of the influence of the earth's magnetic field on the conductibility and on the dielectric constant. If we do so then we get (see "P. R. W.", p. 122) that for

$$\varkappa = 2 \cdot \frac{\omega - h}{\omega} \quad \text{we get} \quad n_{\mathrm{I}\,\min} \cong \frac{\nu}{\omega - h}$$

and for

$$\varkappa = 2 \cdot \frac{\omega + h}{\omega} \quad \text{we get} \quad n_{\mathrm{II}\,\min} \cong \frac{\nu}{\omega + h}.$$

[2] We generally only speak of electrons, because one electron will influence the propagation of the waves about as much as 50 000 ions.

Table 1. The values given are valid for $\nu \ll \omega$, $N_0 = 3.14 \cdot 10^{-10}\,\omega^2$.

$\varkappa = \frac{N}{N_0}$	ε	n	g	u	T_0	γ_0	$e^{-\gamma_0 \lambda}$
$0 \leqq \varkappa < 1$	$1 - \varkappa$	$\sqrt{1-\varkappa}$	1	$c\sqrt{1-\varkappa}$	$\frac{2}{\varkappa \nu}$	$\frac{\nu}{2c} \cdot \sqrt{\frac{\varkappa^2}{1-\varkappa}}$	$e^{-\pi \frac{\varkappa \nu}{(1-\varkappa)\omega}}$
$\varkappa = 1$	0	$\sqrt{\frac{\nu}{2\omega}}$	$\frac{1}{2}$	$c\sqrt{2\frac{\nu}{\omega}}$	$\frac{1}{\nu}$	$\frac{1}{c} \cdot \sqrt{\frac{\omega \nu}{2}}$	$e^{-2\pi}$
$\varkappa = 2$	-1	$n_{\min} = \frac{\nu}{\omega}$	$\frac{\nu^4}{\omega^4}$	$\left(c \cdot \frac{\omega^3}{\nu^3}\right)^*$	$\left(\frac{\nu^3}{\omega^4}\right)^*$	$\frac{\omega}{c}$	$e^{-2\pi \frac{\omega}{\nu}}$

* These expressions are formally in agreement with the formulae (6) and (9) but have no physical significance. See the text.

In table 1 is given a view of the constants which are of interest for the propagation problem here considered. Beside ε, n and g the table contains the formulae for the group-velocity u, for the constant T_0, for the attenuation-constant γ_0 and for the attenuation-factor $e^{-\gamma_0 \lambda}$ which latter indicates the attenuation of the amplitude by its travelling a distance equal to one wave-length in the medium in question.

From the table it appears that for $\varkappa = 2$ we find $u > c$. Since, however, we must always necessarily have $u \leqq c$ the method of determining the group-velocity applied has led to wrong results, and the corresponding value of T_0 is consequently also of no significance. To emphasize these circumstances both of these expressions are placed in brackets.

The reason why we come to such unreasonable results at great densities of electrons — or more correctly, at great values of $\varkappa$ — is quite evident. In the deduction of the formula (6) for the group-velocity we assumed the "signal" to be produced by the superposition of a number of continuous waves having a frequency only very slightly different from the carrier frequency $f = \frac{\omega}{2\pi}$. From the last column in the table it appears, however, that for $\varkappa = 2$ the attenuation will be so great that the waves will lose entirely their periodical character. If for example we put $\omega = 6 \cdot 10^7$ and $\nu = 6 \cdot 10^3$, then by travelling one wave-length the amplitude will have decreased at the rate $e^{-2\pi \cdot 10^4}$, and in this case the formula (6) must consequently lead to unreasonable results. There is, however, no reason to enter further into this question since it is quite evident that the attenuation here is so great that waves entering a medium of the character considered within a period will be reduced practically to zero.

Even for $\varkappa = 1$ where we get $u < c$ it is doubtful whether the values found for u and T_0 will be correct since in this case the attenuation over a wave-length will be $e^{-2\pi}$. The long delayed echoes observed would consequently be unable to pass through a medium corresponding to $\varkappa = 1$, and in the following we will therefore confine ourselves to treat the transmission of signals through such media for which $0 \leqq \varkappa < 1$. For these we get

$$T_0 = \frac{2}{\varkappa \nu} \tag{18}$$

and after t seconds the attenuation of plane waves in such a medium will be

$$e^{-\frac{t}{T_0}} = e^{-\frac{\varkappa \nu}{2} \cdot t}. \tag{19}$$ [1]

In order to get in t seconds a track-length travelled by the wave of $\xi \cdot c \cdot t$ instead of $c \cdot t$ as would be the case in empty space we must have

[1] For $\varkappa \rightarrow 1$ the attenuation deduced here is in agreement with the one given by E. V. Appleton (l. c.).

$$\xi = \sqrt{1-\varkappa} \quad \text{or} \quad \varkappa = 1-\xi^2. \tag{20}$$

During t seconds the amplitude will decrease at the rate

$$e^{-\frac{t}{T_0}} = e^{-\frac{\varkappa\nu}{2}\cdot t} = e^{-\frac{1}{2}(1-\xi^2)\nu t}. \tag{21}$$

If the track traversed by the wave is to be very much shorter than what it would be during an equal time in empty space then we must have $\xi^2 \ll 1$. In that case the attenuation S during t seconds will be approximately

$$S = e^{-\frac{1}{2}\nu t} \tag{22}$$

which is exactly the expression used by Appleton.

To $\xi \rightarrow 0$ corresponds $\varkappa \rightarrow 1$ and the density of electrons $N \rightarrow N_0 = 3 \cdot 14 \cdot 10^{-10}\omega^2$.

Appleton assumes that for a height of 250 km above the surface of the earth we may put $\nu = 1000$. I should be inclined for this height to apply $\nu = 360$[1]. For $t = 10$ seconds the attenuation factors would then be respectively

$$S = e^{-5000} \quad \text{and} \quad S = e^{-1800},$$

and accordingly we would get an attenuation much too great even for a delay of 10 seconds by applying even the value for ν assumed by me. For a height of 400 km Appleton assumes $S = e^{-50}$ for $t = 10$ seconds which he, however, considers as too great an attenuation; but he adds: "But if there were sufficient ionisation at heights of 600 km or more, it is certain that retardation without much absorption could take place, although our inadequate knowledge of the values of ν for such regions precludes a more quantitative statement".

In estimating the value of ν, only the collisions of the

[1] "P. R. W." Appendix, p. 6, fig. IX. 1; compare also table 3 below.

electrons with neutral molecules have, however, so far been considered, and not their collisions with other electrons or ions. The error thus introduced is, however, small, as long as the number of electrons and ions are exceedingly small in comparision to the number of neutral molecules, which will always be the case — at least for the circumstances here considered — as long as the air pressure is not extremely small. The error will, however, be very great if the air pressure approaches zero, or if the number of electrons + ions approaches — or even exceeds — the number of neutral molecules.

We will therefore now consider the free path l and the number of collisions ν of a pure electron atmosphere having N electrons per cm³. According to W. Schottky and H. Rothe[1] the mean free path is determined by

$$l = \frac{1}{\pi \sqrt{2}\, N d^2} \tag{23}$$

where d is the "diameter" of the electron. The latter is determined by the potential electric energy of the two colliding electrons — or of an electron and a positive (or negative) ion — being equal to $\frac{1}{\eta}$ of the kinetic energy of a single electron, when they are at a distance d from each other.

The mean value of the kinetic energy of a single electron will, at a temperature T° abs., be determined by

$$\frac{1}{2} m U^2 \doteq \frac{3}{2} k T = 2.058 \cdot 10^{-16} T, \tag{24}$$

where $k = 1.372 \cdot 10^{-16}$ is the Boltzmann constant.

The number of collisions ν is determined by

[1] Handb. d. Experimentalphysik. Bd. XIII. 2 p. 41, 1928.

$$\nu = \frac{U}{l} = \pi \sqrt{2}\, N d^2 \sqrt{\frac{3kT}{m}}, \tag{25}$$

the mean velocity of the electron — according to (24) — being

$$U = \sqrt{\frac{3kT}{m}} = 6.76 \cdot 10^5 \sqrt{T}. \tag{25'}$$

The diameter d will then be determined by

$$\frac{e^2}{d} = \frac{1}{\eta} \cdot \frac{1}{2} m U^2 = \frac{1}{\eta} \cdot \frac{3}{2} k T = \frac{1}{\eta} \cdot 2.058 \cdot 10^{-16}\, T \tag{26}$$

or

$$d = \frac{\eta e^2}{\frac{3}{2} k T} = 1.107 \cdot 10^{-3} \frac{\eta}{T}. \tag{26'}$$

SCHOTTKY and ROTHE assume as an upper limit $\eta = 100$ to which corresponds

$$d = 0.1107 \cdot \frac{1}{T} \cdot [\mathrm{cm}]. \tag{27}$$

By (23) and (26') the free path will generally be

$$l = 1.84 \cdot 10^5 \frac{T^2}{\eta^2 N} \tag{28}$$

and the number of collisions

$$\nu = 3.68 \cdot \frac{\eta^2}{T^{\frac{3}{2}}} \cdot N. \tag{29}$$

At f. inst. $T = 300°$ abs. we then get $\nu = 7.08 \cdot 10^{-4} \eta^2 N$.

