Det Kgl. Danske Videnskabernes Selskab.
Mathematisk-fysiske Meddelelser. **VI,** 2.

ON THE EFFECT OF PERTURBING ELECTRIC FIELDS ON THE ZEEMAN EFFECT OF THE HYDROGEN SPECTRUM

BY

H. C. UREY

KØBENHAVN
HOVEDKOMMISSIONÆR: ANDR. FRED. HØST & SØN, KGL. HOF-BOGHANDEL
BIANCO LUNOS BOGTRYKKERI
1924

§ 1. Introduction.

According to the quantum theory of line spectra, the effect of a magnetic field on the fine structure of the hydrogen spectrum should be to split each fine structure component into an undisplaced component polarized parallel to the magnetic field and two components displaced equal distances on each side of the first and polarized perpendicular to the field so that three complex lines should appear having the same fine structure as the original line[1]. H. M. HANSEN and JACOBSEN[2] have made an investigation of the effect of a magnetic field on the 4686 line of the helium spark spectrum which has an origin analogous to the hydrogen spectrum and find that their experiments show an agreement with the theory for fields up to about 6000 Gauss. For larger fields they find that the distance between the two stronger components of the fine structure becomes less and especially that the shorter wave length component is shifted toward the red. Recently K. FÖRSTERLING and G. HANSEN have published the results[3] of similar experiments on the Zeemaneffect of the H_α and H_β lines of hydrogen which appear as narrow doublets and have also found that the two members of the doublet seem to move closer together

[1] DEBYE, Phys. Zs. **17**, 507, 1916; SOMMERFELD, Phys. Zs. **17**, 491, 1916; BOHR, Quantum theory of line spectra, Part. II, p. 82, D. Kgl. Danske Videnskabs Selsk. Skrifter, naturvidensk. og mathem. Afd., 8. Række, IV, 1.

[2] H. M. HANSEN and J. C. JACOBSEN, D. Kgl. Danske Vidensk. Selsk. math.-fys. Meddelelser III, 11.

[3] K. FÖRSTERLING and G. HANSEN, Zs. für Phys. **18**, 26, 1923.

as the intensity of the magnetic field is increased. At the highest field intensities used (about 10000 Gauss) the three doublets have merged into three broad lines each with a total width somewhat less than that of the original doublets. Both H. M. HANSEN and JACOBSEN, and FÖRSTERLING and G. HANSEN mention the possibility that these effects may not be due to the direct influence of the magnetic field on the atom and therefore at variance with the theoretical predictions but may be due to the action of small perturbing electric fields in the tube, caused by the change in the discharge due to the magnetic field[1].

In view of the great theoretical interest of this question this possibility will be investigated more closely in the present paper. In the first place we shall consider the possible effect of uniform constant electric fields of different intensities and directions acting on the atoms simultaneously with the magnetic field. To the problem of the origin of these fields and of the possible more complicated effects of the discharge on the spectrum we shall return after the calculations of the mentioned effect.

The effect of an electric field on the fine structure of a hydrogen line has been treated by KRAMERS[2]. His results apply directly also to the case where a magnetic field parallel to the electric field is present, since in this case we obtain a simple superposition of the perturbations due to each field separately. For our purpose it is necessary,

[1] K. FÖRSTERLING and G. HANSEN state that there was no visible change of the discharge when the magnetic field was turned on. Yet, as is evident from their photographs, the intensity of the light emitted in the presence of a magnetic field of 10000 Gauss is less than in the case of no magnetic field. This indicates a change in the character of the excitation even if it is not apparent to the eye.

[2] KRAMERS, Zs. für Phys. **3**, 199, 1920.

however, also to consider the simultaneous effect on the fine structure of an electric and a magnetic field, which make an angle with each other. If, as in our case the electric field is so small that it will only produce comparatively small changes in the fine structure, it will be sufficient to consider an electric field perpendicular to the magnetic field. In fact an electric field can always be resolved in a parallel and a transverse component and in the case under consideration the effects of these components will be simply superposed[1].

§ 2. Calculation of the influence of a transverse electric field on the stationary states of the hydrogen atom under the influence of a magnetic field of such intensity that the relativity and Zeeman effects are of the same order of magnitude.

The Hamiltonian function for this problem disregarding terms proportional to H^2 is[2]

$$= mc^2\left[\left\{1+\frac{1}{m^2c^2}\left(p_r^2+\frac{p_\theta^2}{r^2}+\frac{p_\varphi^2}{r^2\sin^2\theta}\right)\right\}^{1/2}-1\right]-\frac{Ne^2}{r}+\frac{p_\varphi He}{2\pi mc}+Fer\sin\theta\cos\varphi$$

where Ne and $-e$ are nuclear and electronic charge respectively, and m the electronic mass, and r, θ and φ are the polar coordinates of the electron if the origin is taken at the nucleus and p_r, p_θ and p_φ are the conjugated momenta

[1] EPSTEIN (Phys. Rev. **22**, 202, 1923) has announced that the problem of the hydrogen atom under the influence of crossed electric and magnetic fields can be solved by the method of perturbed systems, if the relativity effect is neglected. Recently HALPERN (Zs. für Phys. **18**, 287, (1923) and O. KLEIN (Zs. für Phys. **22**, 109, 1923) have also treated this problem. It is obvious that we cannot use these solutions for our problem since we are primarily interested in the effect of magnetic and electric fields on the fine structure.

[2] A simple calculation shows that the energy term proportional to H^2 is of the order of magnitude $a^2 e^2 H^2/8m c^2$, where a is the semimajor axis of the ellipse, and is therefore quite negligible. (See HALPERN, Zs. für Phys. **18**, 352, 1923).

and H and F are the magnetic and electric field strengths respectively. We shall use the following set of uniformizing variables (angle variables), adapted to the treatment of a perturbed Keplerian motion.

J = twice the absolute value of the energy divided by the frequency of revolution.

$2\pi w$ = mean anomaly multiplied by the frequency of revolution.

$P/2\pi$ = total angular momentum of the electron around the nucleus.

$2\pi\beta$ = angular distance between the line of nodes and the major axis.

$Q/2\pi$ = magnitude of the angular momentum of the electron about the polar axis.

$2\pi\gamma$ = angular distance between the meridian plane $\varphi = 0$ and the lines of nodes.

With these variables the energy function can be written in the form

$$E = E_0 + Fex, \tag{1}$$

where E_0 is a function of J, P and Q only, given by the equation

$$E_0 = -\frac{2\pi^2 N^2 e^4 m}{J^2}\left[1+\left(\frac{\pi N e^2}{c}\right)^2\left(-\frac{3}{J^2}+\frac{4}{JP}\right)\right] \pm \frac{QHe}{4\pi mc}$$

and x represents the displacement of the electron in the direction of the electric field as a function of the J, P, Q, w, β and γ and is expressed by the simple formula

$$x = \sum D'_\tau \cos 2\pi(\tau w + \beta + \gamma) - \sum D''_\tau \cos 2\pi(\tau w - \beta + \gamma) \tag{2}$$

where the D'_τ and D''_τ are functions of the J, P and Q only[1].

[1] Kramers' Dissertation, Intensities of Spectral Lines pp. 16 and 28, D. Kgl. Danske Vidensk. Selsk. Skrifter, naturvidensk. og mathem. Afd., 8. Række III, 3.

The motion of the electron in the unperturbed system for which $F = 0$ can be described as a motion in a Keplerian ellipse with frequency ω on which is superimposed a precession of the ellipse in its plane with frequency σ and of the plane about an axis parallel to the direction of the magnetic field with frequency ϱ. The frequencies ω, σ and ϱ are given by the equations

$$\omega = \frac{dw}{dt} = \frac{\partial E_0}{\partial J}, \quad \sigma = \frac{d\beta}{dt} = \frac{\partial E_0}{\partial P} \text{ and } \varrho = \frac{dp}{dt} = \frac{\partial E_0}{\partial Q} = \pm \frac{He}{4\pi mc},$$

and σ and ϱ are very much smaller than ω. ϱ will be positive or negative depending on whether the angle between the magnetic field and the vector of the angular momentum of the electron about the nucleus is less or greater than a right angle. While in the unperturbed motion, the variables J, P and Q are constant and the variables w, β and γ increase uniformly with the time, this is no longer true, when the system is perturbed by the electric field. In this case the system can now be described by means of the variables J', P', Q', w', β' and γ', related to the former variables by the infinitesimal contact transformation[1] defined by the equations

$$J' = J + Fe\frac{\partial S}{\partial w}, \quad P' = P + Fe\frac{\partial S}{\partial \beta}, \quad Q' = Q + Fe\frac{\partial S}{\partial \gamma},$$

$$w' = w - Fe\frac{\partial S}{\partial J}, \quad \beta' = \beta - Fe\frac{\partial S}{\partial P}, \quad \gamma' = \gamma - Fe\frac{\partial S}{\partial Q},$$

where

$$S = \sum \frac{D'_\tau}{2\pi(\tau\omega + \sigma + \varrho)} \sin 2\pi(\tau w + \beta + \gamma) - \sum \frac{D''_\tau}{2\pi(\tau\omega - \sigma + \varrho)} \sin 2\pi(\tau w - \beta + \gamma).$$