The value $\eta = 100$ assumed by SCHOTTKY and ROTHE[1] for other purposes is no doubt too great for the object here considered. $\eta = 1$ is, on the other hand, no doubt too small. The formula (29) shows, however, the necessity of fixing somewhat narrower limits for η or for the diameter d which depends on η.

[1] l. c.

For reasons which will be given in the following we will, however, not consider collisions between two electrons but between an electron and a positive — or negative — ion, and we assume the latter to be at rest as well before as after the collision.

The angle of deflection, 2θ, of the electron — see fig. 2 — is determined by[1]

$$tg\,\theta = \frac{e^2}{m\,U^2 p}, \qquad (30)$$

where U is the velocity of the electron and p the distance of the ion from the straight path of the electron.

By means of (25′) and (26) equation (30) is reduced to

$$tg\,\theta = \frac{1}{2\eta}\cdot\frac{d}{p} = \frac{\alpha}{p} \qquad (31)$$

where

$$\alpha = \frac{d}{2\eta} = \frac{e^2}{3\,k\,T} = \frac{5.5\cdot 10^{-4}}{T}. \qquad (31')$$

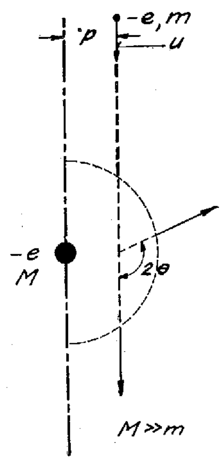

Fig. 2. Collision between an Electron and a fixed negative Ion.

The velocity component of the electron after the collision, in the original direction will be equal to the original velocity multiplied by

$$\zeta' = 1 - 2\sin^2\theta = \frac{p^2 - \alpha^2}{p^2 + \alpha^2}. \qquad (32)$$

That fraction of the original velocity which is deflected from the original direction is consequently

$$\zeta = 1 - \zeta' = \frac{2\,\alpha^2}{p^2 + \alpha^2}. \qquad (32')$$

By integrating this quantity over a circular area at right angles to the original direction of the electron, and having the radius p_0, we get

[1] See f. inst. Handb. d. Physik XXIV, p. 4, 1927.

$$A = \int_0^{p_0} \zeta \cdot 2\pi p\, dp = 2\pi \alpha^2 \cdot lg \frac{p_0^2 + \alpha^2}{\alpha^2}. \qquad (33)$$

If we put

$$A = \pi d_0^2 \qquad (34)$$

then d_0 may be taken as a kind of equivalent-diameter and we then get

$$d_0 = \alpha \sqrt{2\, lg \frac{p_0^2 + \alpha^2}{\alpha^2}}. \qquad (35)$$

Table 2 contains some values of $\frac{d_0}{\alpha}$ for various values of $\frac{p_0}{\alpha}$.

Table 2. Values of $\sqrt{2\, lg \frac{p_0^2 + \alpha^2}{\alpha^2}}$ for various values of $\frac{p^0}{\alpha}$.

$\frac{p_0}{\alpha} =$	1	2	3	4	5	10	20	100	1000
$\sqrt{2\, lg \frac{p_0^2 + \alpha^2}{\alpha^2}} = \frac{d_0}{\alpha} =$	1.18	1.79	2.15	2.38	2.55	3.04	3.46	4.29	5.26

d_0 increases steadily but slowly with increasing values of p_0.

These calculations assume, however, that along the entire curved part of the path the colliding particles are so far away from other charged particles that their movements are practically unaffected by these. Since the density of electrons, according to table 1, must be about $N = N_0 = 3.14 \cdot 10^{-10} (2\pi)^2 \cdot 10^{14} = 1.24 \cdot 10^6$ electrons per cm^3, in order to reduce considerably the group-velocity of the 30 m wave, and since according to (31′) $\alpha \cong 10^{-6}$ cm for $T = 550°$ abs. it will hardly be justifiable to assign to $\frac{p_0}{\alpha}$ a value much higher than 100, to which corresponds $d_0 = 4.29\, \alpha$. On the other hand, the value $\frac{p_0}{\alpha}$ should, no doubt, be higher than 3 to which corresponds $d_0 = 2.15\, \alpha$.

Since for the following it is not essential to know the exact value of d_0 we simply put

$$d_0 = 3\alpha = \frac{1.65 \cdot 10^{-3}}{T} \, [\text{cm}] \qquad (35')$$

which is 67 times smaller than the value assumed by SCHOTTKY and ROTHE.

To (35′) corresponds a length of free path

$$l = 8.3 \cdot 10^4 \frac{T^2}{N} \qquad (36)$$

and a number of collisions

$$\nu = \frac{u}{l} = \frac{6.7 \cdot 10^5 \, T^{\frac{1}{2}}}{8.3 \cdot 10^4 \, T^2} \cdot N = 8.1 \cdot \frac{N}{T^{\frac{3}{2}}}. \qquad (37)$$

Table 3 shows for some values of T the corresponding values of the ratio $\frac{\nu}{N}$.

We cannot, however, insert the values of ν thus obtained in the formula (22) and by this means calculate the attenuation of the waves in an atmosphere consisting solely of electrons. In this case, where the colliding particles are perfectly identical, a collision will exert no influence at all on the propagation of the waves, since we have

Table 3. The ratio $\frac{\nu}{N}$ calculated by means of (37). Values of ν for $N = 1.2 \cdot 10^6$.

$T =$	100°	400°	900°	1600°	2500°	3600°	10 000°	250 000° abs.
$\frac{\nu}{N} =$	$8.1 \cdot 10^{-3}$	$1.0 \cdot 10^{-3}$	$3.0 \cdot 10^{-4}$	$1.3 \cdot 10^{-4}$	$6.5 \cdot 10^{-5}$	$3.8 \cdot 10^{-5}$	$8.1 \cdot 10^{-6}$	$6.5 \cdot 10^{-8}$
$N = 1.2 \cdot 10^6$; $\nu =$	$1 \cdot 10^4$	$1.2 \cdot 10^3$	$3.6 \cdot 10^2$	$1.6 \cdot 10^2$	78	46	9.7	$7.8 \cdot 10^{-2}$

and

$$\left.\begin{array}{c} v_{1x}+v_{2x} = v'_{1x}+v'_{2x}\,; \quad v_{1y}+v_{2y} = v'_{1y}+v'_{2y}\,; \\ v_{1z}+v_{2z} = v'_{1z}+v'_{2z}\,; \\ v_1^2+v_2^2 = v_1'^2+v_2'^2\,, \end{array}\right\} \quad (38)$$

where v_1 and v_2 are the velocities of the electrons before, and v'_1 and v'_2 their velocities after the collision.

The sum of the "current" components, viz. $-e\,(v_{1x}+v_{2x})$, $-e\,(v_{1y}+v_{2y})$, $-e\,(v_{1z}+v_{2z})$, and of the kinetic energy of two electrons is thus not altered. Radio waves are consequently propagated in an atmosphere consisting solely of electrons without suffering any loss and in the same manner as if there were no collisions at all between the electrons[1].

On account of their unstable character such pure electron "atmospheres" ("layers" or "bands") cannot, however, play any important rôle in the cases of propagation of radio waves here considered. To illustrate this we will consider such a large, plane band of electrons or ions.

The mutual repulsion will cause the thickness of the band to increase and consequently the density of electrons to decrease.

It is easily proved that the density q of the charge within a plane layer, see Fig. 3 part I, decreases with increasing time according to the following formula

[1] [In a letter in "Nature" (February 2, 1929, p. 166) L. H. THOMAS has also called attention to the influence which the other electrons and ions, that are present in the neighbourhood of an electron or ion, exert on the length of the effective mean free path of this electron or ion. He does not, however, mention the fundamental difference, with regard to the attenuation of radio waves, between collisions taking place between charged particles of equal charges and equal masses, and collisions in which the two particles have unequal masses or charges (or both).]

$$q_t = \frac{q_0}{1 + 2\pi q_0 \frac{e}{m} \cdot t^2} \quad \text{[E. S. U.]} \tag{39}$$

q_0 being the density at the time $t = 0$, and q_t the density t seconds later.