[1] This was first used in BURGERS' Dissertation, Het Atoommodel van RUTHERFORD-BOHR, Haarlem 1918. For a complete review of the quantum theory of perturbed systems see BOHR, Zs. für Phys. **13**, 117, 1923.

Since σ and ϱ are very small in comparison with ω all terms of this series may be disregarded except those for which τ is zero. If the motion of the perturbed system is expressed in these new variables, the energy E becomes independent of the angular variables w', β', γ' if we disregard terms proportional to the square and higher powers of F and terms of the order of magnitude $F\frac{\sigma}{\omega}$ and $F\frac{\varrho}{\omega}$. With this approximation J', P' and Q' will be constant and w', β' and γ' increase uniformly with the time. The substitution of these variables in the expression for the displacement x gives

$$x = x_0 - Fe\left(\frac{\partial x}{\partial J}\frac{\partial S}{\partial w} + \frac{\partial x}{\partial P}\frac{\partial S}{\partial \beta} + \frac{\partial x}{\partial Q}\frac{\partial S}{\partial \gamma} - \frac{\partial x}{\partial w}\frac{\partial S}{\partial J} - \frac{\partial x}{\partial \beta}\frac{\partial S}{\partial P} - \frac{\partial x}{\partial \gamma}\frac{\partial S}{\partial Q}\right) \quad ($$

where x_0 is the function x (2) in which the new variables have been substituted for the old. The displacement x of the electrical centre of the perturbed atom relative to the nucleus is given by the mean value of x. Since x_0 is zero, $\bar{x}$ is seen to be proportional to F, and by means of Ehrenfest's theorem on adiabatic invariance it is possible to calculate from this displacement the energy term proportional to F^2 in the expression for the energy in the stationary states of the perturbed atom (there occur no terms proportional to F). It is easily shown to be equal to the displacement $\bar{x}$ of the electrical centre multiplied by $Fe/2$.[1]

Since in this calculation we are interested only in the displacement of the electrical centre, we need only find those terms in the expression (3) for x, which do not involve β and γ. When the indicated calculations are performed using the relations

[1] See Kramers' Dissertation p. 78, loc. cit. See also Schrödinger, Zs. für Phys. **11**, 170, 1922; Kramers, Zs. für Phys. **13**, 312, 1923.

$$D_0' = \frac{3}{4}\varkappa\frac{J}{P}(P+Q)\sqrt{J^2-P^2}$$

$$D_0'' = \frac{3}{4}\varkappa\frac{J}{P}(P-Q)\sqrt{J^2-P^2}$$

$$\varkappa = \frac{1}{4\pi^2 Ne^2 m}$$

and substituting for J', P' and Q' from the equations

$$J' = nh, \quad P' = kh, \quad Q' = jh,$$

where n, k and j are integral positive quantum numbers satisfying $n \geqq k \geqq j > o$, the energy expression is found to be

$$E = E_0 - \frac{9}{512}\frac{F^2 h^3}{\pi^4 N^2 e^2 m^2}\frac{n^2}{k^3}\left\{\frac{k+j}{(\sigma+\varrho)^2}\left[(2n^2k-3k^3-k^2j)\sigma + (n^2k-n^2j-2k^3)\varrho\right] + \frac{k-j}{(\sigma-\varrho)^2}\left[(2n^2k-3k^3+k^2j)\sigma-(n^2k+n^2j-2k^3)\varrho\right]\right\}, \tag{4}$$

where

$$E_0 = -\frac{2\pi^2 N^2 e^4 m}{n^2 h^2}\left\{1+\left(\frac{\pi Ne^2}{hc}\right)^2\left(-\frac{3}{n^2}+\frac{4}{nk}\right)\right\} + jh\varrho,$$

and ϱ has a positive or negative value depending on whether the vector of the electronic angular momentum about the nucleus makes an angle less or greater respectively than a right angle with the direction of the magnetic field.

Before concluding this paragraph we shall investigate a little more closely the special cases where the magnetic field is of such an intensity that in a certain stationary state $|\sigma|$ and $|\varrho|$ are exactly equal. Here we have an example of accidental degeneration of the perturbed system. In such a case the ordinary method cannot be used, since the expression (4) for the energy would be infinite, but the so-called Bohlin's[1] method is applicable. Born and Heisen-

[1] K. Bohlin, Über eine neue Annäherungsmethode in der Störungstheorie. Bihang till K. Svenska Vet. Akad. Handlingar **14**, Afd. I, No. 5. Stockholm 1888. Compare Poincaré, Méthodes nouvelles de la mécanique céleste (Paris 1893), Vol. 2, Chap. 9.

BERG[1] have shown how the quantum theory should be applied to such a case. The method can be briefly summarized. It is always possible in such cases to effect a canonical transformation of variables such that with the new uniformizing variables $J_1, J_2, \ldots. J_f, w_1, w_2, \ldots. w_f$ the frequency corresponding to one of the angle variables, say the w_f is zero, i. e. in the stationary state under consideration $\frac{d w_f}{dt} = 0$ in the unperturbed system. If the Hamiltonian function is given in the form

$$H = H_0 + \lambda H_1 + \ldots.,$$

where H_0 is a function of the J's and $\lambda H_1 + \ldots.$, which is a function of the J's and w's, represents the perturbing potential, the energy in the corresponding stationary state of the perturbed system will be of the form

$$E = E_0 + \lambda E_1 + \ldots.,$$

where the additionally energy term λE_1 is secured by taking the mean value of λH_1 over all the w's except w_f. In the corresponding stationary state of the perturbed system w_f will have a stationary value $\overset{0}{w_f}$ which must be determined from the relation

$$\frac{\partial \overline{H}_1}{\partial w_f} = 0$$

and those roots of this equation give mechanically stable motions for which $\overline{H}_1 \Big/ \frac{\partial^2 H_0}{\partial J_f^2}$ is a minium.

For our problem we may use the Hamiltonian function (1) given above after making the canonical transformation defined by the equation

$$J = J, \quad \frac{P+Q}{2} = J_1, \quad \frac{-P+Q}{2} = J_2,$$

$$w = w, \quad \beta + \gamma = w_1, \quad -\beta + \gamma = w_2.$$

[1] M. BORN and W. HEISENBERG, Zs. für Phys. 14, 44, 1923.

Then using the method outlined above we obtain for the case that $\sigma+\varrho=0$

$$\lambda E_1 = \frac{3Fh^2}{16\pi^2 Nem}\frac{n}{k}\sqrt{n^2-k^2}\,(k+j) \qquad (4\text{a})$$

and for $-\sigma+\varrho=0$

$$\lambda E_1 = \frac{3Fh^2}{16\pi^2 Nem}\frac{n}{k}\sqrt{n^2-k^2}\,(k-j) \qquad (4\text{b})$$

Thus for conditions such that accidental degeneration occurs in one of the stationary states of the unperturbed system consisting of the atom in the presence of the magnetic field alone, the energy of the stationary states of the perturbed atom contains a term proportional to the first power of F in contrast to the case when $|\sigma|$ and $|\varrho|$ are not equal. This is due to the circumstance that the atom remains so to speak in oriented positions in the electric field and thus permits the effects of the field to accumulate from one revolution to the next and thus to cause a change in the motion of another order of magnitude. However, the effect can occur for only a few values of the magnetic field and has no general interest for the discussion of the observations of H. M. Hansen and Jacobsen and of Försterling and G. Hansen.

§ 3. Influence of the perturbing electric fields on the Zeeman effect of the hydrogen lines.

We shall now proceed to discuss the theoretical effect of the electric field on the spectrum. The frequency of the emitted light will be given by the Bohr frequency condition