Fig. 3 part II, sections 1, 2 and 3 show the dispersion of a plane homogeneous layer having at the time $t = 0$ a

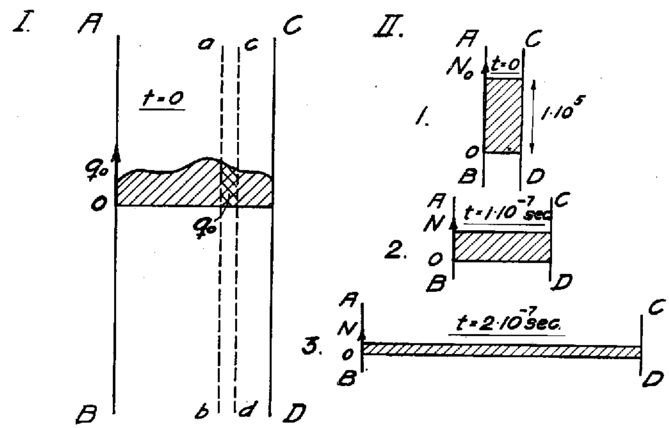

Fig. 3. *ABCD* represent parts of large, plane Bands at Electrons or Ions. In part I q_0 is the Density of the Charge at the time $t = 0$.

density of $N = 1 \cdot 10^5$ electrons per cm^3. Section 2 shows the distribution $1 \cdot 10^{-7}$ sec. and section 3 $2 \cdot 10^{-7}$ sec. later.

From this it appears that a band of electrons having a density of 10^6 can exist only if inside the band there is a positive space charge of practically equal value, and evidently it must be mainly the less mobile positive particles which determine the geometric relations of the band.

If the positive space charge consists of mono-valent positive ions, then the number of "effective" collisions ν is determined simply by means of (37), N being the total density of the positive and negative atomic or molecular ions, and the attenuation of the wave is found by inserting in (22) the value of ν thus obtained. If the positive and negative ions are poly-valent, then ν, and accordingly also the attenuation, will be even greater.

In order that a 30 m wave shall travel at a slow group-velocity the electron density of the medium must — as is shown in the preceding — be about $1.2 \cdot 10^6$ electrons per cm^3. For a temperature of 900° abs., according to table 3, we then get $\nu = 360$. A wave travelling in such a medium for 10 seconds will, according to (22), suffer an attenuation $S = e^{-\frac{1}{2}\nu t} = e^{-1800}$. For $T = 1600$ abs. we get $S = e^{-800}$.

From the preceding it appears that even if we pay no attention to the attenuation suffered by the waves in consequence of collisions between electrons and neutral air molecules, the radio waves will, nevertheless, in consequence of the collisions between the electrons and the necessarily existing positive ions be so greatly attenuated that the long delayed wireless echoes cannot have travelled a considerable part of the time within an electron-ion-atmosphere giving a very small group-velocity[1].

[1] Even if the long delayed echoes cannot be due to propagation in media having a very small group-velocity, such propagation may play an important rôle in the various methods of determining the altitude of the highest point of the path of radio rays, often called the height of the ionized layer, and may have caused some of the discrepancies between the values which various experimenters have found for this altitude in so far as the discrepancies are not due to "reflections" from various "layers" (See "P. R. W." p. 211). This question is treated of in "P. R. W." pp. 171—178, 209—212, and the theory is further developed by E. V. APPLETON (Proc. Phys. Soc. Vol. 41, p. 43—56, 1928) and J. C. SCHELLENG (Proc. Inst. Rad. Eng., Vol. 16, p. 1471—76, 1928), whose papers will be a great help to the investigation of this problem in the future.

2. The Propagation of Electro-magnetic Waves Along the Boundary Surface between an Ionized and a Non-ionized Part of the Earth's Atmosphere.

Having now shown that the long delayed echoes cannot be due to the propagation of the waves in a space containing so many electrons that the group-velocity is many times smaller than the velocity of the waves in free space, because in such a medium the attenuation will be too great, it may be appropriate to investigate whether the long delays may be due to the fact that the waves are propagated along the boundary between such a medium and either empty space or a non-ionized air space. A smaller attenuation would be possible in either case, since the loss of energy in the ionized part could be compensated for by the field energy in the non-ionized part. Something similar occurs in a very pronounced manner in the propagation of the waves along the surface of the earth.[1] We will therefore now consider a little more closely the propagation of the waves along such a plane boundary-surface.

The boundary-surfaces will, of course, generally be curved but the radii of curvature must at all events be very great if propagation along such surfaces shall be of any significance at all for the cases here considered. Further, the concavity of the boundary-surface must face toward the earth and toward the non-ionized part of space and will thus serve to collect the radiation-energy. The boundary between the conductive and the non-conductive part of space will never be perfectly sharp because some electrons and ions will penetrate into the otherwise non-conductive part. Under such conditions the waves will be refracted and show a tendency to follow the curved boundary-surface.

[1] "P. R. W." chap. III. p. 22 and 29—32.

The conditions existing at a plane boundary-surface will, no doubt, give an approximately correct representation of the actual conditions. Considerable mathematical difficulties would also be met with in the treatment of a propagation along a curved surface, while the problem of propagation along a plane boundary surface — as shown by J. ZENNECK[1] — may be solved in a simple manner.

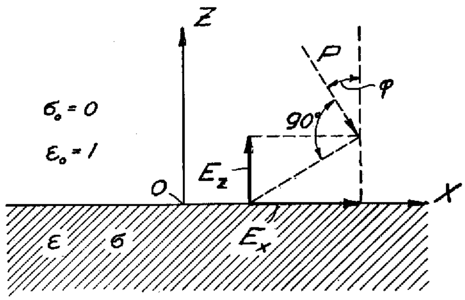

Fig. 4. Propagation of Waves along the Boundary-Surface XY. In the upper Medium the Energy is propagated in the Direction indicated by the Arrow P.

By considering the boundary surface to be an XY-plane and then assuming the plane waves to travel along the plane in the positive direction of the x-axis — see fig. 4 — the attenuation along the boundary surface will be determined by

$$e^{-\gamma_0 x}. \tag{1}$$

If we call the phase-velocity along the surface v then γ_0 and v can be determined by the following equation:[2]

$$\gamma_0 + j\frac{\omega}{v} = j\frac{\omega}{c}\sqrt{\frac{1 + j\frac{\omega}{\sigma}\cdot\frac{\varepsilon}{4\pi c^2}}{1 + j\frac{\omega}{\sigma}\cdot\frac{1+\varepsilon}{4\pi c^2}}} \tag{2}$$

[1] J. ZENNECK: Ann. d. Phys. (4) Bd. 23, p. 846, 1907. "P. R. W." Chap. III.
[2] "P. R. W." p. 17 (the symbols used are different).

provided the medium (0) has the dielectric constant $\varepsilon_0 = 1$ and the conductivity $\sigma_0 = 0$ while the corresponding constants for the ionized medium are ε and σ [E. M. U.].

Let the X- and Z-components of the electric field at the boundary-surface be E_x and E_z then we have

$$\left|\frac{E_x}{E_z}\right|_{z=+0} = \left|\sqrt{\frac{j\frac{\omega}{\sigma}\cdot\frac{1}{4\pi c^2}}{1+j\frac{\omega}{\sigma}\cdot\frac{\varepsilon}{4\pi c^2}}}\right| \tag{3}$$

where the index $z = +0$ indicates that the field components shall be taken in the insulated medium but immediately at the boundary-surface $z = 0$.

If $0 < \varepsilon < 1$ and $\frac{\omega}{\sigma}\cdot\frac{\varepsilon}{4\pi c^2} \gg 1$ then (2) and (3) will give

$$\gamma_0 = \frac{2\pi c \sigma}{(1+\varepsilon)\sqrt{\varepsilon(1+\varepsilon)}}, \quad v = c\sqrt{\frac{1+\varepsilon}{\varepsilon}} = u$$

and (4)

$$\left|\frac{E_x}{E_z}\right|_{z=+0} = \sqrt{\frac{1}{\varepsilon}}.$$

Since here there is no dispersion, the group-velocity is equal to the phase-velocity and both are greater than the velocity c in empty space. This is quite natural, since the energy-propagation does not occur along the boundary surface but in a direction P (see fig. 4) which is perpendicular to the resultant of E_x and E_z. If ε is extremely small, then the energy-propagation will take place practically perpendicular to the boundary-surface.

For the attenuation exponent we get (compare equation (10′) in sect. 1)

$$\gamma_0 x = \gamma_0 u t = \frac{2\pi c^2 \sigma}{\varepsilon(1+\varepsilon)} \cdot t$$
$$= \frac{\frac{1}{2}\varkappa \frac{\omega^2 \nu}{\omega^2+\nu^2}}{\left(1-\varkappa\frac{\omega^2}{\omega^2+\nu^2}\right)\cdot\left(2-\varkappa\frac{\omega^2}{\omega^2+\nu^2}\right)} \cdot t \cong \frac{\frac{1}{2}\varkappa\nu t}{(1-\varkappa)(2-\varkappa)}, \tag{5}$$

where the last term is correct only for $\nu \ll \omega$.