$$\nu = \frac{E'-E''}{h}$$

where E' and E'' represent the energies of the system in the initial and final state respectively. In the presence of a homogeneous magnetic field alone, there will appear in the motion of the atom harmonic components of two types, linear components parallel to the field of frequencies $\tau\omega \pm \sigma$ and circular components perpendicular to the field of frequencies $\tau\omega \pm \sigma \pm \varrho$. From the correspondence principle it follows therefore that, while transitions are possible for which n changes by any number of units, the quantum number k can only increase or decrease by one unit, further that any component of the spectral lines which would appear in the absence of the magnetic field will be split up in three components, one for which j remains unaltered, and two for which j changes by ± 1 unit. The first is polarized parallel to the magnetic field, while the two latter are polarized perpendicular to this field. It has been shown in a general way by Bohr[1] that in the presence of an external electric field new vibrations occur in the motion of the electron whose frequencies are the sums and differences of the frequencies in the undisturbed motion and whose amplitudes are proportional to F and that therefore in accordance with the correspondence principle new transitions would occur giving lines with intensities proportional to F^2. A simple calculation shows that in the case of superimposed parallel electrical and magnetic fields the new frequencies $|\tau\omega|$ and $|\tau\omega \pm 2\sigma|$ occur in the motion parallel to the fields and $|\tau\omega \pm \varrho|$ and $|\tau\omega \pm 2\sigma \pm \varrho|$ in the motion perpendicular to the fields. We must expect therefore in accordance with the correspondence principle new p-components in which k remains unchanged or changes by 2 units and also new s-components in which k changes by 0 or ± 2. These calculations have been made

[1] Bohr, Quantum theory of line spectra, loc. cit. pp. 96—98.

by KRAMERS[1]. In the case of crossed electic and magnetic fields the new frequencies in the motions parallel to the magnetic field are $|\tau\omega \pm \varrho|$ and $|\tau\omega \pm 2\sigma \pm \varrho|$ and those perpendicular to the field $|\tau\omega \pm 2\varrho|$, $|\tau\omega \pm 2\sigma|$, $|\tau\omega \pm 2\sigma \pm 2\varrho|$ and $|\tau\omega|$. We therefore expect new p-components corresponding to changes of k by 0 or ± 2 and of j by ± 1 and new s-components corresponding to a change of k by 0 or ± 2 and of j by 0 or ± 2.

Fig. 1 illustrates these effects diagramatically. The first plot shows the components into which a hydrogen "line" is broken up by the presence of a pure magnetic field. In this we have represented the whole group of possible fine structure components by a single line. The s-components are displaced from the position of the line in the absence of the field by $\pm \varrho$ and the p-component has the position of the original line. The second plot shows the components produced by a strong magnetic field and a weak electric field parallel to it. The solid lines indicate the presence of the components present in the absence of the electric field and the dotted lines superimposed on these indicate the presence of new components corresponding to $k'-k''$ equal to 0 or ± 2 with intensities proportional to F^2. The general appearance of the LORENTZ triplet would be the same as in the absence of the electric field. The third plot shows the components in the presence of a strong magnetic field with a weak electric field perpendicular to it. The solid and dotted lines have the same meaning as in the second plot. The p-components in this case consist of a triplet, one member having the position of the original line and the other displaced from this position by $\pm \varrho$. The s-components consist of five

[1] KRAMERS' Dissertation, Ch. IV and VI.

lines one having the position of the original line and the others displaced from this position by $\pm\varrho$ and $\pm 2\varrho$.

In figure 2 are shown three theoretical plots of the fine structure of the *s*-components of H_α (displaced by the amount ϱ from the original line) in a magnetic field first without a superimposed electric field, second with a superimposed electric field of 200 Volt/cm parallel to the magnetic field and finally with an electric field of 600 Volts/cm perpendicular to a magnetic field of 10,000 Gauss. These plots are labelled unperturbed, parallel and transverse respectively. For H_α we have $n' = 3$ and $n'' = 2$. The values of the k's for the initial and final states are written above the unperturbed components and these are indicated below by k'—k''. The values of the j's for the initial and final states are written at the ends of the components appearing in the presence of the parallel fields and transverse fields. These are indicated below by j'—j'' and $(\pm) j'$—j'' respectively. In the parenthesis beside these numbers in the latter case is placed a plus or a minus sign to indicate whether ϱ has a positive or negative sign in equation (4) for the calculation of the component. The components of the fine structure which do not appear in the presence of a pure magnetic field are represented by dotted lines and all »new« components originating from these so-called »forbidden« components are also represented by dotted lines to indi-

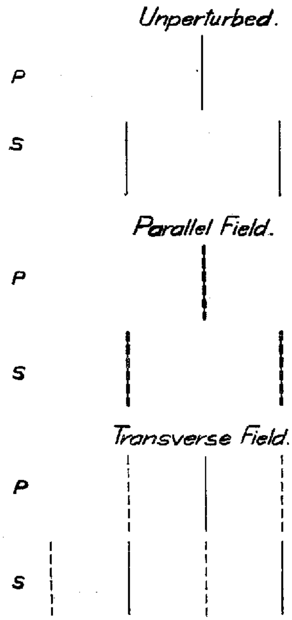


Fig. 1.