In order to obtain a small attenuation exponent the smallest possible value of $\varkappa$ should be chosen. If $\varkappa \ll 1$ then for (5) we get

$$\gamma_0 x = \gamma_0 u t = \frac{1}{4}\varkappa\nu t. \tag{5'}$$

We will come back later to the application of this result.

Next we will consider the case where the density of electrons is so great that $\varkappa > 2$, and we therefore have $\varepsilon = 1 - \varkappa\frac{\omega^2}{\omega^2+\nu^2} < 0$. We then get

$$\frac{\omega}{\sigma}\cdot\frac{\varepsilon}{4\pi c^2} = \frac{\omega^2+\nu^2}{\varkappa\omega\nu} - \frac{\omega}{\nu} \quad \text{and} \quad \frac{4\pi c^2\sigma}{\omega} = \varkappa\frac{\omega\nu}{\omega^2+\nu^2}.$$

For $\nu \ll \omega$ by means of (2) we get

$$\gamma_0 = \frac{\nu}{2c}\cdot\frac{\varkappa}{(\varkappa-2)^{\frac{3}{2}}\cdot(\varkappa-1)^{\frac{1}{2}}} \quad \text{and} \quad v = c\sqrt{\frac{\varkappa-2}{\varkappa-1}} = u. \tag{6}$$

The attenuation exponent will then be

$$\gamma_0 x = \gamma_0 u t = \frac{\varkappa\nu}{2(\varkappa-1)(\varkappa-2)}\cdot t \cong \frac{\nu}{2\varkappa}\cdot t \tag{6'}$$

where the last term holds good for $\varkappa \gg 1$.

Finally, if $\nu \gg \omega$ then

$$\gamma_0 = \frac{\varkappa\omega^2}{4\sqrt{2}\,c\nu}, \quad \nu = c\sqrt{2} = u \tag{7}$$

to which corresponds

$$\gamma_0 x = \gamma_0 u t = \frac{\varkappa}{4} \cdot \frac{\omega^2}{\nu} \cdot t. \tag{7'}$$

For a judgement of the conditions we must know how rapidly the amplitude of the waves decrease in the non-conductive medium and in a direction perpendicular to the boundary-surface.

Using the symbols from fig. 4 this decrease is determined by

$$e^{-r_0 z}, \tag{8}$$

where r_0 and the phase velocity of the waves w in the direction of the Z-axis is determined by[1]

$$r_0 + j\frac{\omega}{w} = j\frac{\omega}{c} \cdot \sqrt{\frac{j\frac{\omega}{\sigma} \cdot \frac{1}{4\pi c^2}}{1 + j\frac{\omega}{\sigma} \cdot \frac{1+\varepsilon}{4\pi c^2}}}. \tag{9}$$

For $\varkappa < 1$ and $\nu \ll \omega$ we consequently get

$$r_0 = \frac{\varkappa \nu}{2c(2-\varkappa)^{\frac{3}{2}}}. \tag{10}$$

The values of r_0 and of the ratio $\left|\frac{E_x}{E_z}\right|_{z=+0}$ and further of $\varphi = \operatorname{arctg}\left|\frac{E_z}{E_x}\right|$ as well as the above obtained results are shown in table 4.

From this table it appears that the attenuation becomes very great for $\varepsilon < -1$ ($\varkappa > 2$) both for $\nu \ll \omega$ and still more for $\nu \gg \omega$. In both cases it is so great that it is out of question that the corresponding wave-propagation can form the basis of the long delayed echoes. A closer consideration further shows that the attenuation-exponent

[1] "P. R. W.", p. 17.

Table 4. The Attenuation-Exponents for the Propagation of plane Waves along a Boundary-Surface between an Ionized and a Non-ionized Part of Space.

Assumptions $\varepsilon_0 = 1;\ \sigma_0 = 0$			Attenuation exponent $\gamma_0 x = \gamma_0 u t$ ν	$\nu = \varkappa \nu_1$	Group-velocity along the boundary-surface u	Attenuation-constant r_0	$\left\lvert \frac{E_x}{E_z} \right\rvert_{z=+0}$	φ $\operatorname{arctg} \left\lvert \frac{E_z}{E_x} \right\rvert = \tan^{-1} \left\lvert \frac{E_z}{E_x} \right\rvert$
$0 < \varepsilon < 1$	$\nu \ll \omega$	$0 < \varkappa < 1$	$\frac{\varkappa \nu}{2(1-\varkappa)(2-\varkappa)} \cdot t$	$\frac{\varkappa^2 \nu_1}{2(1-\varkappa)(2-\varkappa)} \cdot t$	$c \sqrt{\frac{1+\varepsilon}{\varepsilon}}$	$\frac{\varkappa \nu}{2c(2-\varkappa)^{\frac{3}{2}}}$	$\sqrt{\frac{1}{1-\varkappa}}$	$\operatorname{arctg} \sqrt{1-\varkappa} = \tan^{-1} \sqrt{1-\varkappa} < 45°$
	$(\varepsilon \cong 1-\varkappa)$	$0 < \varkappa \ll 1$	$\frac{1}{4} \varkappa \nu t$	$\frac{1}{4} \varkappa^2 \nu_1 t$	$c \sqrt{\frac{1+\varepsilon}{\varepsilon}} \cong c\sqrt{2}$	$\frac{\varkappa \nu}{4\sqrt{2} \cdot c}$	1	45°
$\varepsilon < -1$	$\nu \ll \omega$	$\varkappa \gg 2$	$\frac{\nu}{2\varkappa} \cdot t$	$\frac{\nu_1}{2} \cdot t$	$c \sqrt{\frac{\varkappa-2}{\varkappa-1}}$			
	$\nu \gg \omega$	$\varkappa > 2$	$\frac{\varkappa \omega^2}{4\nu} \cdot t$	$\frac{\omega^2}{4\nu_1} \cdot t$	$c\sqrt{2}$			

becomes very great within the interval $0 > \varepsilon > -1$ so that this assumption also is out of the question.

We then only have the interval $0 < \varepsilon < 1$ $(1 > \varkappa > 0)$. For $\varkappa \to 1$ and consequently $\varepsilon \to 0$ we have $\gamma_0 u t \to \infty$ while for $\varkappa = \frac{1}{2}$ and $\varepsilon = \frac{1}{2}$ the attenuation-exponent $\gamma_0 u t = \frac{\nu_1}{6} \cdot t$. Taking $\nu_1 = 360$ and $t = 10$ seconds we get an attenuation-factor $S = e^{-600}$ while $\varkappa = \frac{1}{4}$ and $\varepsilon = \frac{3}{4}$ gives $S = e^{-85.7}$ for $t = 10$ seconds. In order to obtain by the waves propagated in this manner the long delayed echoes we must consequently have that $0 < \varkappa < \frac{1}{4}$ and accordingly $\frac{3}{4} < \varepsilon < 1$.

From the table 4 we learn that the attenuation-exponent $\gamma_0 u t$ along the boundary-surface will be the smaller the smaller we have the value of $\varkappa$. We cannot, on the other hand, have $\varkappa = 0$, as in that case we should have no boundary-surface at all and therefore the entire energy of the waves would disappear out into space. We are unable to fix any definite limit for the value of $\varkappa$. However, if the waves must be able to travel several hundred times round the earth, which is necessary in order to obtain echoes up to 10 seconds or even more, then the refractive index $n = 1 - \Delta n$ of the ionized layer must assume such small values that waves leaving the transmitter in a horizontal direction are totally reflected at the boundary-surface. At a still smaller value of the reduction Δn of the refractive index practically all of the energy radiated would proceed out into space and get lost. We therefore must have

$$\Delta n \geqq \frac{h}{R} \tag{11}$$

where h is the height of the boundary-surface above the earth and R the radius of the earth.

Since

$$n = 1 - \Delta n \cong \sqrt{\varepsilon} \cong \sqrt{1-\varkappa} \cong 1 - \frac{1}{2}\varkappa$$

we get

$$\varkappa = 2\Delta n \geqq 2\frac{h}{R}. \qquad (12)$$

Taking $h = 160$ km and $R = 6400$ km we find by means of (11) and (12) that $\varkappa \geqq \frac{1}{20}$ and the corresponding value of the attenuation-exponent for $t = 10$ seconds, is, according to table 4.

$$\gamma_0 u t = \frac{1}{4}\varkappa^2 \nu_1 t = 2.25 \qquad (12')$$

for $\nu_1 = 360$. The attenuation-factor will then be $S = e^{-2.25}$.

This attenuation is so small that it will be of no importance in the case here considered; and therefore such a manner of propagation would appear to be a solution of the question and may be so for ionization bands in free space, see sect. 4 below. We shall, however, immediately show that this is not the case for a wave travelling some hundreds of times round the earth. To realise this we must, however, consider the field energy in the non-conductive medium during the wave-propagation in question. According to table 4 the attenuation constant r_0 [compare (8) and (10)] for $\varkappa = \frac{1}{20}$ and for $\nu_1 = 360$ is

$$r_0 = \frac{\varkappa^2 \nu_1}{4\sqrt{2} \cdot c} = 5.3 \cdot 10^{-12} \left[\mathrm{cm}^{-1}\right] = 5.3 \cdot 10^{-7} \left[\mathrm{km}^{-1}\right]$$

and for $\varkappa = \frac{1}{4}$

$$r_0 = 1.3 \cdot 10^{-5} \left[\mathrm{km}^{-1}\right].$$

In the derivation of ZENNECK's formulae for wave-propagation along a plane surface, the non-conductive space is assumed to be of infinite height perpendicular to the boundary-surface. The total electromagnetic field energy in this space is equal to the field energy of a layer of the height H_0 with a uniformly distributed field having all over the same intensity as that found at the boundary-surface, where H_0 is determined by

$$H_0 = \int_0^\infty e^{-2r_0 z} \cdot dz = \frac{1}{2r_0}. \tag{13}$$

For $\varkappa = \frac{1}{20}$ we have $H_0 = 9.4 \cdot 10^5$ km, and for $\varkappa = \frac{1}{4}$ we have $H_0 = 3.8 \cdot 10^4$ km. If we assume the boundary-surface to be at a height of 160 km above the earth, the total energy will be respectively $\frac{940 \cdot 10^3}{160} = 5900$ and $\frac{38 \cdot 10^3}{160} = 240$ times smaller than assumed. The effective attenuation will, therefore, be very much greater than the value 2.25 according to (12′).

The conditions are rather complicated, partly owing to the fact that the boundary-surface is not plane but spherical, and partly owing to the presence of the earth surface. We cannot therefore directly infer that the above found values of the attenuation-exponents, namely, $\gamma_0 u \cdot t_0 = 85.7$ and 2.25 should be multiplied respectively by $\frac{1}{2} \cdot 240 = 120$ and $\frac{1}{2} \cdot 5900 = 2950$ which would raise their values to respectively 10280 and 6700. The correct figures are, however, not essential in this connection but only the fact, which we can easily derive, that the attenuation with this manner of propagation would be so great that a wave-propagation subject to such conditions cannot form the basis of the long delayed echoes.

If the waves were propagated in that manner, then the longer waves, as f. inst. 60 and 90 m, would at all events be propagated with a smaller attenuation and consequently give longer delays. Further, echo should generally be obtained after each passage round the earth. Both of these consequences are contradictory to experience and we consequently consider it as impossible that the long delayed echoes are produced in that manner.

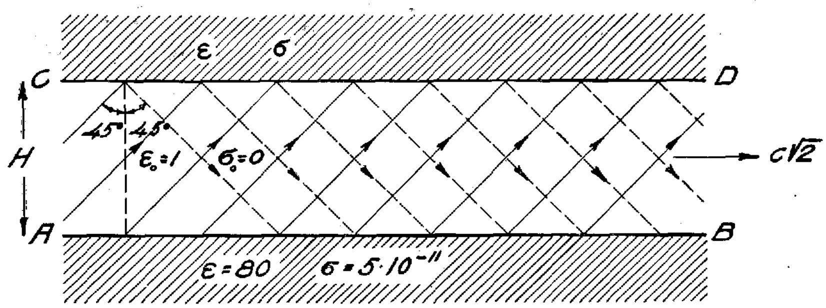

Fig. 5. Schematical representation of the Propagation of Radio Waves between the Surface *A—B* of the Earth and an ionized Layer *C—D*. Assuming the Height *H* to be infinitely great the Waves would travel in the Direction shown in full drawn Lines forming an Angle of incidence $\varphi \cong 45°$ at a very small Value of $\varkappa$.

This result is also verified by a consideration of fig. 5 where the full drawn lines show the direction of the energy-propagation under the assumption that within the limited layer between the earths surface *A—B* and the boundary-surface *C—D* the propagation occurs in the same manner as in unlimited space. The actual propagation will evidently suffer very great losses partly by reflection — at an angle of incidence of about 45°[1] — from the ionized layer, partly by reflection of the wave from — and its propagation along — the surface of the earth.

The most favourable case would be if the propagation occurred between a perfectly even ocean-surface and a per-

[1] Compare “P. R. W ”, p. 140.

fectly conductive outer shell at a height h ($\cong$ 160 km). If we further assume the wave-energy to be uniformly distributed between the earth's surface and the outer shell, then at $\omega = 6.10^7$ ($\lambda \cong 30$ m) we should get an attenuation-constant γ_0 with a value of[1]

$$\gamma_0 \cong \frac{5 \cdot 10^{-3}}{h} \cong 3.1 \cdot 10^{-5} \left[\text{km}^{-1}\right].$$

For $x = 3 \cdot 10^6$ km corresponding to an echo-period of 10 seconds we get $\gamma_0 x = 93$. The actual attenuation-exponent will, no doubt, be considerably greater.

A propagation such as this would only attain a sufficiently low attenuation if the height h were very great — at all event so great that the ionized layer should be located entirely outside the earths atmosphere[2].

On the other hand, a propagation in the manner here considered, and repeated reflections from the surface of the ionized layer contribute in a great measure to the remarkably efficient transmission of short waves in the earths atmosphere which we know from experience. We have in fact that while the direction of the wave-ray in the non conductive medium forms the angle $\psi_0 = 90^\circ - \varphi$ with the boundary surface, where

$$tg\,\psi_0 = \left|\frac{E_x}{E_z}\right|_{z=+0} \cong \sqrt{\frac{1}{\varepsilon}}, \tag{14}$$

the ray will in the ionized medium form an angle ψ with the boundary surface, where

[1] "P. R. W.", p. 32, fig. III 9.

[2] With regard to the limit of the earth's atmosphere we may refer to A. VERONNET: Constitution and Evolution de l'Univers, p. 153 (Paris 1927).

$$tg\,\psi \cong \sqrt{\varepsilon}, \tag{15}$$ [1]

so that $\psi \ll \psi_0$ when $\varepsilon \ll 1$.

For refraction, the direction of the wave-ray in the two media will be determined by

$$\sin(90^\circ - \psi_0) = \sqrt{\varepsilon} \cdot \sin(90^\circ - \psi). \tag{16}$$

In both cases there is thus a pronounced tendency to transfer the electro-magnetic energy from the non-conductive medium to the ionized medium, and further, for the direction of the wave-ray in the latter to become parallel to the boundary-surface. And this tendency must exist even if the transition from the one medium to the other may not be quite sharp but changes gradually.

The well known far-carrying ability of the short waves may thus be satisfactorily explained by means of the propagation here discussed, but this manner of propagation does not afford an explanation of the long delayed echo signals.

3. The Propagation of Radio Waves in a Space Between an Upper Layer, having a Sharply defined Lower Boundary, and the Surface of the Earth.

This problem has partly been treated in the preceding but must be considered a little more closely. A ray SP — see fig. 6 — leaving the transmitter S strikes at the point P a part of the upper boundary-surface which is not quite concentric with the earth's surface but having such

[1] "P. R. W.", p. 18, table 1.
[2] "P. R. W.", p. 197—200.

an acclivity that the reflected ray PQ does not come quite down to the earth. If the upper boundary-surface taken as a whole is otherwise a spherical shell concentric with the earth's surface, then such a ray may travel on in the manner referred to without ever returning again to the earth. If the intervening space is perfectly non-conductive then the rays may travel round the earth several times since comparatively little attenuation will in general be suffered at each reflection from the upper boundary-surface. The number of reflections a during a complete circumscription is

$$a \geqq \pi \sqrt{\frac{R}{2h}}. \qquad (1)$$

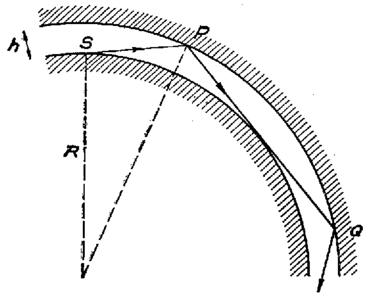

Fig. 6. Schematical Representation of a possible Path of a Ray in the Non-conductive Space between an Upper, Ionized Layer and the Surface of the Earth.

If the boundary-surface is 100 km above the earth and if the radius of the earth is $R = 6400$ km then $a \geqq 17.8 \cong 18$. The height can hardly be estimated as more than 100 km, since otherwise we should have to reckon with a comparatively strong ionization in the intervening space.