cate that their intensities are proportional to the square of the electric force while the intensities of the original components to a first approximation are independent of the field strength. The relative intensities of the components are roughly shown by the widths of the lines and are taken from KRAMERS' dissertation. In Fig. 3 are shown similarly the undisplaced p-components. The methods of representation is the same as for Fig. 2. For the calculations the formula[1]

$$E = E_0 - \frac{9}{4}\left(\frac{h}{2\pi}\right)^8 \frac{F^2 c^2}{N^6 e^{10} m^3} n^5 k \left(n^2 - 2k^2 + j^2\right)$$

has been used for the case of the parallel fields and formula (4) for the case of the transverse fields. For an electric field which makes an angle other than 0° or 90° with the magnetic field, the effects of its parallel and perpendicular components will be simply superimposed as mentioned above.

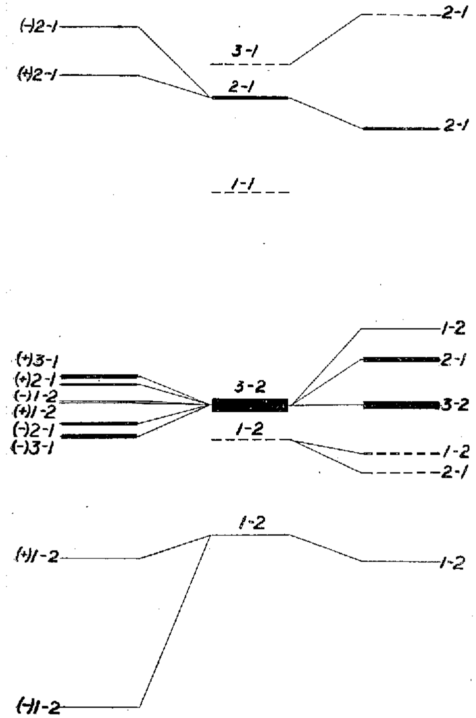
§ 4. Discussion of Experiments.

The experiments in the case of H_α are clearer than for any other line investigated. FÖRSTERLING and G. HANSEN state that the two components move together as the field is increased in a way that is approximately but not exactly symmetrical. The photograph which they reproduce, showing the effects of fields of 4000 and 10000 Gauss respectively on the p-component of H_α, indicates that the stronger component under a field of 4000 Gauss has very nearly the same position as without the field and that the weaker component has shifted toward the stronger component under the influence of the field. The components have quite merged, however, under 10000 Gauss and both components have moved from their original position. Moreover the intensity has become very much less indicating that some change

[1] KRAMERS, Zs. für Phys. 3, 214, 1920.

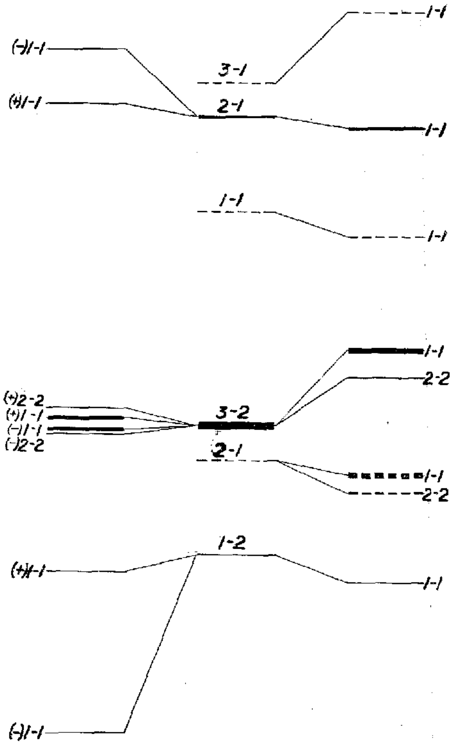
has taken place in the character of the excitation. These authors state that the effect on the s-component is quite similar to the effect on the p-component.

As is evident from the figure the effect of a transverse electric field would be very small compared with that due to a parallel field of the same intensity for magnetic fields of the order 10000 Gauss. Therefore if the effects of the change in the discharge (which originate from the presence of neutral or charged atoms or of electrons) can be actually found from comparison with the effects on the atoms of uniform fields with directions distributed at random the changes in the spectrum due to the transverse component may be neglected in comparison with the changes due to the parallel component. Apart from a random distribution of perturbing fields due to the presence of ions or electrons in the discharge, directed uniform fields may be expected for two reasons. On one hand we may expect an increase of the electric forces in the discharge parallel to



(±) $j'—j''$ $k'—k''$ $j'—j''$
Transverse ℌ Unperturbed Parallel
H_α 3—2 s-Components
Fig. 2.

the current. With the experimental arrangement used this means a perturbing electric force parallel to the magnetic force. On the other hand a transverse electric force may arise from the motion of the radiating atoms in the magnetic field. In fact, if these do not move strictly parallel to the magnetic force, the motion of the charged particles in the atom will on account of the LORENTZ force be affected in a similar way as if the atom was placed in a uniform electric field[1]. A simple calculation shows, however, that it would be neccessary for the atoms to be moving with a velocity component perpendicular to the magnetic field corresponding to the fall of a hydrogen ion through ten volts in order to be acted on by LORENTZ force of two electrostatic units and thus to produce the effect shown in the figures. It seems quite improbable that many atoms could be moving with transverse velocities this large. Moreover no p-components



(±) $j'—j''$ $k'—k''$ $j'—j''$
Transverse Unperturbed Parallel
H_α 3—2 p-Components
Fig. 3.

[1] Comp. W. Wien, Ber. d. Berlin. Akad. d Wiss. **48**, 70, 1914.

are observed in which j changes by ± 1 and no s-components in which j changes by 0 or ± 2, as would be expected in the presence of a relatively strong transverse field (comp. Fig. 1). Therefore we can definitely conclude that the observed effects are not due to transverse fields. The observed effect on the p-component at 4000 Gauss on the other hand might be ascribed to the effect of the components of the perturbing electric field parallel to the magnetic field and this same assumption might explain the effect on the s-components as well. But it seems impossible to explain the observed anomalous effect of larger magnetic fields satisfactorily in this way. If the observations are not essentially influenced by experimental errors due to the difficulties of separating lines which are so near each other, it appears then that the cause for the anomalies observed must be the irregular and varying fields present at encounters and that their effects are not similar to those of homogeneous fields. It may be noted that in the well known broadening of spectral lines due to pressure we meet with a similar problem. As pointed out by STARK[1] this broadening shows a general resemblance with the effect of homegeneous electric fields on these spectra. The attempts to account for the effect quantitatively[2] have proved, however, that in many cases the obervations can be explained only by assuming a particular effect of encounters essentially different from that of homogeneous fields[3].

Another effect which perhaps shows still greater similarity to our case has been observed by H. M. HANSEN, T. TAKAMINE and S. WERNER in their work on the effect of

[1] Compare J. STARK and H. KIRSCHBAUM, Ann. d. Phys. **43**, 1040, 1914.

[2] J. HOLTSMARK, Ann. d. Phys. **58**, 577, 1919.

[3] J. FRANCK, Festschrift der Kaiser Wilhelm Gesellschaft zu ihrem zehnjährigen Jubiläum, pp. 77—81, 1921.

magnetic and electric fields on the mercury spectrum[1]. Thus the line $1S - 2p_1$ (2270 Å) which does not appear in the spectrum of the undisturbed atom, could not be excited in a homogeneous electric field and appeared only as a weak line in the presence of a magnetic field parallel to the direction of the discharge. It was excited quite strongly, however, in a condensed discharge or in a transverse magnetic field. These authors conclude that the line would not be produced in appreciable intensities in homogeneous electric or magnetic fields as is also to be expected from the theory but that the magnetic field modified the discharge in such a way that the conditions were similar to those found in the condensed discharge where the effect of these varying and irregular fields at encounters is greatest. Although unfortunately the present state of the spectral theory does not permit us to describe in detail the effect of these varying fields, it seems quite probable that the observed effect of large magnetic fields on the fine structure of the hydrogen and helium lines is due to the modifying influence of the magnetic field on the discharge rather than to a direct effect of the magnetic field itself.

In conclusion the author wishes to express his best thanks and indebtedness to Dr. H. A. Kramers for his interest and advice during this work.

[1] D. Kgl. Danske Videnskabernes Selskab. Math.-fysiske Meddelelser V, 3, 1923.

Færdig fra Trykkeriet d. 19. Juni 1924.