To simplify the case we will consider the losses by reflection at the upper boundary to be negligible. In that case the ray in fig. 6 will be able to travel very great distances with very small losses; however, in that case we do not receive any signals.

If signals are to be received at the earth's surface we may as the simplest case assume that after each reflection from the upper layer the ray returns to the earth, which will be at least 18 times during each circumscription of the earth. The ray may then be either reflected from the earth's surface at an angle of incidence very little less than 90°, or it must just graze it. In the first case the amplitude will decrease in the ratio of about 0.8[1]. In the last case the ray will travel at least about 16 km along the surface of the earth corresponding to that the earth "dips" 5 m inside of the ray. In this manner we get for each reflection an attenuation-factor[2] of about $e^{-0.3}$. One circumscription of the earth gives thus an attenuation of respectively $0.8^{18} = 10^{-1.75}$ and $e^{-5.4} = 10^{-2.35}$. After 100 circumscriptions the attenuation would be respectively 10^{-175} and 10^{-235}.

If we assume that reception is possible within a zone of 40 km at each reflection from, or grazing along, the earth's surface — according to the assumption made above this width may hardly be estimated at a higher value — then the probability that a signal is received at a given place during one circumscription is equal to $\frac{40 \cdot 18}{40.000} \cong \frac{1}{55}$.

This probability is of the right order of magnitude since, if a certain signal is giving an echo at all, it very often gives two or more[3]; and for 110 circumscriptions we shall then actually have a probability of getting two signals.

On the other hand, the above calculated attenuation is

[1] "P. R. W.", p. 132—135, fig. VIII, 11—18.

[2] "P. R. W.", p. 19, fig. III, 2.

[3] B. v. D. POL: l. c. Of a series of 11 long delayed echoes observed in Oslo the three gave only one echo, six gave two echoes and two gave three echoes. This gives an average of $\frac{21}{11} \cong 2$ echoes for the cases where echoes are obtained at all. The present experimental material is, however, too small to justify a definite conclusion with regard to the average number. In the here considered observations the average of delay was 16 seconds; the longest being 30 seconds, the shortest about 3 seconds.

unreasonably great. The attenuation-exponent should be reduced to at least $\frac{1}{50}$ of its value, and consequently also the number of reflections or grazings at the surface of the earth should be reduced to $\frac{1}{50}$ of that assumed above in order to get a reasonable attenuation. In that case, however, the probability that a signal will give two successive echoes is reduced also to about $\frac{1}{50}$ which is in contradiction to the available experimental material.

Even though the above considerations may not prove the impossibility of obtaining long delayed echoes in the manner last referred to, they show, at all events, that such a probality is extremely small. And another circumstance is decidedly against such a probability.

The losses due to the reflections at both the upper boundary-surface and at the surface of the earth, or by grazing along the earths surface, will decrease with increasing wave-length. Somewhat longer waves, as f. inst. 60 or 90 m, should consequently show extraordinarily long ranges under such conditions as are considered here. Two circumstances may, however, possibly counteract the longer range of the longer waves: Due to the lower frequency the rectilinear radiation would be less pronounced. This, however, is again compensated by a more regular reflection of these longer waves at the uneven surface of the earth. The second circumstance is the attenuation caused by the presence of ions or electrons in the intervening space hitherto considered perfectly non-conductive. No doubt, such ions are present and they will attenuate the longer waves most. The attenuation-constant may namely in this case be determined by[1]

[1] "P. R. W.", p. 121 (8 a).

$$\gamma_0 = N \cdot \frac{2\pi e^2}{m c} \cdot \frac{\nu}{\omega^2 + \nu^2}, \tag{2}$$

which for the 60 m wave will be at most 4 times, and for the 90 m wave at most 9 times greater than for a 30 m wave. Since all the other attenuations, under the conditions postulated, are smaller for the 60 and for the 90 m wave than for a 30 m wave, then the 60 m wave must have a range of at least $\frac{1}{4}$, and the 90 m wave at least $\frac{1}{9}$ that of the 30 m wave. If the latter is able to travel round the earth f. inst. 100 times, then the longer waves should be able to do so at least 25 and 11 times respectively. Any such feature has, as far as the writer is aware, never been observed. In all of the cases where complete circumscription of the earth has been observed the wave-length was between 15 and 25 m.

Presumably therefore we may consider it as an established fact that the long delayed echoes are not obtained in the manner last referred to.

Also, presumably, we have now tried every conceivable possibility of explaining the long delayed echoes by means of the propagation of radio waves within the earth's atmosphere.

4. The Reflection of Radio Waves From — or Propagation Along — Bands of Ions out in Space.

Since, according to the preceding, the long delayed echoes cannot arise either by the propagation of radio waves within the earths atmosphere, or by the waves travelling outside the latter in a medium so strongly ionized that the group-velocity approaches zero, they must be due to the fact that the waves have travelled very great distances outside the earth's atmosphere. I shall not enter too deeply into the astro-physical problems connected with the present case but shall only consider some of the relevant wave propagation problems.

The first problem in that respect is: which radio-waves are able to penetrate the earth's atmosphere and proceed

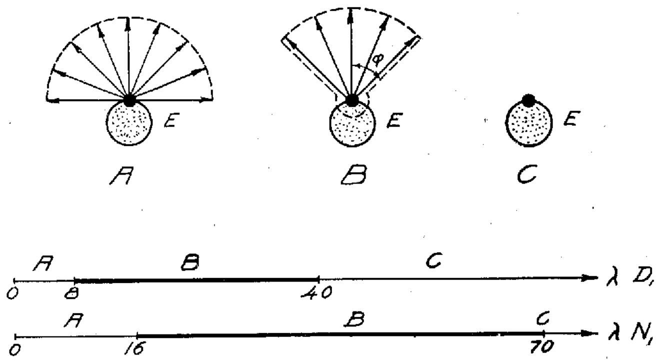

Fig. 7. All Wave-lengths within the Interval C (D_1 represents the Conditions at Noon, N_1 the Conditions at Midnight; about 40° Northern Latitude) cannot penetrate out to Space but are refracted or reflected back to the Earths Surface. All Waves within the Interval A penetrate the atmosphere and go out into Space. Within the Interval B it depends on the angle of ascent of the Wave whether it leaves the Atmosphere or is reflected back again to the Earth's Surface.

out into space? This question is thoroughly treated elsewhere[1], and according to what is set forth there we assume — see fig. 7 — that all waves shorter than about 8 m will always penetrate out into space with comparatively small attenuation. At midnight this will be the case for all waves up to about 16 m. All waves longer than 40 m are completely refracted or reflected back to the earth at noon, and at midnight all waves longer than about 70 m. The given

[1] "P. R. W.", chap. XI especially section (i), p. 214—218. See also "Radiofoniens Aarbog 1929", p. 16—20 (1928, Copenhagen).

limits are, however, not fixed values but vary according to the varying state of ionization of the upper atmosphere.

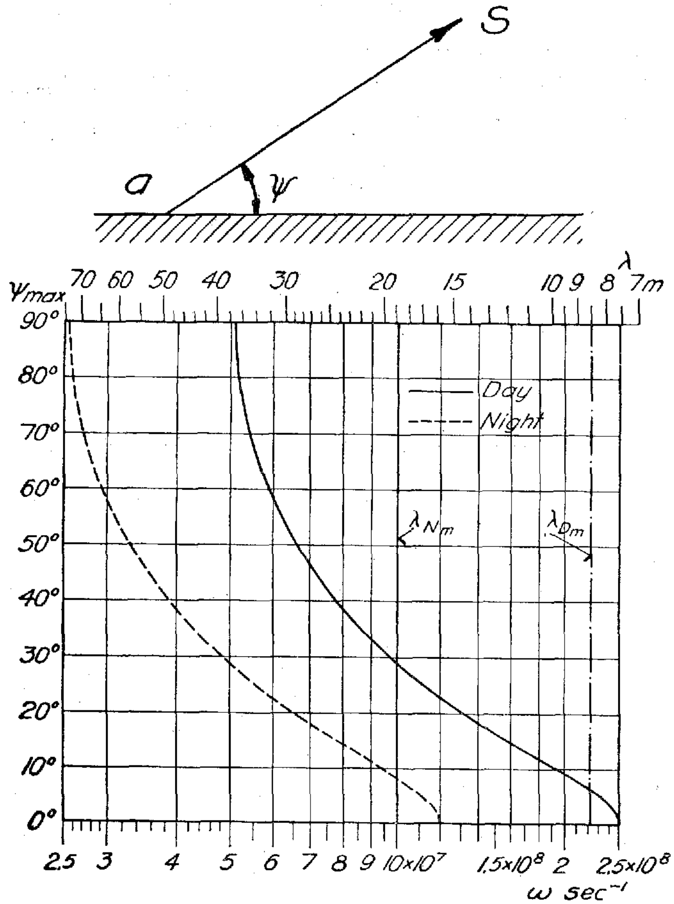

Fig. 8. The Earth Angle $\psi_{\max}$ as Function of the Wave-length λ. The full drawn Curve represents the Conditions at Noon, the dotted Curve at Midnight.

All experimental data indicate, however, that the figures given are reasonable mean values.

For waves between 8 and 40 m at noon it will consequently depend upon their angle of ascent ψ (see fig. 8) whether they penetrate out into space or return to the earth. The greatest angle of ascent at which a wave does not penetrate out into space is called $\psi_{\max}$, and the dependency of this angle on the wave-length at noon is shown in the full drawn curve in fig. 8 while the dotted curve shows the same relation at midnight. These curves also depend of course on the state of ionization in the upper atmosphere, but represent reasonable mean values.

According to the above, the long delayed echoes can arise only with waves shorter than 70 m. Waves longer than 70 m can generally neither penetrate out into space nor from the outside penetrate the upper ionized layer and come down to the earth.

The relations for these long waves are indicated in part *C* fig. 7 where *E* indicates the earth and the black point the transmitter. No wave-rays go from the transmitter out to space.

The relations for very short waves are indicated at *A*. Here emission occurs to the whole of the hemisphere which has for lower boundary the tangential plane to the surface of the earth at the point of transmission.

Of most interest are the relations within the active short-wave interval *B*. Here the emission to space occurs within a cone having the apex angle

$$\varphi = 90^\circ - \psi_{\max}. \tag{1}$$

If the transmitter radiates with equal strength in all directions then the ratio between that fraction η which penetrates out into space and the total radiation within the whole hemisphere will be determined by

$$\eta_1 = 1 - \cos\varphi = 2\sin^2\frac{\varphi}{2}. \tag{2}$$

If the radiation occurs from at vertifical, linear aerial the length of which is small in comparison to the wave-length then the radiation intensity is proportional to $\sin^2 \varphi$ and we then get[1]

$$\eta_2 = 1 + \frac{1}{8} \cos 3\varphi - \frac{9}{8} \cos \varphi. \tag{3}$$

In fig. 9 are shown the values of η_1 and η_2 as functions

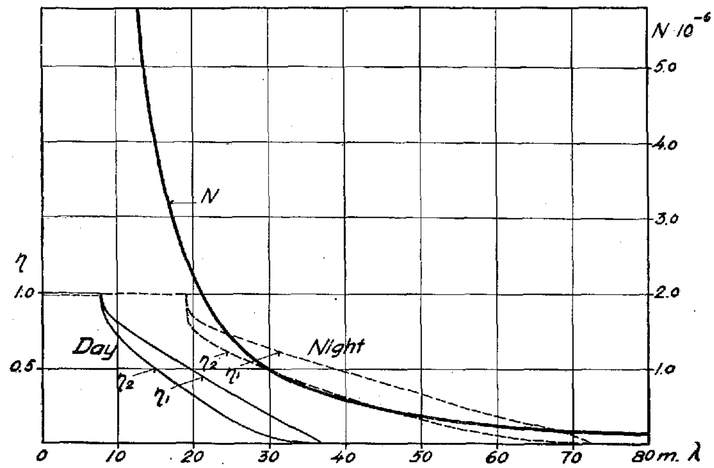

Fig. 9. The Values of the Constants η_1 and η_2 at Day and at Night, calculated on the Basis of fig. 8. The N-curve shows the Values of N calculated by means of Equation (4).

of the wave-length λ. Corresponding values of λ and ψ_{max} are taken from fig. 8 and then φ is found by means of equation (1).

[1] Beside the direct radiation to space calculated by means of (2) and (3), where the wave-rays out in space proceed in nearly the same direction as the one at which they left the transmitter, a more or less regular radiation will take place from — and in a direction nearly parallel to — the upper surface of the ionized layer. This radiation is mainly due to those rays which leave the transmitter at an earth angle in the vicinity of ψ_{max}. Rays may be emitted from any point of the upper surface of the ionized layer on wave-lengths within the interval B fig. 7 but the intensity of such rays is generally rather small.

From Fig. 9 it appears that at wave-lengths appreciably more than 40 m, η_1 and η_2 decrease to very small values. Above 30 m the radiation out to space will consequently be comparatively very small, and it appears from the figure at all events that the probability that waves may penetrate out into space in a favourable direction decreases rapidly with increasing wave-length for waves from about 30 m and upwards.

On the other hand, we have shown above that for total reflections at normal incidence of a 30 m wave a density of electrons of about $N_0 = 1.2 \cdot 10^6$ per cm^3 is necessary. At other wave-lengths the necessary density of electrons is determined with an accuracy sufficient in this connection by

$$N = N_0 \left(\frac{30}{\lambda}\right)^2 \cong 1.2 \cdot 10^6 \left(\frac{30}{\lambda}\right)^2. \qquad (4)$$

The N-curve in fig. 9 shows the values of the density of electrons determined in this manner, and we see that these values increase rapidly with decreasing wave-lengths. The probability of finding a band of ions having a density of electrons equal to, or greater than N is of course smaller the greater the value of N.

From the preceding it appears that there must be a certain wave-length for which the probability of getting long delayed echoes is the greatest, and from figs. 7 and 9 we may draw the conclusion that this most favourable wave-length must be about 30 m; but the possibility of course exists that echoes may be observed on somewhat shorter as well as on somewhat longer waves; but to the best of our knowledge, long delayed echoes so far have been observed only on waves of about 30 m.

If we were to judge solely from fig. 9 we would be apt to assume that a somewhat longer wave, f. inst. about 40 m, would be the most favourable for the observation of echoes. By calculating the η_1- and η_2-curves we have, however, not considered the loss suffered by the waves by their passage through the ionized part of the upper atmosphere and since this loss increases with increasing wavelength, the said curves for the effective radiation out to space should decrease somewhat more rapidly than shown in fig. 9.

Another circumstance of some importance in this connection may be mentioned. All of the preceding considerations are based upon a state of ionization of the atmosphere which is assumed by the writer in consequence of the experimental material at hand with regard to the propagation of radio waves in general. Considering this very comprehensive material there is some reason to believe that the ionization-distribution assumed as a whole is approximately correct from the earth's surface up to the altitude of maximum ionization *i. e.* up to about 130—150 km. As to the density of electrons and the conductivity above that height where the ionization is maximum, our ordinary terrestrial experience with propagation of radio waves is unable to give any information. The density of electrons at greater heights may consequently very well exceed somewhat the values indicated in "P. R. W.". Hitherto it has been possible only to judge about the state of ionization in these regions above the altitude of maximum ionization by means of multiple echoes of short delays[1] and by means of the terrestrial magnetic conditions[2]. The investigation of the ionization of these very high layers

[1] "P. R. W.", p. 209—212.

[2] [The bearing of the magnetical evidence on the value of the total ionization of the atmosphere is discussed in a valuable paper by S. CHAPMAN just published ("On the Theory of the Solar Diurnal Variation of

may possibly in the future profit by the experiences gained from the long delayed echoes.

With regard to the character and the location of the bands of ionization which cause the echoes we may just mention that presumably they may be divided into two groups, namely, those along which the waves are propagated, and those from which the waves are reflected.

Of the first kind are presumably those bands of electrons which according to STÖRMER are due to the invasion of electrons into the magnetic field of the earth; see f. inst. fig. 10. As long as it is a question of relatively few electrons only, the classical work of STÖRMER offers the necessary information with regard to the form of the tracks possible, and to the bounding of those spaces to which such tracks are limited. Above we have shown, however, that in those bands or layers which may be of importance for the phenomena here considered of the propagation of radio waves, the density of electrons is so great that the electrostatic forces will prevent the development of pure electron-bands or of dense bands consisting exclusively of ions of one sign. In the ionized bands which are effective echo-reflectors the positive and the negative space charges

the Earth's Magnetism", "Proc. Roy. Soc." (A). Vol. 122, p. 369—386, 1929). This author comes to the conclusion that the total ionization required from a magnetical point of view is of the same order of magnitude as that assumed by the writer in "P. R. W.", the magnetic evidence indicating a somewhat higher value. CHAPMAN adheres to the "dynamo" theory eventually combined with the "drift current theory" which he suggests in the above mentioned paper. Upon numerical considerations he concludes that the "diamagnetic" theory of Ross GUNN ("Phys. Rev.", Vol. 32, p. 133—141, 1928) can hardly be right. It may be added that N. BOHR ("Studier over Metallernes Elektronteori", Chap. IV, Copenhagen 1911) has proved that such a diamagnetic effect does not exist. The remarks of R. GUNN (l. c. p. 136) do not meet the main points of BOHR's arguments and do not invalidate his general conclusions.]

must be of nearly equal value, *i. e.* the resulting space charge must be comparatively small. Such a band of ions will also have a tendency to follow the tracks calculated by STÖRMER, and the shape of the track and the velocity will in the main be determined by the heavy ions.

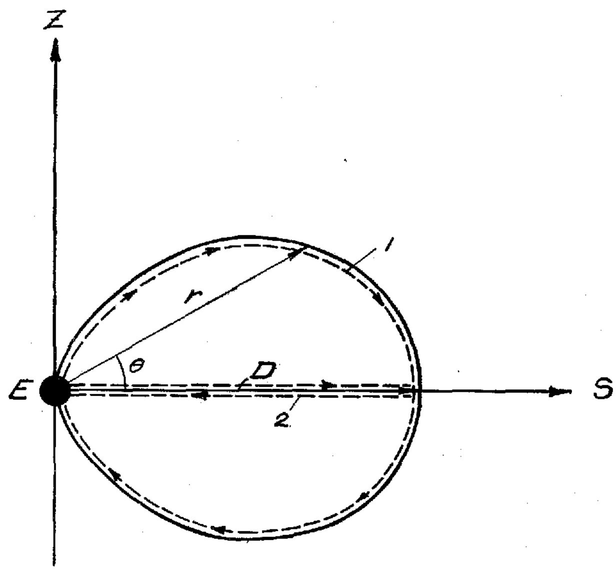

Fig. 10. The full drawn Curve indicates the inner Limits of the Paths of charged Particles according to Equation (5). E represents the Earth with its magnetical axis along the Z-axis. The dotted Curves 1 and 2 indicate the two Echo-Tracks possible. The direction S pointing toward the Sun.

In fig. 10 we have shown as an example the boundaries of the tracks of the corpuscular rays in a particular case for which the equation of the bounding-curve is[1]

$$r = \frac{\cos^2\theta}{1+\sqrt{1+\cos^3\theta}} \cdot \sqrt{\frac{M}{\frac{cm}{e}\cdot U}} \tag{5}$$

where r is the radius vector, $\left(\frac{\pi}{2}-\theta\right)$ its angle with the

[1] C. STÖRMER, Arch. sc. phys. et nat. t. XXIV, p. 129, 1907.

magnetic axis Z of the earth. $M = 8.6 \cdot 10^{25}$ = the magnetical moment of the earth, $c = 3 \cdot 10^{10}$ cm sec.$^{-1}$ = the velocity of light in empty space, m and e the mass and the charge of the ion [E. S. U.].

According to STÖRMER the bounding-surfaces which may give long delayed echoes constitute the boundary-surfaces between those parts of space into which the ions in question may penetrate at a given velocity, and those parts of space into which these ions cannot come. Such boundary surfaces may be efficient as guiding planes for a propagation of radio waves, and such a propagation is indicated by the dotted curve 1 in fig. 10.

For such a propagation along a band of electrons or ions, which connects the northern and southern polar areas, and which has very great radii of curvature, only a comparatively small density of electrons is demanded and, according to table 4, the propagation may occur with comparatively small losses.

The other manner of obtaining echoes is indicated by the track 2 in fig. 10. Here the wave-ray is reflected from the bounding-surface and returns to the surface of the earth. This manner of propagation may occur with only very little attenuation, but demands in the bounding surface the density of electrons determined by equation (4).

These bands of ions, formed by the magnetic field of the earth, presumably play an important rôle in the production of wireless echoes up to 30 seconds and possibly even up to 60 seconds, as was originally suggested by STÖRMER[1,2].

[1] C. STÖRMER: "Nature", Vol. 122, p. 681, 1928; "C. R." tome 187, p. 811, 1928.

[2] For this kind of echo the probability of obtaining good echo signals will, as shown by STÖRMER ("Nature", Vol. 123, p. 16, 1929), depend

But there is also the possibility that outside that space in which the magnetic field of the earth exerts its influence in this manner, bands of ions having sufficient density of electrons may attain such forms that they act as reflectors which after one or more reflections return the radio waves to the earth. If a wave-ray is to return to the earth with

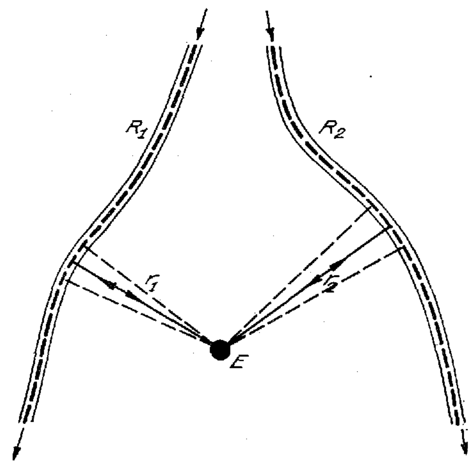

Fig. 11. Two curved Ionization-Bands acting as Concave Mirrors at the Places indicated. E represents the Earth.

sufficient intensity after a single reflection from a very distant ionization-band the centre of the curvature of that band must necessarily be located at or near the earth. Such curvatures may partly be due directly to the influence of the earth's magnetic field and, outside that space where the latter appreciably influences the ionization-bands, partly to the electric field from charges directly on the earth and particularly from such charges which may be

upon the angle between the magnetic axis of the earth and the direction to the sun, being greatest when these two directions are at right angles to each other. [The latest evidence seems to be in agreement with Störmer's prediction.]

"arrested" by the magnetic field of the earth. Such bent ionization-tracks are outlined in fig. 11.

Long delayed echoes may further be obtained in the manner indicated in fig. 12, where R_1 and R_2 are two bands of ions extending from the sun and which at a and c reflect a wave-ray coming from the earth E. Between a and c the wave-ray traverses the curved path abc the curved form of which is due to increasing density of electrons in the direction toward the sun.

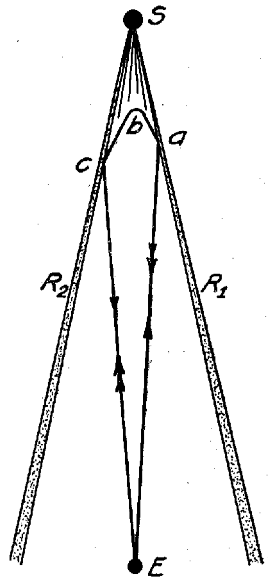

Fig. 12. Schematical Representation of a possible Manner of obtaining Echoes after two Reflections, at a and c, and of a curved Path abc of a Wave-Ray between the two Points of Reflection. E represents the Earth.

Since, generally, the bands of ions are not perfectly electrically neutral they will mutually act upon one another, thus forming more or less curved bands. In this case the centre of curvature will, however, in general not be located near the earth and consequently the bent part of the band will not be efficient in producing echoes. If, however, the radiation from the earth is emitted at a comparatively great space-angle an exceedingly large space within our solar system will be searched and consequently the probability of finding a favourable constellation may be not altogether vanishingly small, although such echo-phenomena presumably are rather seldom.

In consequence of the above considerations the writer therefore anticipates that in the future, as the study of the echoes is carried on more systematically and on a more extensive scale there will be occasionally reported echoes of very long delays, possibly up to 10 to 15 minutes or even more[1].

From the considerations set forth in this paper, it presumably appears that:

(1) Echoes delayed more than 10 seconds cannot be due to the propagation of radio waves entirely within the atmosphere of the earth, nor to a propagation of the waves outside this, in a medium so densely crowded with electrons that the group-velocity decreases to such small values that the distance travelled will be comparatively short.

(2) Echoes delayed up to 30 (possibly to 60) seconds are probably due to propagation along — or reflections from — "Störmer bands" of electrons within the magnetic field of the earth.

(3) Occasionally echoes may be obtained with such great delay that those bands of ions to which the echoes are due must be located at such great distances from the earth (more, for example, than 40.000.000 km), that they are outside the space in which the magnetic field of the earth exerts any appreciable direct influence.

[1] This prediction, set forth some time ago by the writer ("Radiofoniens Aarbog 1929, p. 22—24, October 1928 — the above figs. 8, 11 and 12 are taken from this paper) has received an unexpectedly quick confirmation, since in a letter dated February the 2nd 1929 Mr. Jörgen Hals of Oslo communicates that he has observed echoes up to 4 min. 20 sec. corresponding to a path-length of 78.000.000 km. This observation — if correct — confirms the above considerations, namely, that the long delayed echoes are not due to propagation entirely within the earth's atmosphere, and further that echoes may occur with such long delay that they must be due to ionization-bands located outside that space in which the magnetic field of the earth directly exerts its influence. [Another echo, having a retardation of 3 min. 15 sec. was observed by Mr. Hals February 14, 1929.]

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